Trajectories All the Way Down

Movement as the Generative Principle of Geometry, Cognition, and Computation

Flyxion

Independent Scholar

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Abstract

This essay advances a single unifying claim: complex structures in mathematics, neuroscience, cognition, and computation are not static objects stored in systems but the outcomes of trajectories through spaces generated by simple transformations. Scaling, rotation, and oscillation function as universal geometric primitives. When composed into chains of operators, these primitives produce mathematical functions, neural dynamics, semantic reasoning, and computational representations indistinguishable in their underlying architecture. The essay traces this principle from classical sweep geometry and complex analysis through Lie group theory, central pattern generators, hippocampal navigation, information geometry, and deep learning, arguing that dimension, thought, and meaning all arise from the same source: movement through representational space. A philosophical consequence follows. Cognition does not store representations; it generates them. Knowledge is not retrieved but traversed. The mind is less an archive than a vehicle, and its objects less locations than roads.

Contents

Introduction: Against the Static View of Structure

Modern science has inherited a tendency to treat the objects it studies as things that exist rather than as processes that occur. A point in mathematics is defined by its coordinates. A memory in neuroscience is assumed to reside somewhere in neural tissue. A concept in cognitive psychology is imagined as a node in a network, a discrete element connected to other discrete elements. A representation in machine learning is a vector of real numbers frozen at a moment in computation. In each case, the dominant metaphor is storage: knowledge, form, and meaning are housed in structures that persist, waiting to be accessed.

This essay argues that the storage metaphor is systematically misleading. It produces accurate descriptions at a surface level while obscuring the generative dynamics that make structures possible in the first place. A triangle is not a set of three points; it is the minimal network of traversable paths among three locations. A memory is not a pattern inscribed in synaptic weights; it is a trajectory the brain can reliably reconstruct through its own dynamics. A concept is not a node but a region in a high-dimensional associative manifold, defined by the attractor geometry of the cognitive system rather than by any fixed content.

The unifying thesis of this essay can be stated simply. Complex functions, concepts, and behaviors emerge from compositions of simple transformations generated by movement—scaling, rotation, and oscillation—applied to representational spaces. What appears as structure in mathematics, perception, or semantic reasoning can therefore be interpreted as trajectories produced by chained operators. The structure does not precede the motion; it is the motion, stabilized and made readable.

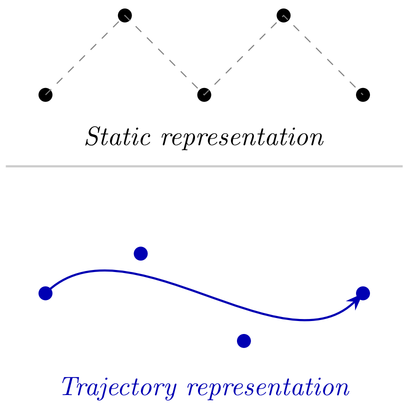


Figure 1: Static node-based representations contrasted with trajectory-based representations. The same points appear in both, but the trajectory view privileges the paths that generate them.

This thesis is not merely metaphorical. It draws on a deep family of mathematical results showing that rotation and scaling, as geometric primitives, generate universal approximators when composed. It connects to empirical findings in systems neuroscience showing that the same hippocampal circuits used for spatial navigation are reused for conceptual reasoning. It aligns with the architecture of modern deep learning, where stacked layers of simple linear and nonlinear operators approximate arbitrarily complex functions. And it resonates with the philosophy of embodied cognition, which has long argued that abstract reasoning is grounded in sensorimotor dynamics.

The essay develops this argument across fifteen sections. It begins with classical geometry, where the generative role of motion in producing higher-dimensional objects is already explicit. It then introduces Lie theory and the amplitwist interpretation of complex differentiation. From there it moves through Fourier decomposition, biological oscillators, spatial navigation, semantic cognition, control systems, image analysis, information geometry, category theory, and the evolutionary origins of trajectory-based cognition, showing that the same structural principle reappears at every level. The essay also includes a section on empirical predictions, a mathematical appendix proving the amplitwist decomposition, and a philosophical conclusion. If successful, the reader should finish the essay unable to think of a line, a memory, or a neural activation without also thinking of a path.

Sweep Geometry and the Generative Role of Motion

Classical Euclidean geometry presents its objects as if they were discovered rather than constructed. A point is defined as that which has position but no magnitude. A line is defined as that which has length but no breadth. These definitions are static. They describe objects by what they are, not by how they come to be.

Yet a rival tradition runs continuously through the history of geometry, one that treats shape as the outcome of movement. In this tradition, a line is what a point traces when it moves. A surface is what a line traces when it moves in a direction not contained in itself. A volume is what a surface traces when it moves in a direction not contained in itself. Each higher-dimensional object is the swept trajectory of a lower-dimensional one under a motion that adds a new independent direction to the system.

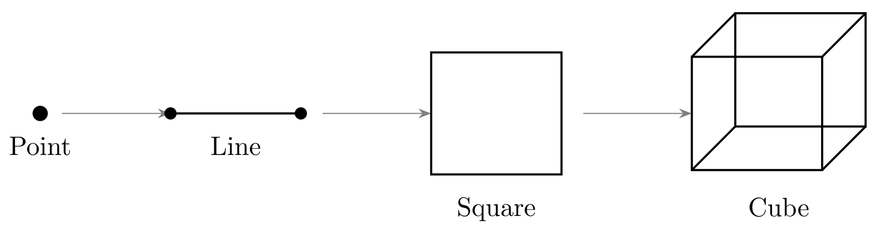


Figure 2: Higher-dimensional shapes as swept trajectories of lower-dimensional ones. Each transition adds one new independent direction of motion.

This construction can be formalized with elementary parametric notation. A line segment from point $\mathbf{a}$ to point $\mathbf{b}$ is the image of the map

$$\mathbf{x}(t) = (1 - t)\mathbf{a} + t\mathbf{b}, \quad t \in [0, 1]. \tag{1}$$

As the parameter t varies continuously from zero to one, a point moves along the segment. The segment is not a collection of static points; it is the record of a motion. Introducing a second independent parameter s that translates the segment in a direction perpendicular to $\mathbf{b} - \mathbf{a}$ produces a square. A third independent parameter translates the square in a direction orthogonal to its plane and produces a cube. In each case, the transition from one object to the next corresponds not to an increase in the number of points stored but to the addition of a new independent direction of motion.

