EE-559 – Deep learning

10.1. Generative Adversarial Networks

François Fleuret
https://fleuret.org/ee559/
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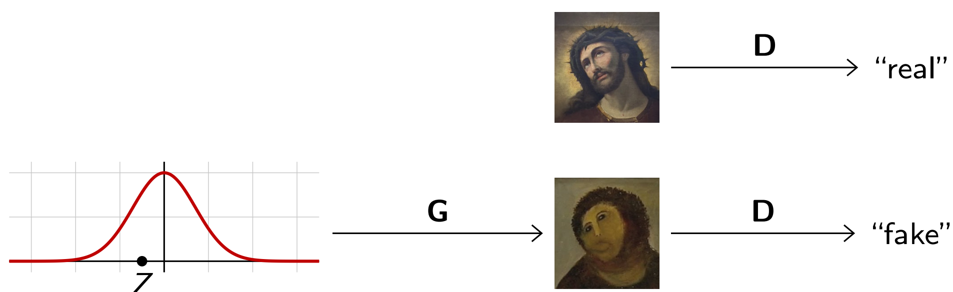
A different approach to learn high-dimension generative models are the **Generative Adversarial Networks** proposed by Goodfellow et al. (2014).

The idea behind GANs is to train two networks jointly:

- A **discriminator D** to classify samples as "real" or "fake",
- a **generator G** to map a [simple] fixed distribution to samples that fool **D**.



The approach is **adversarial** since the two networks have antagonistic objectives.

A bit more formally, let $\mathscr{X}$ be the signal space, and D the latent space dimension.

- The **generator**

$$\mathbf{G} : \mathbb{R}^D \to \mathscr{X}$$

is trained so that [ideally] if it gets a random normal-distributed Z as input, it produces a sample following the data distribution as output.

- The **discriminator**

$$\mathbf{D} : \mathscr{X} \to [0, 1]$$

is trained so that if it gets a sample as input, it predicts if it comes from from $\mathbf{G}$ or from the real data.

If $\mathbf{G}$ is fixed, to train $\mathbf{D}$ given a set of "real points"

$$x_n \sim \mu, \;\; n = 1, \dots, N,$$

we can generate

$$z_n \sim \mathscr{N}(0, I), \;\; n = 1, \dots, N,$$

build a two-class data-set

$$\mathscr{D} = \Big\{ \underbrace{(x_1, 1), \dots, (x_N, 1)}_{\text{real samples} \sim \mu}, \underbrace{(\mathbf{G}(z_1), 0), \dots, (\mathbf{G}(z_N), 0)}_{\text{fake samples} \sim \mu_{\mathbf{G}}} \Big\},$$

and minimize the binary cross-entropy

$$\begin{aligned} \mathscr{L}(\mathbf{D}) &= -\frac{1}{2N} \left(\sum_{n=1}^{N} \log \mathbf{D}(x_n) + \sum_{n=1}^{N} \log(1 - \mathbf{D}(\mathbf{G}(z_n))) \right) \\ &= -\frac{1}{2} \left(\hat{\mathbb{E}}_{X \sim \mu} \Big[\log \mathbf{D}(X) \Big] + \hat{\mathbb{E}}_{X \sim \mu_{\mathbf{G}}} \Big[\log(1 - \mathbf{D}(X)) \Big] \right), \end{aligned}$$

where μ is the true distribution of the data, and $\mu_{\mathbf{G}}$ is the distribution of $\mathbf{G}(Z)$ with $Z \sim \mathscr{N}(0, I)$.

The situation is slightly more complicated since we also want to optimize **G** to *maximize* **D**'s loss.

Goodfellow et al. (2014) provide an analysis of the resulting equilibrium of that strategy.

Let's define

$$\mathscr{L}_{\mathbf{G}}(\mathbf{D},\mathbf{G}) = \mathbb{E}_{X\sim\mu}\Big[\log \mathbf{D}(X)\Big] + \mathbb{E}_{X\sim\mu_{\mathbf{G}}}\Big[\log(1-\mathbf{D}(X))\Big]$$

which is high if **D** is doing a good job (low cross entropy), and low if **G** fools **D**.

