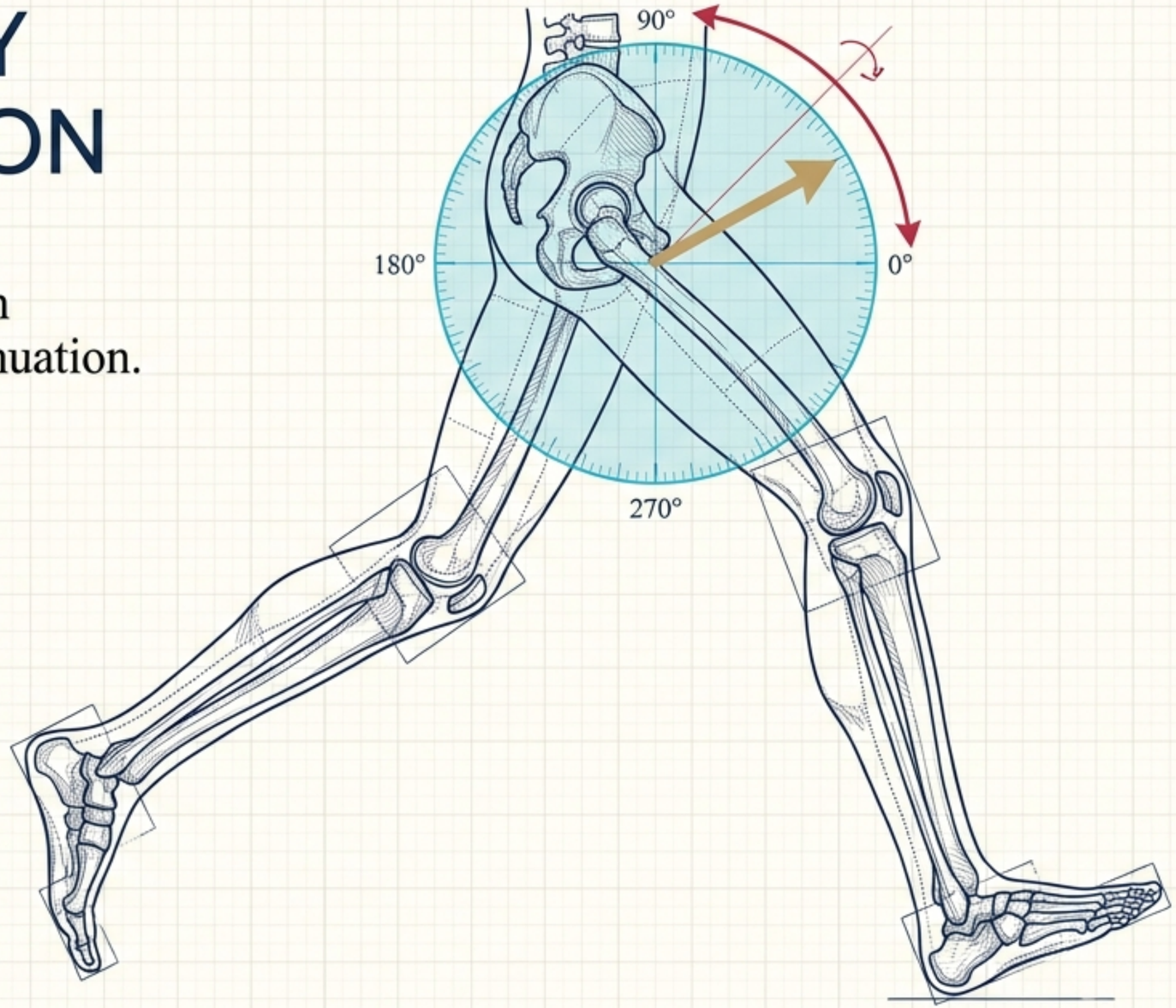
THE GEOMETRY OF LOCOMOTION

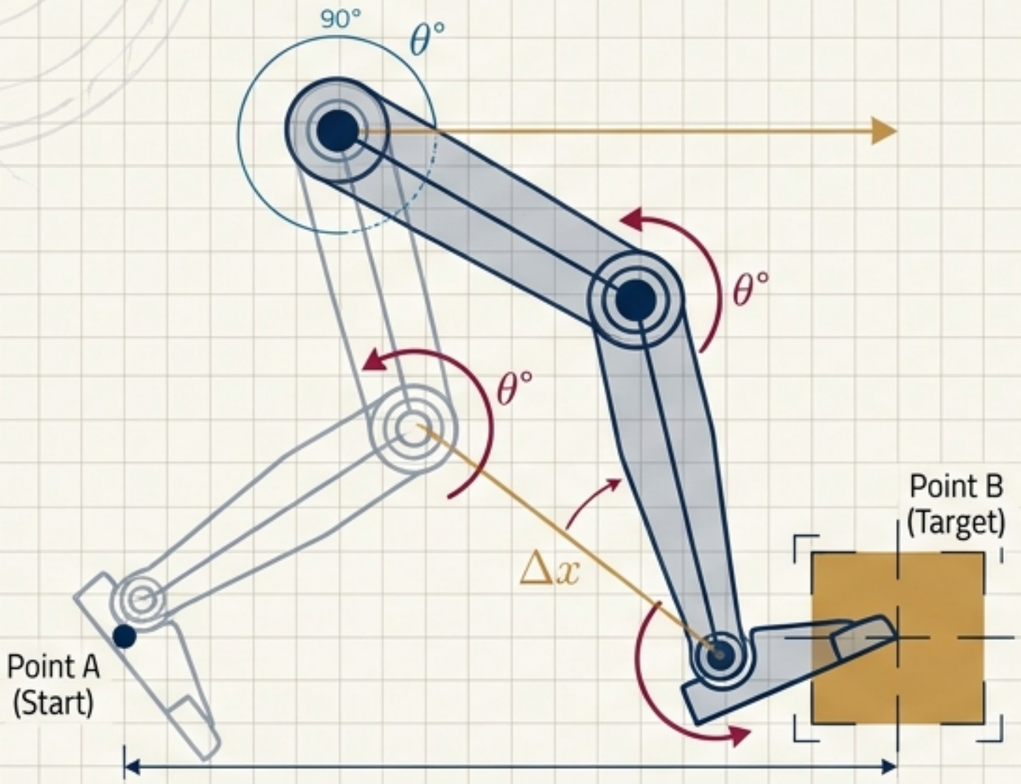
A falsifiable framework for central-pattern-generator-driven movement