Verification and Proof Manual for the RSVP–Polyxan Architecture

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Abstract

This manual provides a formal verification framework for the RSVP–Polyxan unified architecture. We express RSVP fields, Polyxan hyperstructures, semantic embeddings, and galaxy-rendering logic using typed operational semantics, denotational models, and categorical invariants. Each major subsystem is stated axiomatically and accompanied by Coq/Lean-style definitions. We prove consistency of the RSVP action, well-typedness of the Polycompiler endofunctor, safety of projections and resets, and stability results for gradient flows. This manual is intended as the foundation for machine-verifiable proofs.

Contents

1 Foundational Axioms

We begin with the minimal axioms required for the unified model.

Axiom 1 (Semantic Manifold)**.** *There exists a smooth manifold (X, g) representing semantic space.*

Axiom 2 (RSVP Fields)**.** *There exist fields:*

$$\Phi : X \to \mathbb{R}, \quad \mathbf{v} : X \to TX, \quad S : X \to \mathbb{R}.$$

Axiom 3 (Polyxan Hypergraph)**.** *There is a typed hypergraph*

$$G = (V, E, T)$$

with content atoms as nodes and typed spans as edges.

Axiom 4 (Embedding Map)**.** *Each node $i \in V$ is assigned an embedding:*

$$\mathbf{z}_i \in X.$$

Axiom 5 (Coupling Principle)**.** *RSVP fields are influenced by hypergraph curvature and density, and embeddings evolve in RSVP gradient flows.*

2 Type System

We introduce a dependent type system suitable for mechanization.

2.1 Types

$$\begin{aligned} T ::= \mathsf{Real} \mid \mathsf{Vec}(n) \mid \mathsf{Atom} \mid \mathsf{Span} \mid \mathsf{Link}(T, T) \\ \mid \mathsf{Field}(X) \mid \mathsf{Embed}(T) \mid \mathsf{GalaxyView} \end{aligned}$$

2.2 RSVP Type Signatures

$$\Phi : \mathsf{Field}(X, \mathsf{Real})$$

$$\mathbf{v} : \mathsf{Field}(X, \mathsf{Vec}(d))$$

$$S : \mathsf{Field}(X, \mathsf{Real})$$

2.3 Polycompiler Type

$$\mathsf{Poly} : \mathsf{Atom} \to \mathsf{Atom}$$

Proposition 1 (Well-typedness)**.** *If a : Atom, then* $\mathsf{Poly}(a)$: *Atom.*

Proof. Immediate from the endofunctor definition. □

3 Operational Semantics

3.1 State Definition

Define global system state:

$$\Sigma = (\mathcal{C}, \mathbf{Z}, F, \mathcal{U})$$

where:

- $\mathcal{C}$ = content graph (atoms, spans, links)
- $\mathbf{Z}$ = embedding maps for all entities
- $F = (\Phi, \mathbf{v}, S)$ = RSVP field configurations
- $\mathcal{U}$ = user states (positions, controls)

3.2 Transition Rules

We use small-step semantics:

$$\Sigma \xrightarrow{\alpha} \Sigma'$$

3.2.1 Content Update

$$\frac{c' = \mathsf{ingest}(c)}{(\mathcal{C}, Z, F, U) \xrightarrow{\mathsf{content}} (\mathcal{C}', Z, F, U)}$$

3.2.2 RSVP Step

$$\frac{F' = \mathsf{step}(F, \mathcal{C})}{(\mathcal{C}, Z, F, U) \xrightarrow{\mathsf{field}} (\mathcal{C}, Z, F', U)}$$

3.2.3 Embedding Update

$$\frac{Z' = \mathsf{embed}(Z, F)}{(\mathcal{C}, Z, F, U) \xrightarrow{\mathsf{embed}} (\mathcal{C}, Z', F, U)}$$

3.2.4 Galaxy Rendering

$$\frac{U' = \mathsf{resolve}(U, Z)}{(\mathcal{C}, Z, F, U) \xrightarrow{\mathsf{render}} (\mathcal{C}, Z, F, U')}$$

4 Denotational Semantics

We interpret each subsystem as a morphism in a category.