A precise consequence follows from this perspective. Dimension is not an intrinsic property of objects but a measure of the number of independent directions of motion required to generate them. A curve is one-dimensional because it takes one parameter to trace it. A surface is two-dimensional because it takes two independent parameters. This interpretation shifts geometry from static description to generative dynamics. The question is not what an object is but how it is produced.

The philosophical import of this shift is considerable. If dimension corresponds to degrees of generative freedom, then mathematical objects encode motion rather than merely extending through space. Geometry, understood in this way, is a science of possible trajectories rather than a catalog of fixed forms.

The same logic applies in topology, where paths and homotopies—continuous deformations of paths—are more fundamental than the spaces themselves in many contexts. The fundamental group of a topological space is defined entirely in terms of based loops: paths that begin and end at the same point. The space reveals its topology through the structure of its possible motions, not through any intrinsic property of its points. Sweep geometry, Lie theory, and homotopy theory all converge on the same intuition. Structure is the record of transformation.

Lie Groups and the Algebra of Motion

The generative interpretation of geometry introduced in the previous section finds its most systematic mathematical expression in Lie theory. Where sweep geometry emphasizes the construction of objects from motion, Lie groups formalize the structure of the motions themselves.

A Lie group is a smooth manifold whose elements represent transformations and whose group operation corresponds to the composition of those transformations.

Rotations of three-dimensional space form the group $SO(3)$, translations form $\mathbb{R}^3$, and rigid body motion combines both into the Euclidean group $SE(3)$.

The key insight of Lie theory is that global transformations can be generated from infinitesimal ones. The tangent space at the identity element of a Lie group forms its associated Lie algebra, whose elements act as generators of motion. Exponentiating these generators produces finite transformations. For example, rotations about the three coordinate axes generate all possible orientations of a rigid body:

$$R(\theta) = e^{\theta A}, \tag{2}$$

where A is an element of the Lie algebra $\mathfrak{so}(3)$. The exponential map carries infinitesimal twists into finite rotations. The generators of the algebra are precisely the primitive directions of movement in transformation space.

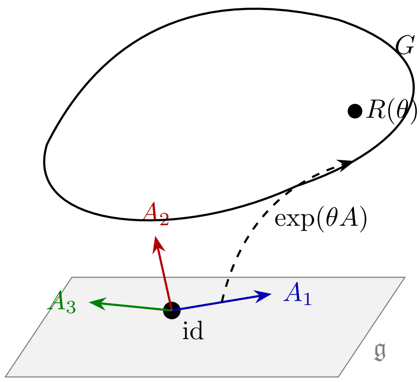


Figure 3: The Lie algebra $\mathfrak{g}$ sits at the identity of the Lie group G as a tangent space of generators. Exponentiating a generator A produces a one-parameter family of finite transformations on the group manifold.

From the perspective advanced in this essay, Lie algebras provide the mathematical language for describing primitive motions. Each generator corresponds to a fundamental direction of movement in transformation space. Complex structures arise from exponentiating and composing these generators. This algebraic view of motion connects directly to the amplitwist principle of the next section: rotation generators correspond to twists in the local tangent space, while symmetric generators correspond to scaling along principal axes. Together they span the transformation algebra that produces structure through composition.

It is worth observing that many of the structures encountered in physics, geometry, and machine learning carry implicit Lie group symmetries. The rotation group $SO(3)$ governs the orientation of rigid bodies. The unitary group $U(n)$ governs the symmetries of quantum states. The general linear group $GL(n)$ governs the full space of invertible linear transformations applicable to neural representations. In each case, the group's generators are the primitive directions through which the space of transformations is navigated, and the exponential map converts those generators into traversable paths. The abstract machinery of Lie theory is, in this sense, a precise formalization of the essay's central intuition: global structure emerges from the composition of infinitesimal motions.

The Amplitwist Principle: Complex Differentiation as Local Geometry

Complex analysis offers the most elegant instance of the essay's central thesis. A function from the complex plane to itself is called holomorphic at a point if it is complex-differentiable there. The geometric content of holomorphicity can be expressed with disarming simplicity.

Consider a holomorphic function $f : \mathbb{C} \to \mathbb{C}$ that is differentiable at a point z_0. In a sufficiently small neighborhood of z_0, the function behaves approximately as

$$f(z_0 + \varepsilon) \approx f(z_0) + f'(z_0)\,\varepsilon. \tag{3}$$

The derivative $f'(z_0)$ is itself a complex number. Writing it in polar form,

$$f'(z_0) = \rho\, e^{i\theta}, \tag{4}$$

reveals its geometric content precisely. Multiplying the displacement ε by ρ scales it by the factor ρ. Multiplying by $e^{i\theta}$ rotates it by the angle θ. This is Needham's *amplitwist*: an amplitude change and a twist occurring simultaneously.

The amplitwist interpretation has a direct consequence for the global behavior of holomorphic functions. Because every holomorphic map acts locally as a rotation and scaling, it is conformal: it preserves the angles between curves meeting at a point. The global analytic structure of a holomorphic function emerges from the continuous variation of these local amplitwist operations across the domain.

This establishes a mathematical prototype for the essay's central thesis. Complex, globally-structured analytic mappings are built from chains of simple geometric operations: local rotations and scalings that vary smoothly from point to point. The complication of the global map arises not from any single large transformation but from the accumulated composition of many small ones. Structure is the record of trajectories through operator space.

The deeper significance of the amplitwist principle becomes apparent when one

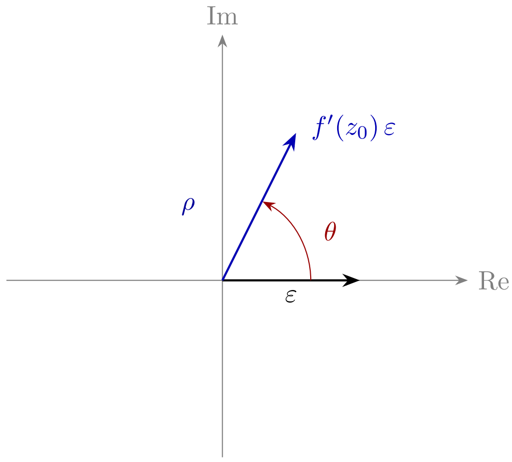


Figure 4: Local amplitwist transformation in the complex plane. The derivative $f'(z_0) = \rho e^{i\theta}$ simultaneously rotates ε by angle θ and scales it by ρ.

considers conformal mappings and Riemann's mapping theorem. The theorem states that any simply connected proper open subset of the complex plane is conformally equivalent to the open unit disk. Such maps can be extremely complicated in explicit form, yet all of them can be analyzed as compositions of local amplitwists. The Möbius transformations, which form the group of conformal automorphisms of the Riemann sphere, are the simplest such maps; they are generated by translations, scalings, rotations, and inversion, each of which is an elementary geometric transformation. More complicated conformal mappings are built from compositions of these generators.