as hierarchical continuation.



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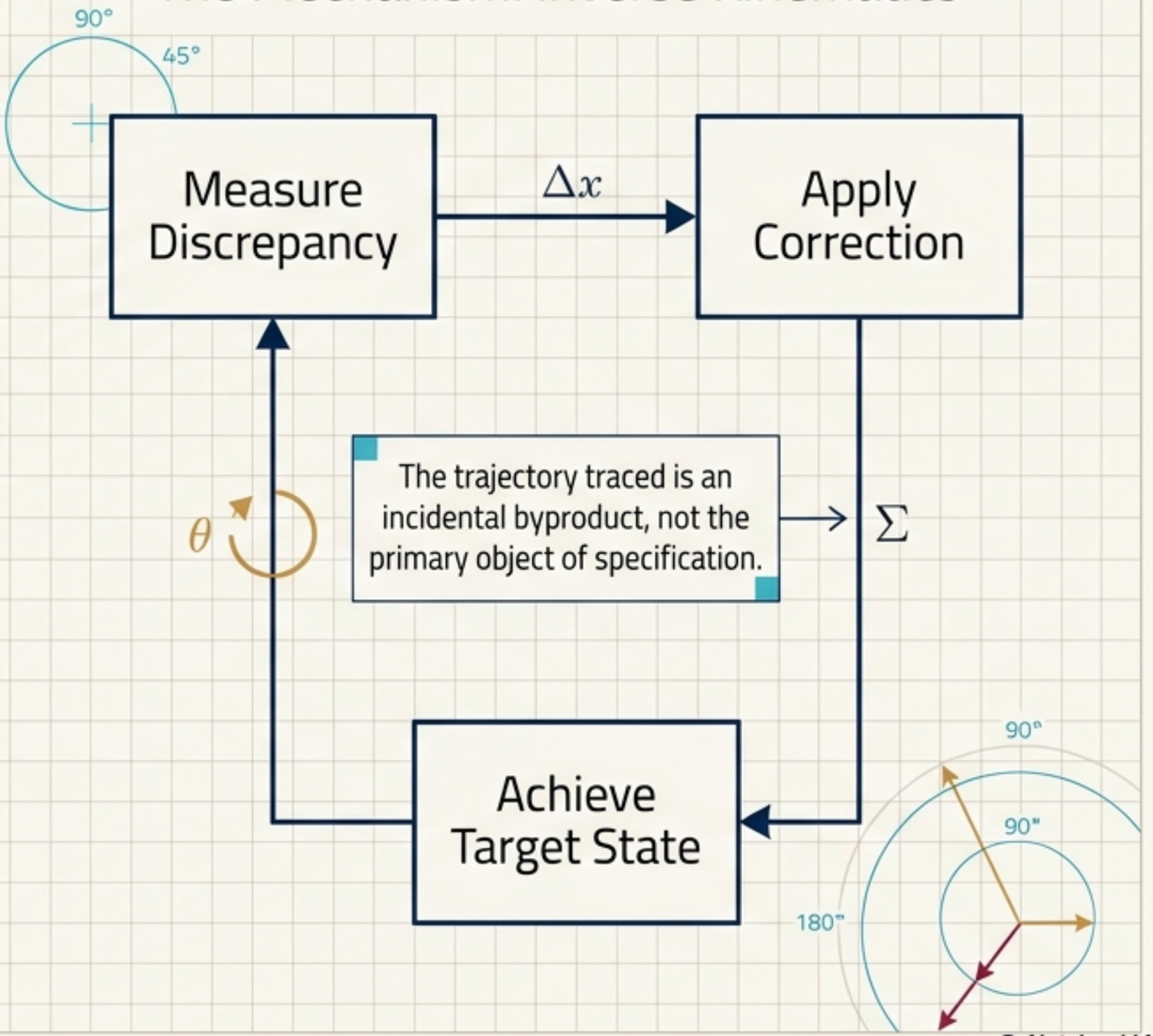
The Pose is Primary, the Path is a Byproduct