Our ultimate goal is a $\mathbf{G}^*$ that fools *any* **D**, so

$$\mathbf{G}^* = \underset{\mathbf{G}}{\operatorname{argmin}} \max_{\mathbf{D}} \mathscr{L}_{\mathbf{G}}(\mathbf{D},\mathbf{G}).$$

If we define the optimal discriminator for a given generator

$$\mathbf{D}^*_{\mathbf{G}} = \underset{\mathbf{D}}{\operatorname{argmax}} \mathscr{L}_{\mathbf{G}}(\mathbf{D}, \mathbf{G}),$$

our objective becomes

$$\mathbf{G}^* = \underset{\mathbf{G}}{\operatorname{argmin}} \mathscr{L}_{\mathbf{G}}(\mathbf{D}^*_{\mathbf{G}}, \mathbf{G}),$$

that is:

Find a G whose loss against the best D is low.

We have

$$\begin{aligned}\mathscr{L}_{\mathbf{G}}(\mathbf{D}, \mathbf{G}) &= \mathbb{E}_{X \sim \mu}\Big[\log \mathbf{D}(X)\Big] + \mathbb{E}_{X \sim \mu_{\mathbf{G}}}\Big[\log(1 - \mathbf{D}(X))\Big] \\ &= \int_x \mu(x) \log \mathbf{D}(x) + \mu_{\mathbf{G}}(x) \log(1 - \mathbf{D}(x)) dx.\end{aligned}$$

Since

$$\underset{d}{\operatorname{argmax}} \;\; \mu(x) \log d + \mu_{\mathbf{G}}(x) \log(1 - d) \quad = \quad \frac{\mu(x)}{\mu(x) + \mu_{\mathbf{G}}(x)},$$

and

$$\mathbf{D}^*_{\mathbf{G}} = \underset{\mathbf{D}}{\operatorname{argmax}} \mathscr{L}_{\mathbf{G}}(\mathbf{D}, \mathbf{G}),$$

if there is no regularization on $\mathbf{D}$, we get

$$\forall x, \;\; \mathbf{D}^*_{\mathbf{G}}(x) = \frac{\mu(x)}{\mu(x) + \mu_{\mathbf{G}}(x)}.$$

So, since

$$\forall x,\ \mathbf{D}^*_{\mathbf{G}}(x) = \frac{\mu(x)}{\mu(x) + \mu_{\mathbf{G}}(x)}.$$

we get

$$\begin{aligned}
\mathscr{L}_{\mathbf{G}}(\mathbf{D}^*_{\mathbf{G}}, \mathbf{G}) &= \mathbb{E}_{X\sim\mu}\Big[\log \mathbf{D}^*_{\mathbf{G}}(X)\Big] + \mathbb{E}_{X\sim\mu_{\mathbf{G}}}\Big[\log(1 - \mathbf{D}^*_{\mathbf{G}}(X))\Big] \\
&= \mathbb{E}_{X\sim\mu}\left[\log \frac{\mu(X)}{\mu(X) + \mu_{\mathbf{G}}(X)}\right] + \mathbb{E}_{X\sim\mu_{\mathbf{G}}}\left[\log \frac{\mu_{\mathbf{G}}(X)}{\mu(X) + \mu_{\mathbf{G}}(X)}\right] \\
&= \mathbb{D}_{KL}\left(\mu \,\middle\|\, \frac{\mu + \mu_{\mathbf{G}}}{2}\right) + \mathbb{D}_{KL}\left(\mu_{\mathbf{G}} \,\middle\|\, \frac{\mu + \mu_{\mathbf{G}}}{2}\right) - \log 4 \\
&= 2\,\mathbb{D}_{JS}\left(\mu, \mu_{\mathbf{G}}\right) - \log 4
\end{aligned}$$

where $\mathbb{D}_{JS}$ is the Jensen-Shannon Divergence, a standard dissimilarity measure between distributions.