The amplitwist decomposition—$f'(z_0) = \rho e^{i\theta}$—is also the content of the polar decomposition theorem for linear transformations. Any invertible complex linear map can be written as the product of a rotation and a positive scaling. The amplitwist is therefore not a special property of complex numbers but a universal decomposition available for any linear transformation. This universality will become important in the sections on image analysis and latent representations.

Oscillatory Primitives and Universal Approximation

The amplitwist principle identifies rotation and scaling as the generators of complex analytic structure. A parallel story unfolds in signal processing and computational mathematics, where the generative role of oscillatory components is made explicit through Fourier analysis.

The central result of classical Fourier theory is that any sufficiently well-behaved periodic function can be expressed as a superposition of sinusoidal components:

$$f(x) = \sum_{n=-\infty}^{\infty} c_n \, e^{inx}. \tag{5}$$

Each term e^{inx} is a pure oscillation, a rotation in the complex plane at angular frequency n. The coefficient c_n controls the amplitude. Complex signals arise from compositions of simple oscillatory primitives.

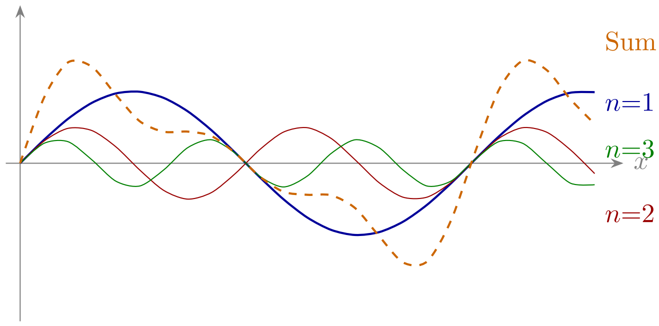


Figure 5: A complex signal (dashed) arising from superposition of three oscillatory primitives at frequencies $n = 1, 2, 3$. Each component is a rotation in the complex plane at the corresponding angular rate.

What does a single layer of a neural network do, geometrically? It applies a linear transformation W to an input vector $\mathbf{x}$, shifts the result by a bias $\mathbf{b}$, and then applies a pointwise nonlinear function σ:

$$\mathbf{h} = \sigma(W\mathbf{x} + \mathbf{b}). \tag{6}$$

By the polar decomposition, the linear transformation W can be analyzed as a rotation followed by a scaling in each principal direction. The nonlinearity σ then folds and warps the resulting configuration, creating new curvature in the feature space. Stacking many such layers composes many small geometric transformations until the cumulative effect approximates a complicated mapping.

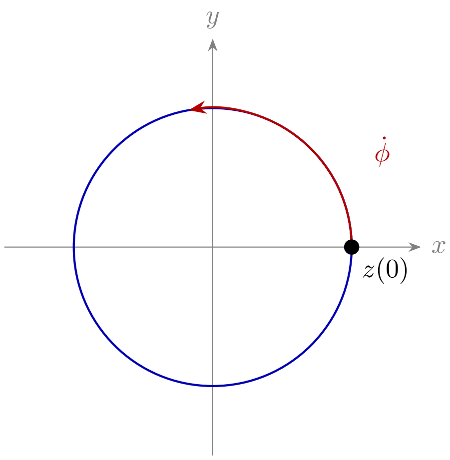
This architecture is the computational analogue of the amplitwist cascade. Just as a complex analytic function is locally a rotation and scaling at every point, and its global behavior emerges from composing these local operations, a deep neural network computes a complex input-output mapping by composing many layers of small geometric transformations. The universality of the approximation corresponds to the completeness of the Fourier expansion: given enough layers and units, the chain of operators can approximate any target function.

Central Pattern Generators and Phase-Space Geometry

The previous sections established that composition of geometric primitives produces universal approximators in mathematics and computation. The same principle appears in biological motor systems, but now the primitives are physically implemented by networks of coupled oscillators.

Central pattern generators are neural circuits that produce rhythmic, patterned motor output without requiring continuous sensory input. Their dynamics have been extensively studied through coupled-oscillator models in which each unit contributes a simple periodic rhythm and interactions among units produce coordinated patterns.

A single oscillator in a CPG circuit can be modeled as a two-dimensional dynamical system with a stable limit cycle. If its state is represented as $z(t) = x(t) + iy(t)$, its evolution corresponds to circular motion in the complex plane—a rotation at a fixed radius, with amplitude modulated by coupling to other units.



CPG limit cycle in phase space

Figure 6: A CPG oscillator traces a stable limit cycle in the complex phase plane. Amplitude modulation scales the radius; phase coupling rotates the state along the cycle. Together they implement the two components of an amplitwist.

When two or more such oscillators are coupled, their phase relationships evolve according to coupling dynamics, canonically described by the Kuramoto model. In a network of N coupled oscillators with natural frequencies ω_i, the phase ϕ_i evolves as

$$\dot{\phi}_i = \omega_i + \frac{K}{N} \sum_{j=1}^{N} \sin(\phi_j - \phi_i), \tag{7}$$

where K is the coupling strength. When K exceeds a critical threshold, the oscillators synchronize. Partial synchrony at intermediate coupling strengths produces a rich variety of phase-locking patterns, each encoding a different spatial or temporal structure in the motor output.

A CPG network is therefore a physical implementation of amplitwist cascades. Each oscillator contributes a rotation in its local phase plane; coupling translates phase information across the network. The motor pattern that emerges corresponds to a trajectory in the high-dimensional phase space of the full network—stable, reproducible, and parameterized by coupling strengths and intrinsic frequencies.

An animal that varies its gait from walking to trotting to galloping does not switch to a completely different neural circuit; it modulates the coupling strengths and phase offsets of the same oscillator network, moving from one stable trajectory to another. The transitions between gaits correspond to bifurcations in the coupled-oscillator dynamics. The body's movement repertoire is a collection of stable trajectories in CPG phase space, navigated through the same control parameters that generate the individual oscillations.