The Default Controller



State-centric control views balance as an achieved condition. Locomotion is the repeated application of error correction.

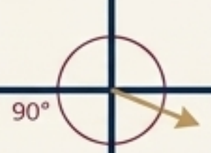
The Mechanism: Inverse Kinematics



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Histories Generate States, Not the Reverse

Diagnostic Comparison Matrix







	State-Centric Default	Trajectory-Centric Alternative
Primary Object	Pose (Targets)	Path (Histories)
Definition of Balance	Achieved condition	Regenerated coherence
Mathematical Engine	Inverse Kinematics	Coupled Oscillators
Biological Analogue	Reflex Chains (Sherrington)	Central Pattern Generators (Graham Brown)

In biology and CPG-robotics, rhythm is intrinsic to the network. The trajectory is what the system does by virtue of its coupling structure; joint angles are simply read off this ongoing activity.

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The Mathematics of Spontaneous Synchronization

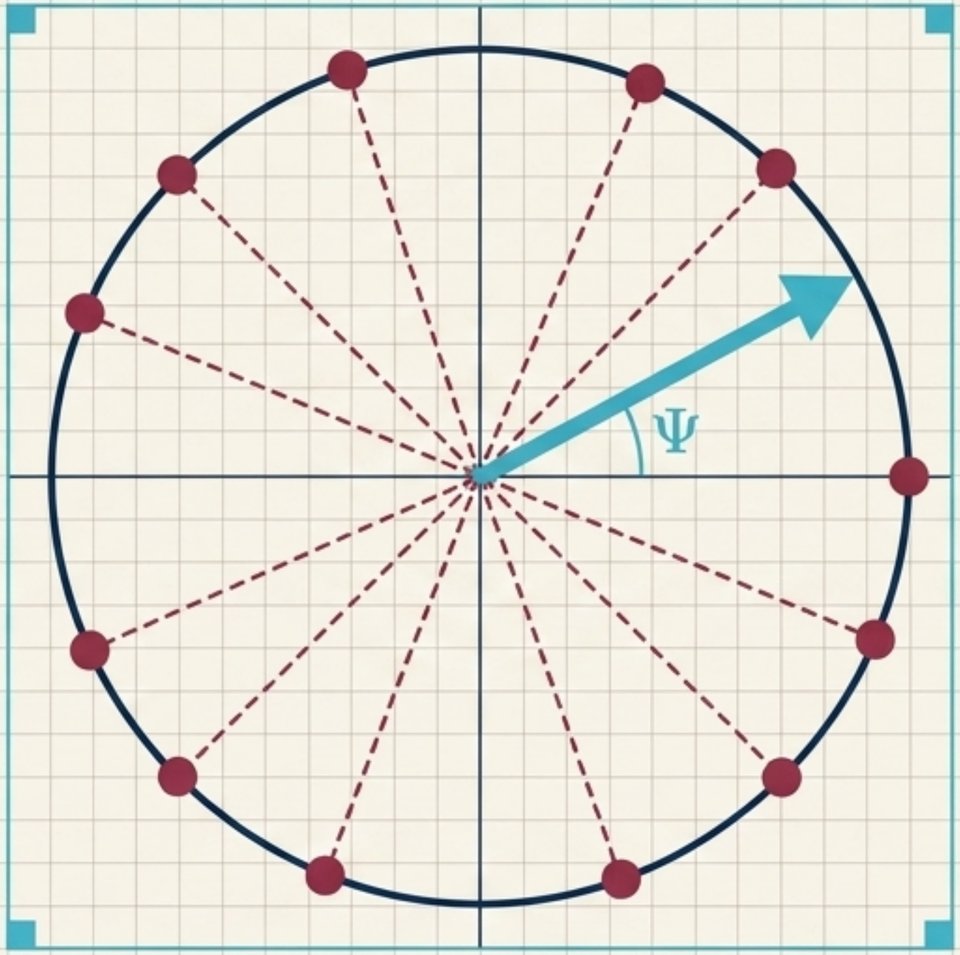
Kuramoto Model Equation

$$\frac{\mathrm{d}\theta_i}{\mathrm{d}t} = \omega_i + \sum K_{ij} \sin(\theta_j - \theta_i - \Delta\varphi_{ij})$$

- ω_i: Natural frequency
- K_{ij}: Coupling strength
- $\Delta\varphi_{ij}$: Target phase offset

A CPG does not specify a joint-angle trajectory. It specifies a coupling structure among phase oscillators.

Coupled Phase Field



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The Order Parameter is an Amplitwist Field

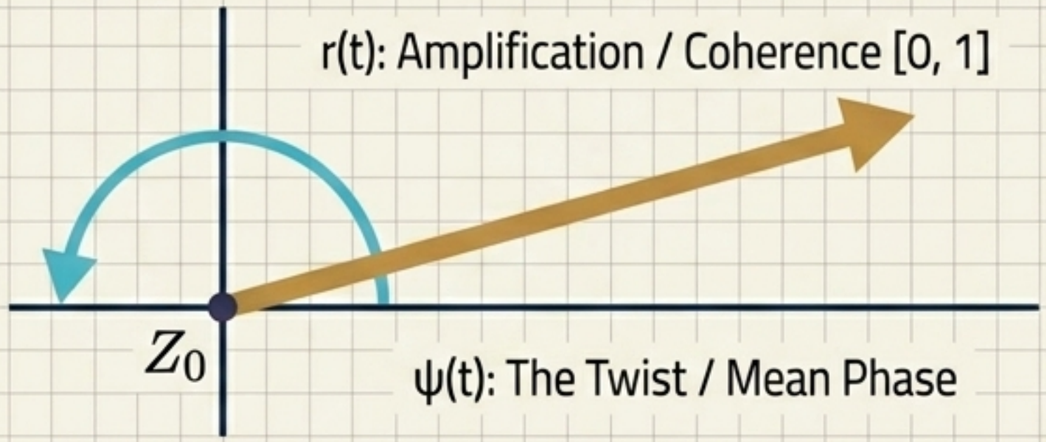
"This is not an analogy. The complex order parameter is, without modification, an amplitwist operator."

Oscillator Math

$$Z(t) = \underbrace{r(t)}_{\text{Mean field of N oscillators}} \cdot e^{i\psi(t)}$$