To recap: if there is no capacity limitation for **D**, and if we define

$$\mathscr{L}_{\mathbf{G}}(\mathbf{D}, \mathbf{G}) = \mathbb{E}_{X\sim\mu}\Big[\log \mathbf{D}(X)\Big] + \mathbb{E}_{X\sim\mu_{\mathbf{G}}}\Big[\log(1 - \mathbf{D}(X))\Big],$$

computing

$$\mathbf{G}^* = \underset{\mathbf{G}}{\operatorname{argmin}} \max_{\mathbf{D}} \mathscr{L}_{\mathbf{G}}(\mathbf{D}, \mathbf{G})$$

amounts to compute

$$\mathbf{G}^* = \underset{\mathbf{G}}{\operatorname{argmin}}\, \mathbb{D}_{JS}(\mu, \mu_{\mathbf{G}}),$$

where $\mathbb{D}_{JS}$ is a reasonable dissimilarity measure between distributions.

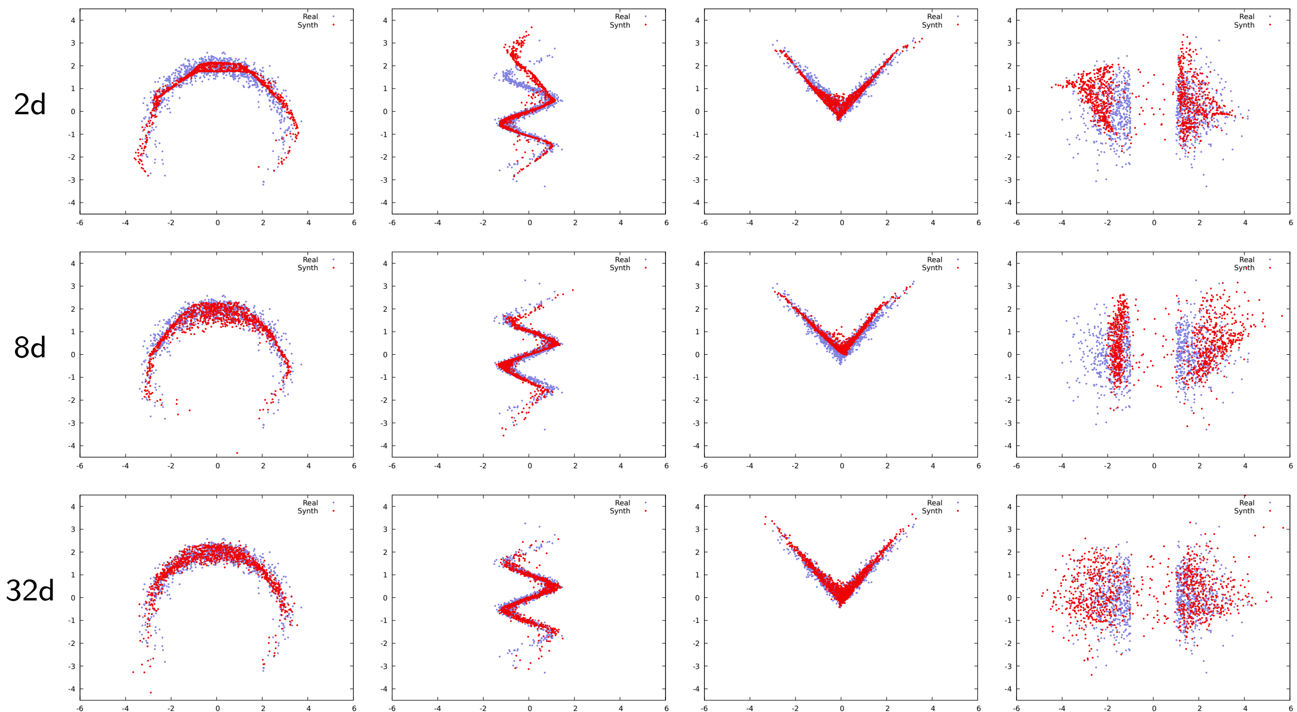
Although this derivation provides a nice formal framework, in practice **D** is not “fully” optimized to [come close to] $\mathbf{D}^*_{\mathbf{G}}$ when optimizing **G**.