Neuromodulatory signals—dopamine, serotonin, norepinephrine—adjust the excitability of CPG neurons, effectively changing the gain of the oscillation and scaling the amplitude of the output rhythm. Phase coupling combined with neuromodulatory gain control implements, at the biological circuit level, the precise operations identified in complex analysis as the generators of conformal transformations.

Hippocampal Navigation and the Geometry of Cognitive Maps

The hippocampal formation provides the most empirically detailed example of a biological system that generates structure through trajectories rather than storage. Place cells in the hippocampus proper fire selectively when an animal occupies a specific region of its environment. Grid cells in the medial entorhinal cortex fire at the vertices of a regular hexagonal lattice that tiles the environment.

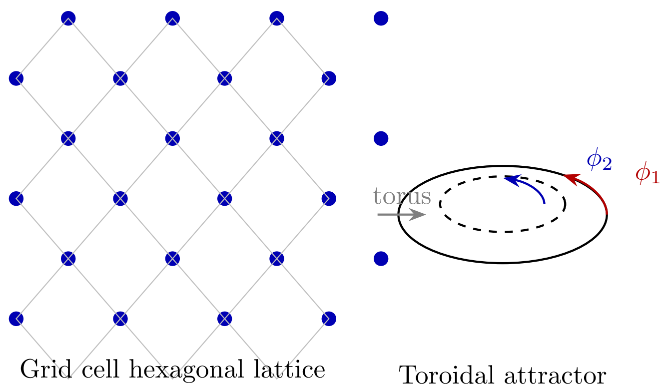


Figure 7: Left: Grid cell hexagonal lattice tiling an environment. Right: The corresponding toroidal attractor structure. Grid codes correspond mathematically to periodic coordinates on a torus, with two independent phase variables ϕ_1, ϕ_2.

The mathematical relationship between grid cells and place cells can be understood as a transformation between different coordinate representations of the same space. Grid cells provide a Fourier-like basis for spatial location: the hexagonal periodicity at multiple spatial scales and orientations constitutes a set of oscillatory components in two-dimensional space. Place fields can be constructed by summing the activity of grid cells at appropriate phases and scales, in a manner formally analogous to the synthesis of a spatial pattern from frequency components.

Path integration is the mechanism by which the grid and place cell system maintains a continuous estimate of position through movement. As the animal moves, velocity signals are integrated over time to update the phase of each grid cell oscillator, moving the animal's represented position through the cognitive map. The map is not retrieved from storage but continuously reconstructed from movement.

The extension of this spatial navigation machinery to non-spatial cognition is one of the most significant recent developments in cognitive neuroscience. Experiments using functional neuroimaging have shown that grid-cell-like signals appear in the entorhinal cortex when human subjects navigate abstract spaces. When participants learned relationships among objects arranged in a two-dimensional conceptual grid defined by abstract features, entorhinal activity exhibited the hexagonal periodicity characteristic of spatial grid cells. The same neural machinery computed both spatial and conceptual coordinates.

Andrea Hiott has developed a complementary account in recent work, arguing that the neural machinery of spatial navigation constitutes a scaffold for the formation of semantic memory. Her proposal is that *way-making*—the activity of establishing paths through an environment—underlies both physical navigation and cognitive processes such as remembering, planning, and conceptual reasoning. In this view, the hippocampus is not a memory storage device but a way-making engine: it generates trajectories through structured spaces, whether those spaces are geographical or conceptual in character. Representations, on this account, are communicative assessments of regularities in embodied action—the ways in which the system has learned to move through its world.

Semantic Cognition as Navigation Through Attractor Landscapes

The static view of semantic memory treats concepts as nodes in a network, connected by labeled edges encoding semantic relationships. A more geometrically explicit alternative draws on the theory of conceptual spaces developed by Gärdenfors. In this framework, concepts correspond to convex regions in a high-dimensional quality space, where dimensions correspond to perceptual or functional properties.

The attractor-landscape model extends this framework by adding dynamics. In a sufficiently rich associative system, activating one concept creates a gradient of activation that spreads to nearby regions of the conceptual manifold, preferentially activating concepts sharing contextual, co-occurrence, or causal structure with the initial concept. This spreading activation defines attractor basins: regions of the manifold toward which nearby trajectories converge. A schema—a structured scenario such as RESTAURANT, HOSPITAL, or ARGUMENT—corresponds to a basin in this landscape, where many conceptually related items cluster under a common contextual geometry.

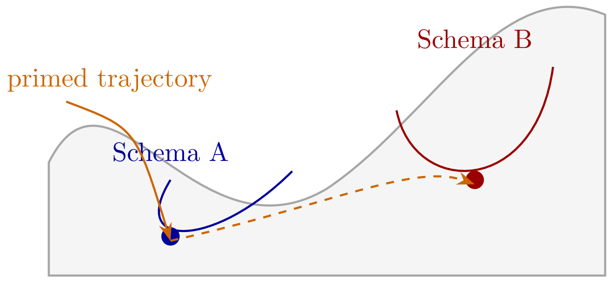


Figure 8: Concepts as attractor basins in a semantic landscape. A priming stimulus shifts the trajectory toward Schema A; subsequent associations can lead the trajectory into Schema B. Reasoning follows gradient descent through the landscape, not retrieval from storage.

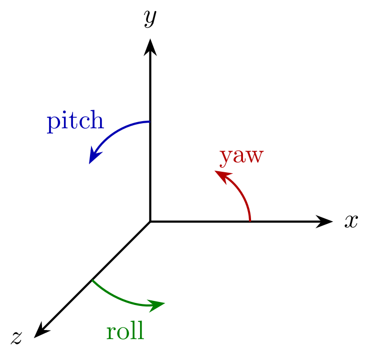
Semantic priming can be understood in these terms as the perturbation of the cognitive system's position in conceptual space. When a prime word is presented, it shifts the system's representational state toward the region associated with that word, making trajectories into nearby attractor basins more probable. The concept reached after priming is not retrieved; it is arrived at through a shortened trajectory facilitated by the initial perturbation.