Needham's Geometry

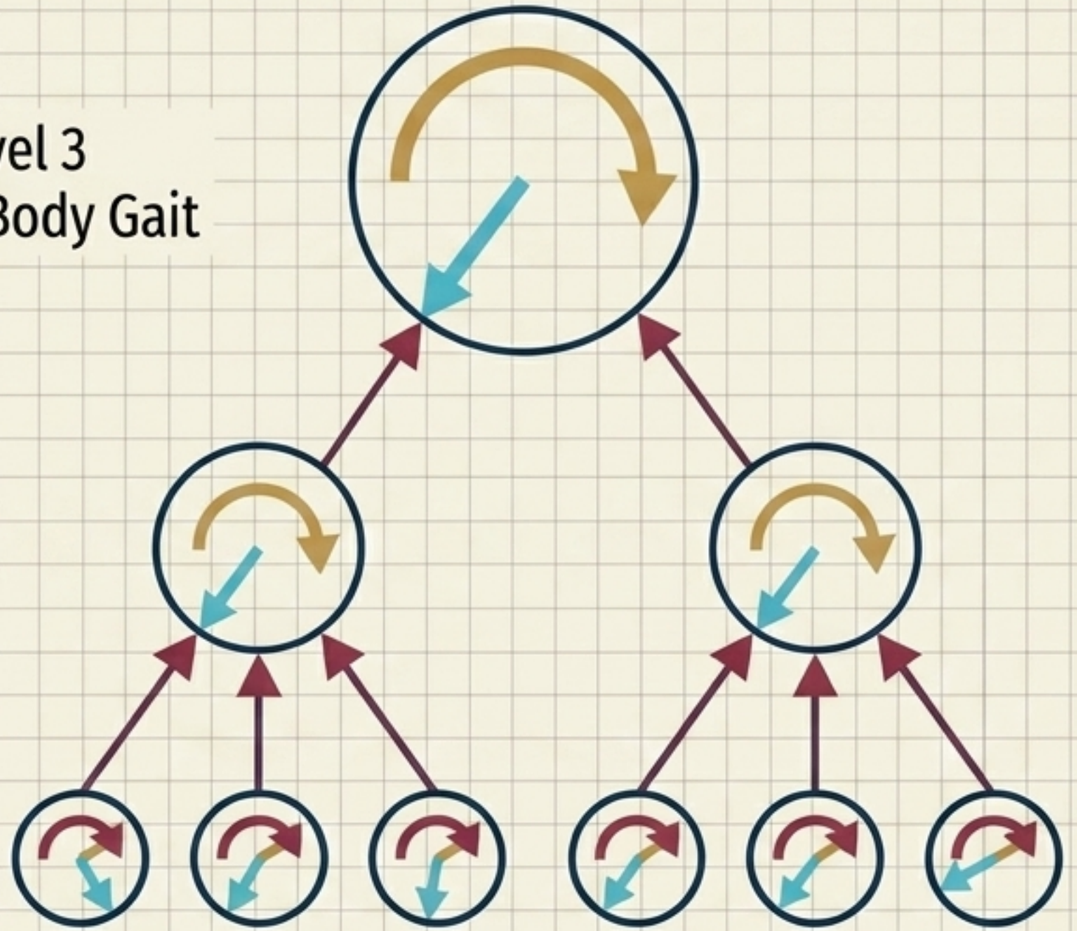
Amplitwist Operator Diagram



An instantaneous generator of a trajectory—scaling infinitesimal displacements by r and rotating them by ψ.

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Hierarchical Composition of Amplitwist Chains



Coupling Functional:

$$A^{k+1}(t) = C_K\left(A_1^k, \ldots, A_n^k\right)$$

The **Assembly Index:** $A = n - 1$

A bipedal walker mapping joints directly to a whole-body gait has $A = 1$.

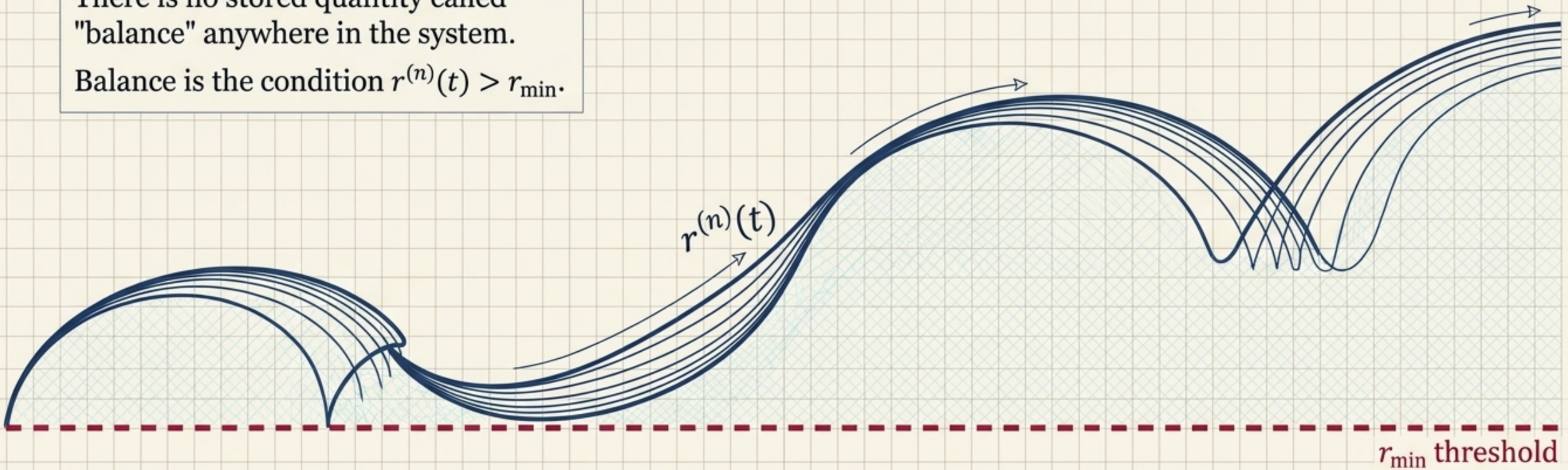
Walk vs. Trot are not different laws. They are different nominal parameters (K and phase offsets) passed to the identical underlying coupling functional.