In our minimal example, we alternate gradient steps to improve **G** and **D**.

```
z_dim, nb_hidden = 8, 100

model_G = nn.Sequential(nn.Linear(z_dim, nb_hidden),
                        nn.ReLU(),
                        nn.Linear(nb_hidden, 2))

model_D = nn.Sequential(nn.Linear(2, nb_hidden),
                        nn.ReLU(),
                        nn.Linear(nb_hidden, 1),
                        nn.Sigmoid())
```



```
batch_size, lr = 10, 1e-3

optimizer_G = optim.Adam(model_G.parameters(), lr = lr)
optimizer_D = optim.Adam(model_D.parameters(), lr = lr)

for e in range(nb_epochs):

    for t, real_batch in enumerate(real_samples.split(batch_size)):
        z = real_batch.new(real_batch.size(0), z_dim).normal_()
        fake_batch = model_G(z)

        D_scores_on_real = model_D(real_batch)
        D_scores_on_fake = model_D(fake_batch)

        if t%2 == 0:
            loss = (1 - D_scores_on_fake).log().mean()
            optimizer_G.zero_grad()
            loss.backward()
            optimizer_G.step()
        else:
            loss = - (1 - D_scores_on_fake).log().mean() \
                   - D_scores_on_real.log().mean()
            optimizer_D.zero_grad()
            loss.backward()
            optimizer_D.step()
```



In more realistic settings, the fake samples may be initially so "unrealistic" that the response of **D** saturates. That causes the loss for **G**

$$\hat{\mathbb{E}}_{X \sim \mu_{\mathbf{G}}} \Big[\log(1 - \mathbf{D}(X)) \Big]$$

to be far in the exponential tail of **D**'s sigmoid, and have zero gradient since $\log(1 + \epsilon) \simeq \epsilon$ does not correct it in any way.

Goodfellow et al. suggest to replace this term with a **non-saturating** cost

$$-\hat{\mathbb{E}}_{X \sim \mu_{\mathbf{G}}} \Big[\log(\mathbf{D}(X)) \Big]$$

so that the log fixes **D**'s exponential behavior. The resulting optimization problem has the same optima as the original one.

The loss for **D** remains unchanged.