Navigation through this landscape exhibits properties that distinguish it from symbolic retrieval. Trajectories have duration: they take time to traverse attractor basins and to transition between them. They have inertia: the system tends to continue in the direction of current activation rather than jumping discontinuously to arbitrary locations. They are sensitive to context: the same starting point can lead to different destinations depending on the current gradient of activation across the manifold. These are the hallmarks of dynamical navigation, not database lookup.

The amplitwist operators identified in previous sections provide the local transformation primitives that deform and navigate this terrain: rotations in representational space shift the alignment of concepts relative to one another, scalings modulate the relative salience of different conceptual dimensions, and oscillatory dynamics provide the carrier for maintaining coherent trajectories over time. A schema is not a template superimposed on incoming information but an attractor basin whose geometry shapes the trajectories of activation that pass through it. Inference is navigation: following the gradient of the schema basin to its expected terminus.

Control Parameters and Degrees of Freedom in Transformation Space

The mathematical structure of a navigation system, whether physical or cognitive, can be understood through the concept of degrees of freedom. A degree of freedom is an independent parameter whose variation produces a distinct mode of change in the system's state. A rigid body moving freely in three-dimensional physical space has six degrees of freedom: three translational parameters specifying position and three rotational parameters specifying orientation.



6 degrees of freedom: 3 translation, 3 rotation

Figure 9: Degrees of freedom in a rigid body navigation system. Three translational and three rotational generators span the full Euclidean group $SE(3)$. Any reachable configuration is reached by composing motion along these axes.

The video game Descent provides an unusually pure illustration of this structure. Unlike most navigation games restricted by gravitational constraints, Descent places the player's vehicle in a fully three-dimensional microgravity environment where all six degrees of freedom are simultaneously available. Navigation consists of continuously adjusting these six control parameters, composing small local transformations to produce complex global trajectories. The player does not compute a path; they navigate by applying simple local controls that accumulate into whatever trajectory the task requires.

The analogy to the amplitwist framework is precise. In complex analysis, the generating operations—rotation and scaling—provide two degrees of freedom at each point of the domain. A conformal mapping is navigated through the continuous adjustment of these two parameters. The global mapping emerges from the accumulated composition of local adjustments, just as a global trajectory through three-dimensional space emerges from the accumulated composition of local thrust and rotation commands.

The space of all transformations of a given type is itself a geometric space, often a Lie group. A point in transformation space represents a specific transformation. Moving through transformation space corresponds to composing transformations in a continuous way. The generating operations of the transformation group—the Lie algebra generators—play the role of control axes: they are the minimal set of infinitesimal motions from which all transformations in the group can be reached.

Image Analysis and the Latent Geometry of Representations

An image can be formally represented as a vector $\mathbf{x} \in \mathbb{R}^n$, where n is the total number of pixel values. The images that arise from natural scenes occupy a small, structured subset of this space: a manifold of much lower intrinsic dimension, curved through the ambient space in a way that reflects the statistical regularities of natural image structure.

A geometric transformation applied to an image corresponds to movement along a specific direction in this manifold. The basic linear transformation can be written as

$$\mathbf{y} = A\mathbf{x}, \tag{8}$$

where A is a transformation matrix. By the polar decomposition theorem, $A = RS$, where R is orthogonal and S is symmetric positive definite. An orthogonal transformation preserves all distances and angles; the image is reoriented without being distorted. The symmetric factor stretches or compresses the image along its principal axes. Every linear image transformation is, therefore, an amplitwist: a composition of rotation and scaling.

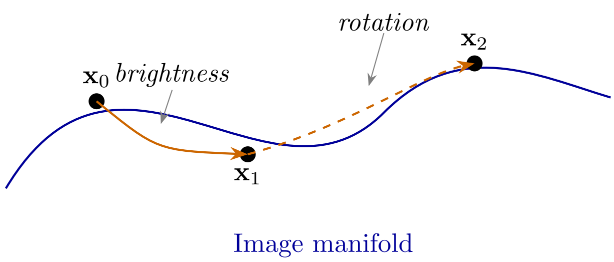


Figure 10: Images as points on a curved manifold in high-dimensional space. Adjustable parameters (brightness, rotation) correspond to navigable directions along the manifold. Each slider is literally a coordinate in transformation space.

In practical image editing, this structure manifests as the set of available adjustment controls. Brightness, contrast, gamma, exposure, hue, saturation, and sharpness each correspond to a specific combination of linear and nonlinear transformations in the image representation space. The slider for each control moves the image along a specific direction in that space. Adjusting multiple controls simultaneously navigates a multi-dimensional subspace, tracing a trajectory through the image manifold parameterized by the control values.

In latent representations learned by autoencoders or diffusion models, directions in latent space correspond to interpretable image transformations: pose change, lighting variation, identity in a face recognition system. Moving along these directions produces smooth, semantically meaningful variations in the reconstructed image. In diffusion models, the generation of a new image from noise is literally a trajectory: a path through image space guided by a learned vector field at each point. The composition of many small denoising steps accumulates into a coherent image through the same principle of generating complex structure from chained simple operators that runs through this entire essay.

Information Geometry and Statistical Manifolds

The trajectory view of cognition and computation acquires additional mathematical clarity through information geometry, a field that studies statistical models as geometric manifolds. In information geometry, a probability distribution $p(x|\theta)$ is treated as a point on a manifold whose coordinates are the parameters θ. Distances between distributions are measured using the Fisher information metric,

$$g_{ij}(\theta) = \mathbb{E}\left[\frac{\partial \log p(x|\theta)}{\partial \theta_i}\frac{\partial \log p(x|\theta)}{\partial \theta_j}\right]. \tag{9}$$

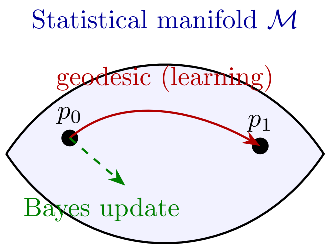


Figure 11: The statistical manifold of probability distributions parameterized by θ, equipped with the Fisher information metric. Learning corresponds to geodesic paths; Bayesian updating moves the distribution along shorter locally curved paths determined by the metric geometry.

Learning and inference correspond to trajectories on this manifold. Gradient descent follows the steepest path toward a minimum of the loss function, while Bayesian updating moves the system along geodesics determined by new data. The training of a neural network is literally navigation through a statistical manifold. Each update step is a local movement in parameter space guided by the geometry of the Fisher metric. Natural gradient methods—which replace the Euclidean gradient with the

Fisher-metric gradient—make this geometric interpretation operationally precise: they follow the geodesic path of steepest descent on the information manifold rather than in raw parameter space.