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Balance is Continuously Regenerated Coherence

There is no stored quantity called "balance" anywhere in the system.

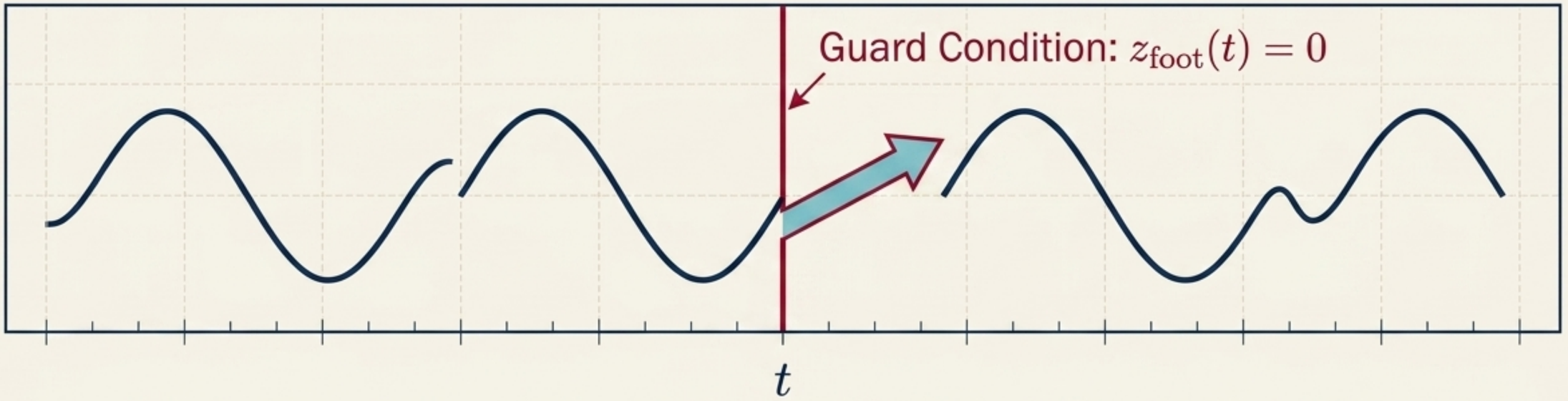
Balance is the condition $r^{(n)}(t) > r_{\min}$.



Balance is not a target state re-attained; it is an unbroken continuation, continuously recomputed from the phase relationships of the level below, evaluated *now*.

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Hybrid Systems and the Limits of Smoothness



The Problem

Kuramoto coupling cannot natively represent instantaneous mechanical changes like foot-strikes.