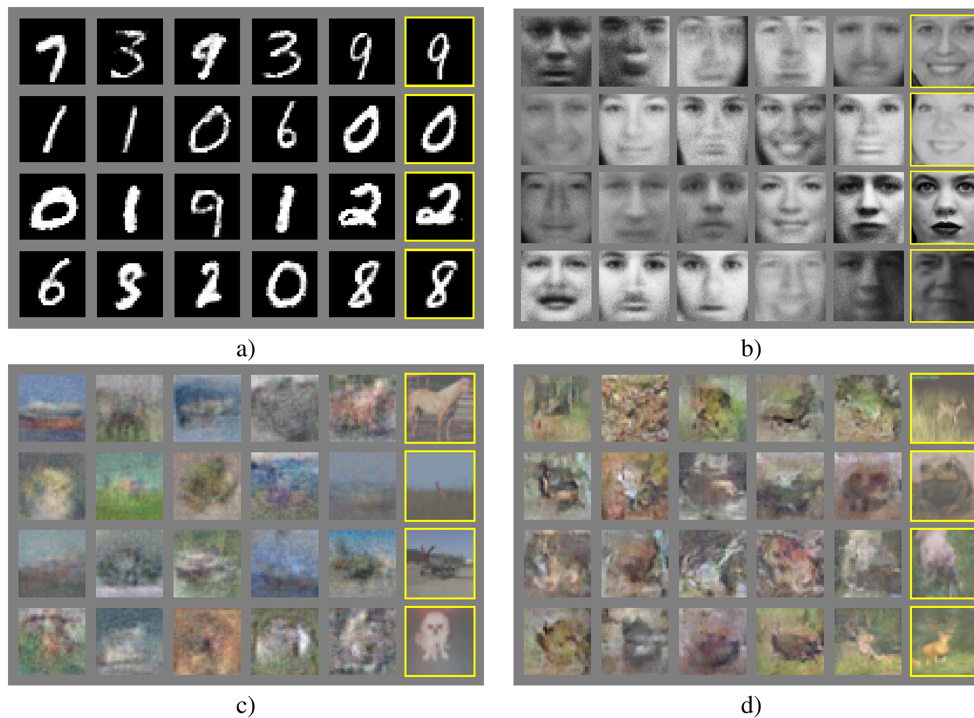


Figure 2: Visualization of samples from the model. Rightmost column shows the nearest training example of the neighboring sample, in order to demonstrate that the model has not memorized the training set. Samples are fair random draws, not cherry-picked. Unlike most other visualizations of deep generative models, these images show actual samples from the model distributions, not conditional means given samples of hidden units. Moreover, these samples are uncorrelated because the sampling process does not depend on Markov chain mixing. a) MNIST b) TFD c) CIFAR-10 (fully connected model) d) CIFAR-10 (convolutional discriminator and "deconvolutional" generator)

(Goodfellow et al., 2014)

Deep Convolutional GAN

"We also encountered difficulties attempting to scale GANs using CNN architectures commonly used in the supervised literature. However, after extensive model exploration we identified a family of architectures that resulted in stable training across a range of datasets and allowed for training higher resolution and deeper generative models."

(Radford et al., 2015)

Radford et al. converged to the following rules:

- Replace pooling layers with strided convolutions in **D** and strided transposed convolutions in **G**,
- use batchnorm in both **D** and **G**,
- remove fully connected hidden layers,
- use ReLU in **G** except for the output, which uses Tanh,
- use LeakyReLU activation in **D** for all layers.

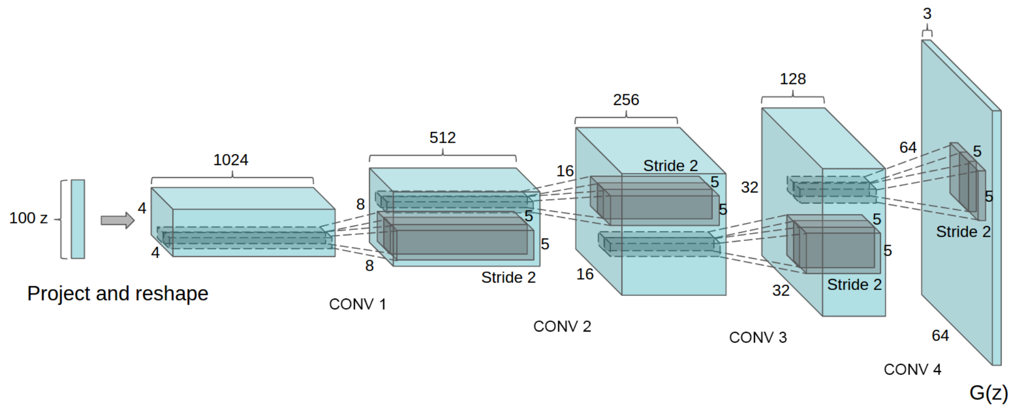


Figure 1: DCGAN generator used for LSUN scene modeling. A 100 dimensional uniform distribution Z is projected to a small spatial extent convolutional representation with many feature maps. A series of four fractionally-strided convolutions (in some recent papers, these are wrongly called deconvolutions) then convert this high level representation into a 64×64 pixel image. Notably, no fully connected or pooling layers are used.

(Radford et al., 2015)