The connection to the trajectory framework developed in this essay is direct. Just as spatial navigation traces paths through physical environments, statistical learning traces paths through information space. The operators applied during learning are geometric transformations that reshape the representation manifold until trajectories converge on stable attractors corresponding to learned solutions. The Fisher metric governs the local geometry, playing a role analogous to the Riemannian metric on the cognitive manifolds described in earlier sections.

Seen in this light, overfitting corresponds to a trajectory that descends too aggressively into a narrow well of the loss landscape without finding a broad, stable attractor. Regularization corresponds to the imposition of geometric constraints on the allowable trajectories, analogous to the entropy field S in the RSVP framework that prevents runaway amplification. In both biological and artificial learning systems, the geometry of the learning trajectory shapes what can be generalized and what cannot.

Toward a Unified Geometric Model

The preceding sections have traced the essay's central thesis through ten distinct domains. It is now possible to state their convergence as a unified geometric model of cognition and computation.

Let $\mathcal{M}$ be a smooth manifold representing the state space of some system. A *representational state* of the system is a point $p \in \mathcal{M}$. A *primitive operator* is a smooth map $T : \mathcal{M} \to \mathcal{M}$ that implements a local rotation, scaling, or oscillatory phase shift in the tangent space at each point. A *trajectory* is a smooth curve $\gamma : [0, 1] \to \mathcal{M}$ produced by the continuous application of a sequence of primitive operators. A *structure* is a trajectory or family of related trajectories that the system reliably produces under given conditions.

The universality of the model follows from three classical results taken together. By the polar decomposition theorem, any linear transformation can be expressed as a composition of rotation and scaling. By the universal approximation theorem for neural networks, any continuous function can be approximated by a composition of linear transformations and nonlinear activations. By the completeness of the Fourier basis, any square-integrable function can be expressed as a superposition of oscillatory components, each of which is a rotation in the complex plane. Together these results imply that any smooth map between manifolds can be approximated to arbitrary precision by compositions of primitive rotations and scalings. The generating primitives are universal.

A further unification comes from considering the role of entropy in constraining trajectories. In the RSVP framework developed elsewhere, a scalar entropy field S regulates the behavior of a scalar field Φ and a vector field $\mathbf{v}$, where Φ corresponds to amplitude (scaling) and $\mathbf{v}$ to directional flow (rotation). The entropy field prevents runaway amplification and constrains the set of achievable trajectories to a thermodynamically stable manifold. In neural terms this corresponds to the metabolic and homeostatic limitations of the nervous system; in computational terms, to regularization conditions imposed during learning. In all cases, entropy provides a thermodynamic regulator that shapes the geometry of the trajectory space, determining which paths are stable and which decay.

This unified model does not reduce all cognitive processes to a single mechanism. Different systems implement the trajectory-generation principle with different geometries of state space, different transformation groups, and different regulatory constraints. What the model provides is a common mathematical language for describing all of them.

Category Theory and the Composition of Transformations

If trajectories are the fundamental units of structure, then composition becomes the fundamental operation. Category theory provides a formal language for describing such compositional systems.

A category consists of objects and morphisms between them. Morphisms represent transformations, and the central rule of category theory is that morphisms can be composed:

$$f : A \to B, \quad g : B \to C \quad \Rightarrow \quad g \circ f : A \to C. \tag{10}$$

The significance of this structure is that objects are defined not by their internal composition but by the transformations that connect them to other objects. A morphism corresponds to a path through transformation space. Composing morphisms corresponds to chaining trajectories. The identity morphism corresponds to remaining stationary in the manifold.

Higher category theory extends this framework by allowing morphisms between morphisms, capturing the relationships among transformations themselves. In such systems, structure emerges from the network of possible trajectories rather than from the intrinsic nature of the objects involved. A functor is a structure-preserving map between categories: it carries objects to objects and morphisms to morphisms, preserving composition. In the essay's terms, a functor is a transformation of trajectory spaces that respects the compositional structure.

This categorical perspective illuminates many of the connections drawn in earlier sections. The relationship between CPG dynamics and hippocampal navigation, for instance, can be understood as a functor that carries phase-space trajectories of oscillatory circuits to trajectories through spatial or conceptual manifolds, preserving the compositional structure of the trajectory algebra. The relationship between image manifolds and semantic manifolds can be understood similarly. In each case, the relevant mathematical object is not the spaces themselves but the morphisms between them—the transformations that carry structure from one domain to another.

Category theory therefore provides a natural mathematical language for the generative view of structure advanced throughout this essay. Objects become nodes in a web of transformations, and meaning arises from the ways in which those transformations compose. This is the precise mathematical form of the essay's central thesis: it is not what things are that matters but how they transform.

Evolutionary Origins of Trajectory-Based Cognition

The trajectory model of cognition is not merely a mathematical abstraction. It reflects the evolutionary history of nervous systems.

Early nervous systems evolved to coordinate movement. Locomotion requires integrating sensory information, predicting environmental changes, and adjusting motor commands in real time. These tasks are inherently dynamical: they cannot be performed by a system that merely retrieves static stored representations.

Central pattern generators provided the earliest neural mechanisms for producing coordinated motion. The oscillatory dynamics of these circuits created stable trajectories in phase space that could be modulated by sensory input. As nervous systems became more complex, the same dynamical mechanisms were recruited for new purposes. Spatial navigation reused locomotor oscillators to track position. Conceptual reasoning reused spatial navigation circuits to traverse abstract spaces of meaning. This hypothesis of neural reuse has gained substantial empirical support: the same areas of the hippocampal formation that process spatial coordinates also process abstract conceptual relationships, and the same grid-cell periodicity that tiles physical space also tiles conceptual space.

From an evolutionary perspective, abstract cognition did not arise by inventing entirely new mechanisms. Instead, existing trajectory-generating systems were repurposed for increasingly abstract forms of navigation. The abstract structures of semantic memory, planning, and inference inherit their geometry from the oscillatory and navigational dynamics of far more ancient motor systems. The mind therefore retains the dynamical architecture of its origins. Even the most abstract reasoning processes can be understood as descendants of the mechanisms that once coordinated movement through physical environments. The philosopher's claim that thought is inner speech has an older predecessor: thought is inner locomotion, the navigation of a terrain made internal.