Conjecture 1

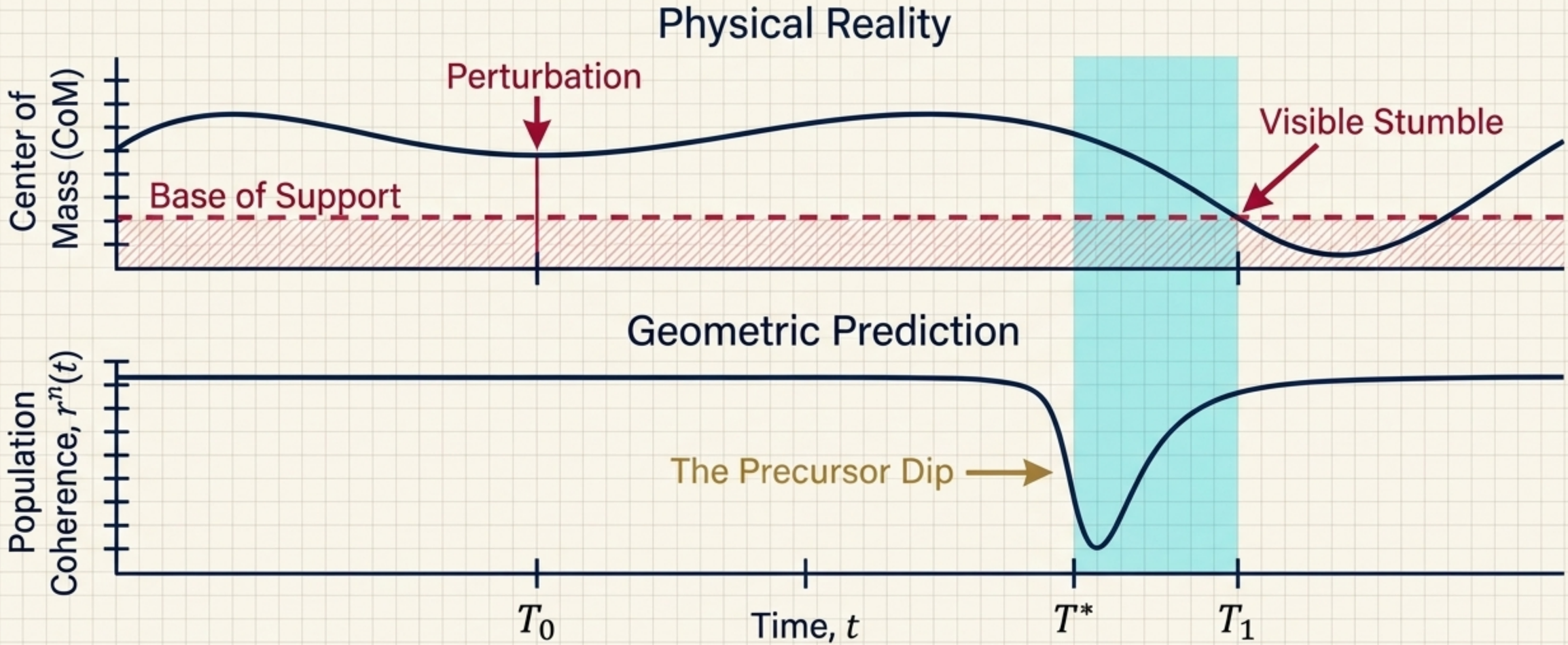
The reset map acts as an instantaneous discrete amplitwist operator applied to the pre-impact order parameter:

$$Z(t_+^*) = A_e \cdot Z(t_-^*)$$

Valid only for planar, single-point contact gaits.

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The Decisive Signature of Balance Recovery



If hierarchical continuation is true, a collapse in population coherence $r^n(t)$ strictly precedes any externally visible loss of balance.

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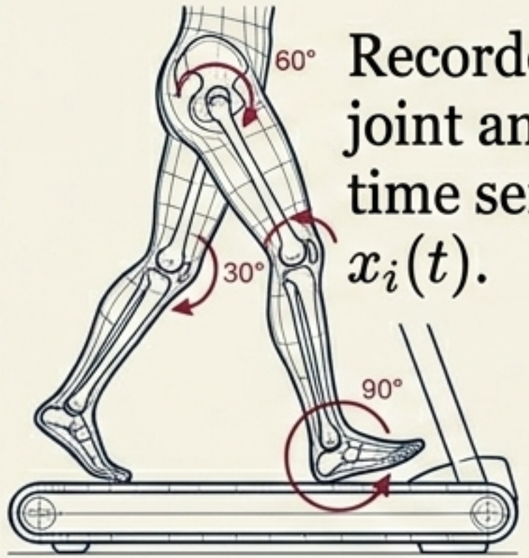
The Falsification Crucible

The Test	Passes If	Fails If
1. Compactness Criterion	Chain parameters $\leq$ joint-angle description for steady gaits.	Chain requires more parameters or deeper hierarchy than direct kinematics.
2. Ground Contact	Reset map at foot-strike is expressible as a single complex multiplication on Z.	Physically correct reset requires structure outside the amplitwist language.
3. Balance Recovery	Coherence drops before visible imbalance and monotonically re-locks.	No coherence precursor is observed; recovery leaves no trace in $r^{(n)}(t)$.

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Operationalizing the Test Tomorrow

Raw Data



Recorded joint angular time series $x_i(t)$.

→ **Hilbert Transform** →

Analytic Signal

$$x_i(t) + i\hat{x}_i(t) = a_i(t)e^{i\hat{\theta}_i(t)}$$

→

Empirical Estimator

$$r_n^{(n)}(t) = \left| \frac{1}{N} \sum e^{i\hat{\theta}_i(t)} \right|$$

No new data collection is required. Applying the Hilbert transform to existing treadmill/slip-perturbation datasets yields an empirical coherence estimator, allowing immediate testing of the balance-recovery conjecture.