We can have a look at the reference implementation provided in

`https://github.com/pytorch/examples.git`


```
# default nz = 100, ngf = 64

class Generator(nn.Module):
    def __init__(self, ngpu):
        super(Generator, self).__init__()
        self.ngpu = ngpu
        self.main = nn.Sequential(
            # input is Z, going into a convolution
            nn.ConvTranspose2d(     nz, ngf * 8, 4, 1, 0, bias=False),
            nn.BatchNorm2d(ngf * 8),
            nn.ReLU(True),
            # state size. (ngf*8) x 4 x 4
            nn.ConvTranspose2d(ngf * 8, ngf * 4, 4, 2, 1, bias=False),
            nn.BatchNorm2d(ngf * 4),
            nn.ReLU(True),
            # state size. (ngf*4) x 8 x 8
            nn.ConvTranspose2d(ngf * 4, ngf * 2, 4, 2, 1, bias=False),
            nn.BatchNorm2d(ngf * 2),
            nn.ReLU(True),
            # state size. (ngf*2) x 16 x 16
            nn.ConvTranspose2d(ngf * 2,     ngf, 4, 2, 1, bias=False),
            nn.BatchNorm2d(ngf),
            nn.ReLU(True),
            # state size. (ngf) x 32 x 32
            nn.ConvTranspose2d(    ngf,      nc, 4, 2, 1, bias=False),
            nn.Tanh()
            # state size. (nc) x 64 x 64
        )
```

```
# default nz = 100, ndf = 64

class Discriminator(nn.Module):
    def __init__(self, ngpu):
        super(Discriminator, self).__init__()
        self.ngpu = ngpu
        self.main = nn.Sequential(
            # input is (nc) x 64 x 64
            nn.Conv2d(nc, ndf, 4, 2, 1, bias=False),
            nn.LeakyReLU(0.2, inplace=True),
            # state size. (ndf) x 32 x 32
            nn.Conv2d(ndf, ndf * 2, 4, 2, 1, bias=False),
            nn.BatchNorm2d(ndf * 2),
            nn.LeakyReLU(0.2, inplace=True),
            # state size. (ndf*2) x 16 x 16
            nn.Conv2d(ndf * 2, ndf * 4, 4, 2, 1, bias=False),
            nn.BatchNorm2d(ndf * 4),
            nn.LeakyReLU(0.2, inplace=True),
            # state size. (ndf*4) x 8 x 8
            nn.Conv2d(ndf * 4, ndf * 8, 4, 2, 1, bias=False),
            nn.BatchNorm2d(ndf * 8),
            nn.LeakyReLU(0.2, inplace=True),
            # state size. (ndf*8) x 4 x 4
            nn.Conv2d(ndf * 8, 1, 4, 1, 0, bias=False),
            nn.Sigmoid()
        )
```



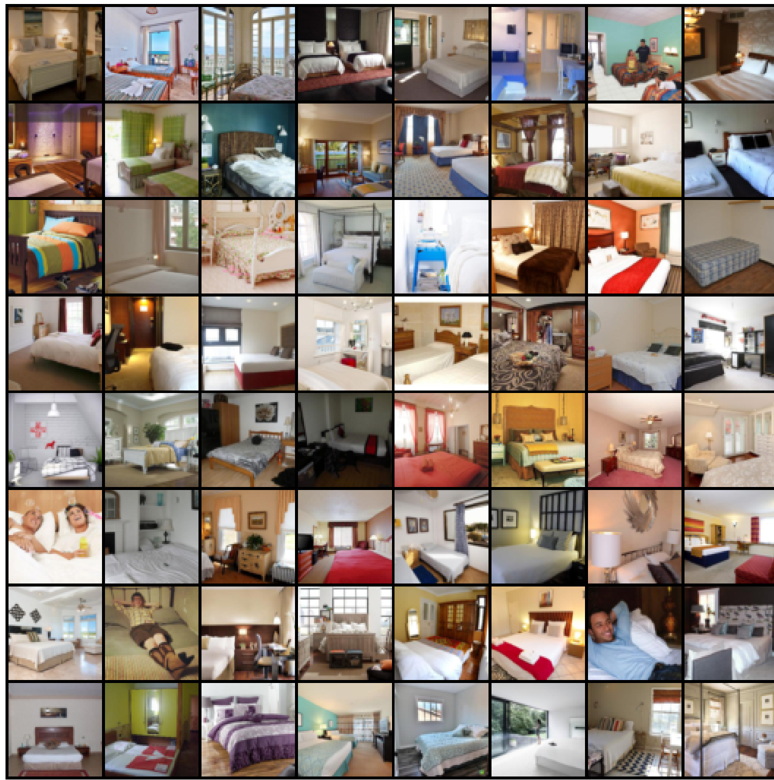
```
# custom weights initialization called on netG and netD
def weights_init(m):
    classname = m.__class__.__name__
    if classname.find('Conv') != -1:
        m.weight.data.normal_(0.0, 0.02)
    elif classname.find('BatchNorm') != -1:
        m.weight.data.normal_(1.0, 0.02)
        m.bias.data.fill_(0)

criterion = nn.BCELoss()

fixed_noise = torch.randn(opt.batchSize, nz, 1, 1, device=device)
real_label = 1
fake_label = 0

# setup optimizer
optimizerD = optim.Adam(netD.parameters(), lr=opt.lr, betas=(opt.beta1, 0.999))
optimizerG = optim.Adam(netG.parameters(), lr=opt.lr, betas=(opt.beta1, 0.999))
```

```
############################
# (1) Update D network: maximize log(D(x)) + log(1 - D(G(z)))
###########################
# train with real
netD.zero_grad()
real_cpu = data[0].to(device)
batch_size = real_cpu.size(0)
label = torch.full((batch_size,), real_label, device=device)

output = netD(real_cpu)
errD_real = criterion(output, label)
errD_real.backward()
D_x = output.mean().item()

# train with fake
noise = torch.randn(batch_size, nz, 1, 1, device=device)
fake = netG(noise)
label.fill_(fake_label)
output = netD(fake.detach())
errD_fake = criterion(output, label)
errD_fake.backward()
D_G_z1 = output.mean().item()
errD = errD_real + errD_fake
optimizerD.step()
```