Empirical Predictions

The trajectory framework developed in this essay generates several testable predictions. Stating them explicitly distinguishes the framework from a purely philosophical thesis and opens it to scientific evaluation.

First, neural population activity during conceptual reasoning should exhibit rotational dynamics similar to those observed in motor cortex and hippocampal navigation circuits. Dimensionality reduction methods such as jPCA should reveal phase-space rotations corresponding to conceptual transitions. Rotational dynamics have already been observed in motor cortex during movement preparation; if the trajectory model is correct, analogous rotations should appear in areas associated with semantic processing and abstract reasoning, and their phase structure should correspond to the semantic distance between concepts being traversed.

Second, learned representations in artificial neural networks should organize transformations along low-dimensional manifolds whose generators correspond to interpretable geometric operations such as rotation, scaling, or translation. Specifically, the Fisher information metric on the parameter manifold of a well-trained network should exhibit a low effective dimensionality, reflecting the fact that the network has found a compact set of generators for its input-output mapping.

Third, perturbations of oscillatory phase relationships in neural circuits should disrupt both motor coordination and conceptual navigation, reflecting their shared dynamical substrate. Targeted disruption of hippocampal theta oscillations, for instance, should impair not only spatial navigation performance but also the speed and accuracy of semantic inference on structured conceptual spaces, particularly for inferences that require traversing multiple attractor basins in sequence.

Fourth, behavioral measures of reasoning should exhibit temporal characteristics consistent with trajectory traversal, including inertia, gradient-following dynamics, and attractor basin transitions. Specifically, the time required to make a semantic inference should increase with the conceptual distance between the starting concept and the target concept, measured along the geodesic of the semantic manifold, not simply as a function of the number of associative steps between them as counted in a static network.

Fifth, subjects trained in environments that impose oscillatory structure—rhythmic locomotion, music, breathing exercises with controlled phase—should show enhanced performance on conceptual navigation tasks that require traversal of multiple attractor basins, if the hypothesis is correct that the shared dynamical substrate can be recruited across domains.

Testing these predictions would provide empirical support for the hypothesis that cognition is fundamentally a process of navigating representational manifolds, and would distinguish the trajectory model from the storage-retrieval model with which it competes.

The Trajectory Ladder of Dimension and Cognition

The argument of this essay can be compressed into a single diagram. Each level in the following ladder arises from the trajectory of the level below it, and the same generative logic that carries a point to a line carries an oscillatory phase space to a conceptual manifold.

The first four steps—point, line, square, cube—correspond to the sweep geometry of the second section. The transition from cube to torus corresponds to the move from static polyhedra to oscillatory dynamical systems, where two independent phase variables generate a toroidal phase space. The transition from torus to manifold corresponds to the move from simple periodic dynamics to the rich, curved representational geometry of hippocampal and cortical systems. The transition from manifold to concept corresponds to the formation of attractor basins within that manifold. And the transition from concept to reasoning corresponds to navigation through those basins, following trajectories shaped by contextual gradients and associative structure.

At each step, the entity at the new level is not a new kind of thing in a categorically different realm; it is the record of a trajectory through the space generated by the previous level. This is the deepest form of the essay's claim, and the diagram makes it visual. Structure is generated, not stored. Dimension is not intrinsic but earned through motion. And cognition, at its most abstract, is geometry at its most dynamic.

Philosophical Consequences: From Storage to Generation

The geometric model developed in the preceding sections has philosophical consequences that reach beyond the specific scientific domains it addresses. These consequences concern the nature of representation, the structure of knowledge, and the relationship between cognition and world.

The prevailing model of mental representation is what might be called the storage-retrieval model. Representations are encoded, stored in memory, and retrieved when needed. Knowledge is a warehouse. Reasoning is the manipulation of stored representations according to rules. The mind is analogous to a library: it contains information, organized for access.

The trajectory model proposed in this essay is fundamentally at odds with this picture. What the storage-retrieval model calls a representation is not a stored entity but a reproducible trajectory. A memory is not a pattern inscribed in synaptic weights but a trajectory the system can reconstruct from partial initial conditions. A concept is not a node with a fixed content but an attractor basin that shapes the trajectories of activation that pass through it. Knowledge is not a set of stored facts but a configuration of the system's trajectory space—a set of paths the system can reliably follow through its representational manifold.

The trajectory model aligns with, and provides mathematical precision for, the tradition of embodied cognition in philosophy and cognitive science. Embodied cognition holds that cognition is not the manipulation of abstract symbols inside an isolated brain but a process constitutively involving the body and its environment. The hippocampal findings reviewed in earlier sections provide direct neural evidence: the spatial representations constructed by place and grid cells are not stored in the cells but generated through movement, through the path integration of self-motion signals over time. The map is made, not found.

Hiott's way-making account extends this insight explicitly to semantic cognition. Understanding a concept is not accessing a stored definition; it is traversing a region of conceptual space, navigating among associated ideas, following the gradient of an attractor basin to its center. The meaning is the trajectory, not the destination.

A further philosophical implication concerns the relationship between individual and social cognition. Language provides a set of external landmarks in conceptual space—words and sentences that function as attractors, orienting the trajectories of individual cognitive systems toward socially coordinated destinations. The meaning of a word is not a private representation stored in each speaker's mind but a convergence of trajectories in a shared conceptual manifold, maintained through the ongoing practice of using language together.

If structures arise from trajectories rather than from storage, then knowledge, meaning, perception, and reasoning all need to be reconceived as processes of navigation through structured spaces rather than as operations on fixed representations. The mind is not a library but a vehicle, and its knowledge is not stored in it but enacted by it.

Conclusion: Movement as the Generative Principle

This essay has traced a single geometric insight across thirteen domains. The insight is this: simple transformations—scaling, rotation, and oscillation—compose to generate complex structure, and what appears as a stable object in mathematics, neuroscience, or computation is, at a deeper level, the stabilized record of a trajectory produced by chaining such transformations.