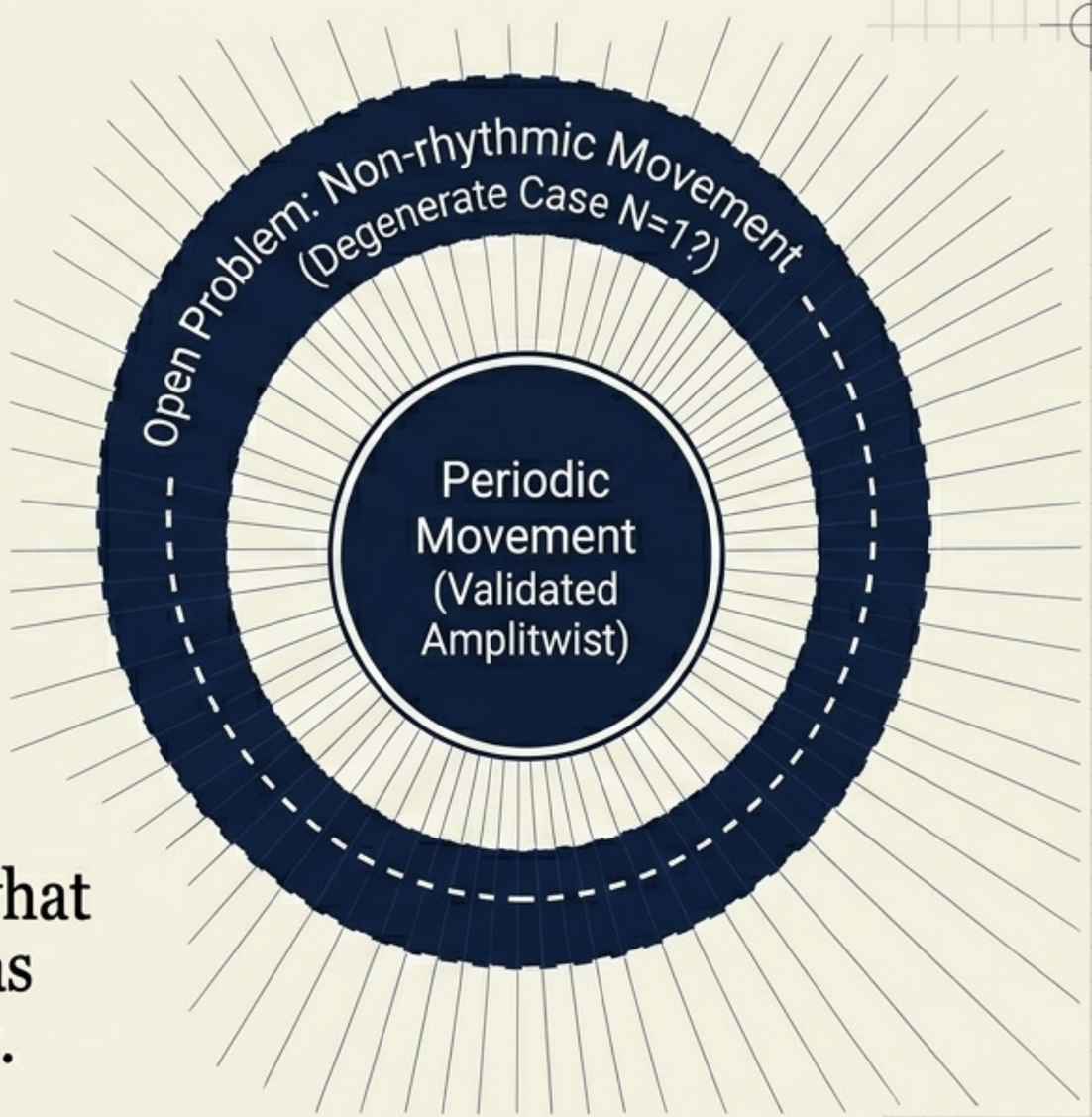
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Synthesis & Open Horizons

Recursive Continuation:
Locomotion joins Vim macros and repaired telemetry as systems that persist by continuously regenerating the conditions of their own further operation.

The Uncharted Frontier: Does a discrete action (reaching, tool manipulation) admit an amplitwist description without requiring a population order parameter?

Final Word: If the geometry fails its tests, what remains—CPGs as trajectory generators—was true before this essay and remains true after.



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