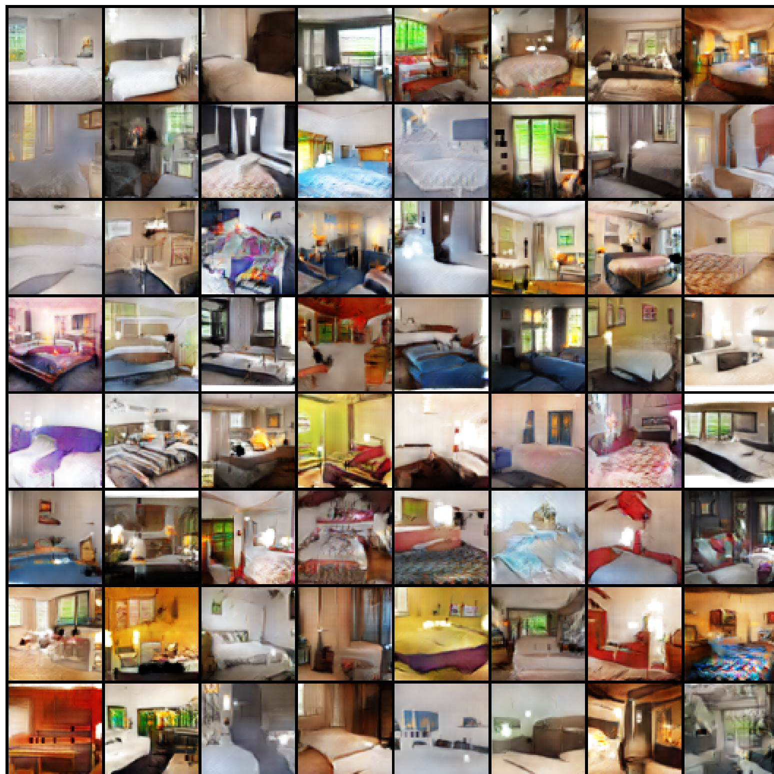


```
############################
# (2) Update G network: maximize log(D(G(z)))
###########################
netG.zero_grad()
label.fill_(real_label)  # fake labels are real for generator cost
output = netD(fake)
errG = criterion(output, label)
errG.backward()
D_G_z2 = output.mean().item()
optimizerG.step()
```