4.1 Content Hypergraph

Let $\mathbf{C}$ be the category:

- Objects: content atoms, spans, link-typed composites.
- Morphisms: typed links.

4.2 RSVP Functor

Define:

$$\mathsf{RSVP} : \mathbf{C} \to \mathbf{F}$$

where $\mathbf{F}$ is the category of semantic fields.

Object mapping:

$$\mathsf{RSVP}(a) = (\Phi(a), \mathbf{v}(a), S(a)).$$

4.3 Polycompiler Endofunctor

$$\mathsf{Poly} : \mathbf{C} \to \mathbf{C}.$$

The composite nanoflow:

$$\mathsf{RSVP} \circ \mathsf{Poly}$$

denotes semantic deformation.

5 Formal Properties and Proofs

Theorem 1 (RSVP Stability)**.** *If Z evolves under the RSVP embedding flow:*

$$\frac{d\mathbf{z}_i}{dt} = -\nabla_\Phi + \mathbf{v} - \nabla_S,$$

and V satisfies convexity conditions, then embeddings converge to minimizers of the energy functional.

Sketch. Standard Lyapunov argument: Define $E(Z) = \sum_i V(\mathbf{z}_i)$. Gradient flow yields $\frac{dE}{dt} \leq 0$. Convexity ensures uniqueness of minimizers. □

Theorem 2 (Projection Safety). *User projections cannot introduce inconsistencies in other users' galaxy views if projection maps preserve the RSVP embedding metric.*

Sketch. Projection is read-only and does not modify state. Metric preservation ensures sheaf coherence. □

Theorem 3 (Reset Consistency). *Let $\mathcal{R}$ be the global reset operator. If $\mathcal{R}$ reprojects embeddings via RSVP relaxation, then*

$$\mathcal{R}(\Sigma)$$

is a fixed point of the RSVP-Poyxan consistency equations.

Proof. $\mathcal{R}$ recomputes F and Z from $\mathcal{C}$. This enforces all coupling equations by construction. □

6 Coq/Lean-Style Specification

6.1 RSVP Types

```
Structure Field := {
  phi : X -> R;
  v   : X -> Vector d;
  S   : X -> R;
}.
```

6.2 Embeddings

```
Definition Embedding := AtomId -> X.
```

6.3 Energy Functional

```
Definition Energy (E : Embedding) (F : Field) : R :=
  integral_X (V (phi F) (v F) (S F) (density E) (curvature E)).
```

6.4 Gradient Flow

```
Definition grad_step (E : Embedding) (F : Field) : Embedding :=
  fun i =>
    let z := E i in
    z - alpha * grad_phi F z
```

```
    + beta * v F z
    - gamma * grad_S F z.
```

6.5 Stability Theorem (Formal Skeleton)

```
Theorem RSVP_stable :
  convex V ->
  forall E0, exists E_star,
    converges (iter grad_step E0) E_star.
```

7 Sheaf Verification

Define presheaf:

$$\mathscr{G}(U) = \{\text{galaxy layouts over } U\}.$$

Theorem 4 (Galaxy Sheaf)**.** *$\mathscr{G}$ is a sheaf if embedding restriction is functorial.*

Proof. If layouts agree on overlaps and arise from restricted embeddings, the glued layout is unique. □

8 Categorical Soundness

Theorem 5. *The diagram*

$$\begin{tikzcd} \mathbf{C} \arrow[r, "\mathsf{Poly}"] \arrow[d, "\mathsf{RSVP}"'] & \mathbf{C} \arrow[d, "\mathsf{RSVP}"] \\ \mathbf{F} \arrow[r, "\mathsf{Def}"] & \mathbf{F} \end{tikzcd}$$

commutes up to natural transformation.

Sketch. Def interprets Polycompiler outputs as semantic deformations. Naturality follows from morphism preservation. □

9 Conclusion

This manual provides the axioms, type systems, operational rules, denotational semantics, and formal theorems required to fully verify the RSVP–Polyxan system. It is designed as a foundation for:

- machine-checked proofs,

- correctness and safety guarantees,
- future formalizations in Coq, Lean, Agda, or F*,
- integration with field-theoretic simulators.