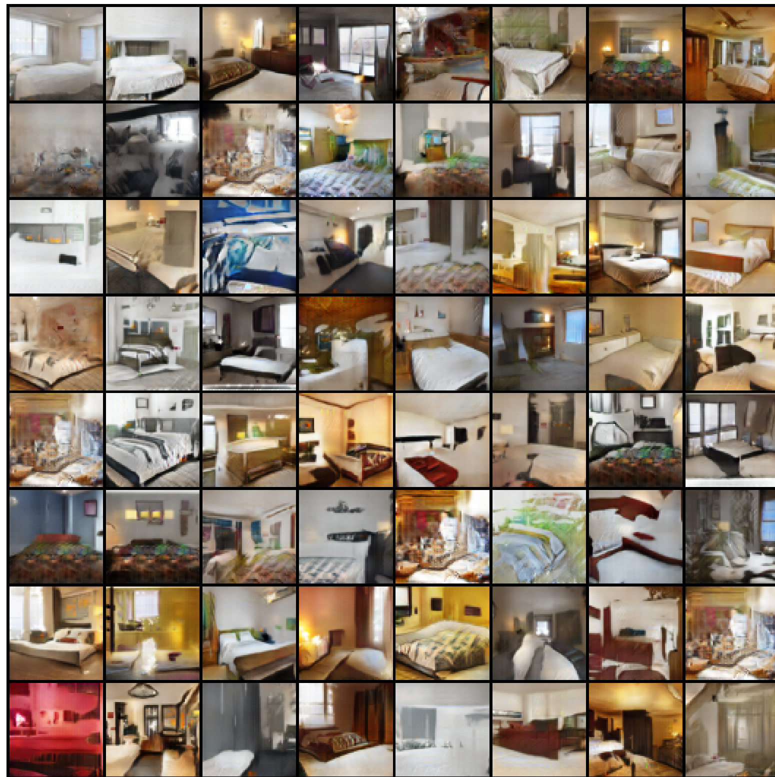
Note that this update implements the $-\log(D(G(z)))$ trick.



Real images from LSUN's "bedroom" class.



Fake images after 1 epoch (3M images)



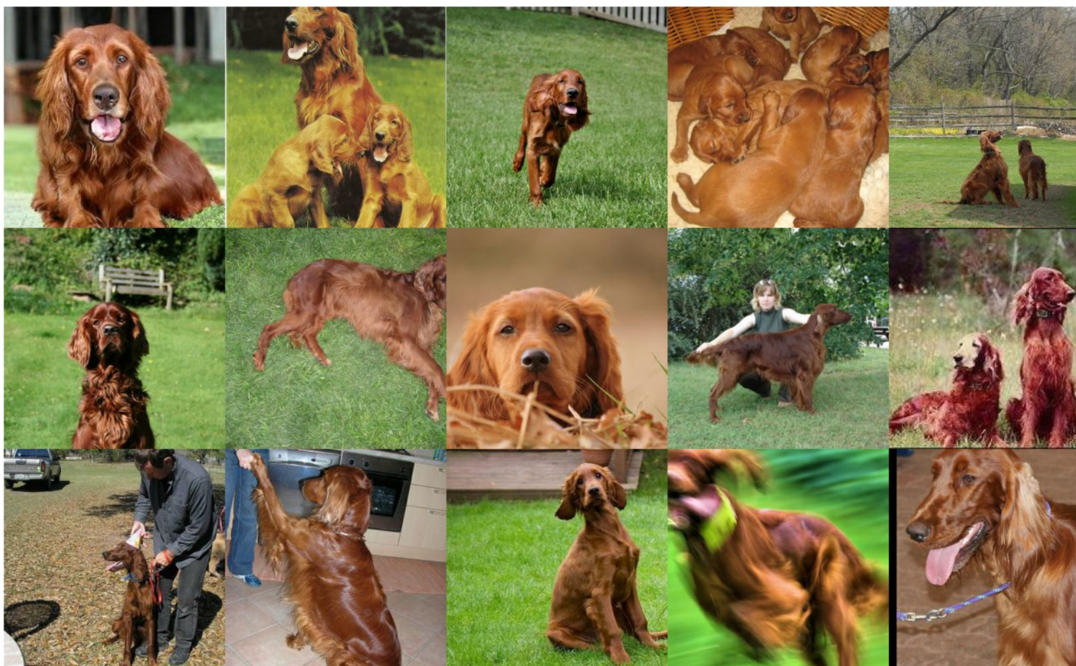
Fake images after 20 epochs

Training a standard GAN often results in two pathological behaviors:

- Oscillations without convergence. Contrary to standard loss minimization, we have no guarantee here that it will actually decrease.
- The infamous "mode collapse", when **G** models very well a small sub-population, concentrating on a few modes.

Additionally, performance is hard to assess and is often a "beauty contest".

(Brock et al., 2018)



(Brock et al., 2018)

(Karras et al., 2018)

References

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