In classical geometry, dimension is the number of independent directions of motion required to generate an object. In Lie theory, global transformations are generated from infinitesimal ones; the Lie algebra provides the minimal vocabulary of motions from which the entire transformation group is navigated. In complex analysis, holomorphic functions are locally amplitwists—simultaneous rotations and scalings—and complicated conformal mappings arise from composing many such local operations across a domain. In signal processing and machine learning, oscillatory primitives generate universal approximations to arbitrary functions. In biological motor systems, central pattern generators implement oscillatory rotation in phase space. In the hippocampal formation, spatial and conceptual representations are constructed through path integration of movement signals. In semantic cognition, concepts are attractor basins and reasoning is navigation. In transformation spaces and latent representations, adjustable parameters correspond to directions in a geometrically structured space. In information geometry, learning is geodesic descent on the statistical manifold. In category theory, composition of morphisms is the fundamental operation and objects derive meaning from their transformation relations.

Across all these domains, the pattern is the same. Simple primitives compose. The composition generates structure. The structure is a trajectory, not a location. The system is a vehicle, not a warehouse.

The evolutionary picture reinforces this conclusion: the nervous system did not invent abstract cognition from nothing. It repurposed the trajectory-generating machinery of motor systems for increasingly abstract navigation. The same oscillatory circuits that once guided a worm through a gradient of nutrients now, in elaborated form, navigate the conceptual gradients of a mathematical proof.

This view demands a revision of the metaphors through which cognition is usually imagined. The library metaphor should give way to the navigation metaphor: knowledge as a structured terrain, thinking as movement through it. The computer metaphor should give way to the dynamical systems metaphor: the brain as a trajectory generator, producing behavior by navigating a high-dimensional phase space shaped by the geometry of the organism's embodied history. These are not merely aesthetic choices. They determine what scientific questions are asked, what experiments are designed, and what models are built.

Scaling, rotation, and oscillation are not merely tools of mathematics; they are the generative principles by which the universe, and the minds that inhabit it, produce structure from motion.

The line is a moving point. The cube is a moving square. The concept is a moving activation. The thought is a moving mind.

Mathematical Appendix: Amplitwist as a Conformal Polar Decomposition

This appendix formalizes the geometric claim used throughout the essay: the derivative of a holomorphic function acts locally as a rotation together with a uniform scaling, and this local amplitwist is a special two-dimensional conformal case of the general polar decomposition for invertible linear maps.

Complex differentiation as local linearization

Let $f : \mathbb{C} \to \mathbb{C}$ be complex differentiable at z_0. By definition, there exists a complex number $f'(z_0)$ such that

$$\lim_{\varepsilon \to 0} \frac{f(z_0 + \varepsilon) - f(z_0) - f'(z_0)\varepsilon}{\varepsilon} = 0. \tag{11}$$

Equivalently, there exists a remainder term $r(\varepsilon)$ with

$$f(z_0 + \varepsilon) = f(z_0) + f'(z_0)\varepsilon + r(\varepsilon), \qquad \frac{r(\varepsilon)}{\varepsilon} \to 0 \text{ as } \varepsilon \to 0. \tag{12}$$

Theorem A.1 (Local amplitwist decomposition)**.** *Let $f : \mathbb{C} \to \mathbb{C}$ be holomorphic at z_0 with $f'(z_0) \neq 0$. Then there exist unique numbers $\rho > 0$ and $\theta \in \mathbb{R}$ such that*

$$f'(z_0) = \rho e^{i\theta}, \tag{13}$$

and the first-order action of f near z_0 is a composition of a uniform scaling by ρ and a rotation by angle θ.

Proof. Write the derivative in polar form,

$$f'(z_0) = \rho e^{i\theta}, \quad \rho = |f'(z_0)| > 0, \quad \theta = \arg f'(z_0).$$

Substituting into (12) gives

$$f(z_0 + \varepsilon) = f(z_0) + \rho e^{i\theta}\varepsilon + r(\varepsilon).$$

Multiplication by $e^{i\theta}$ rotates every vector in the complex plane by angle θ, while multiplication by ρ scales every vector length by ρ. Since $r(\varepsilon) = o(\varepsilon)$, the remainder is negligible for sufficiently small ε. Hence the local action of f is, to first order, exactly a rotation followed by a uniform scaling. □

Corollary A.2 (Conformality)**.** *If f is holomorphic at z_0 and $f'(z_0) \neq 0$, then f preserves oriented angles at z_0.*

Proof. A rotation preserves all angles, and a uniform scaling preserves all angles. Their composition therefore preserves all angles. □

Real Jacobian form and the Cauchy–Riemann constraint

Write $z = x + iy$ and $f(z) = u(x, y) + iv(x, y)$. If f is holomorphic, the Cauchy–Riemann equations hold:

$$u_x = v_y, \qquad u_y = -v_x. \tag{14}$$

Hence the real Jacobian takes the special form

$$J_f(z_0) = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}, \qquad a = \Re f'(z_0), \qquad b = \Im f'(z_0). \tag{15}$$

Proposition A.3 (Similarity form of the holomorphic Jacobian)**.** *If f is holomorphic at z_0 and $f'(z_0) \neq 0$, then*

$$J_f(z_0)^T J_f(z_0) = |f'(z_0)|^2 I.$$

Therefore $J_f(z_0)$ is a similarity transformation: it preserves angles and scales all directions equally.

Proof. From (15),

$$J_f(z_0)^T J_f(z_0) = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} a & -b \\ b & a \end{pmatrix} = (a^2 + b^2)I = |f'(z_0)|^2 I.$$

□

General polar decomposition

Theorem A.4 (Polar decomposition)**.** *Let $A \in GL(n, \mathbb{R})$. Then there exist unique matrices $R \in O(n)$ and S symmetric positive definite such that*

$$A = RS, \qquad S = (A^T A)^{1/2}, \qquad R = AS^{-1}. \tag{16}$$

Proof. Since A is invertible, $A^T A$ is symmetric positive definite with unique positive square root $S = (A^T A)^{1/2}$. Define $R = AS^{-1}$. Then

$$R^T R = S^{-T} A^T A S^{-1} = S^{-1} S^2 S^{-1} = I,$$

so R is orthogonal. Uniqueness follows from the uniqueness of the positive square root of $A^T A$. □

Corollary A.5 (Amplitwist as conformal polar decomposition)**.** *If A is the real Jacobian of a holomorphic map at a point, then its polar factor $S = \rho I$ for some $\rho > 0$, and its orthogonal factor R is a planar rotation. Thus*

$$A = \underbrace{\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}}_{R} \underbrace{\begin{pmatrix} \rho & 0 \\ 0 & \rho \end{pmatrix}}_{S}. \tag{17}$$

So the amplitwist decomposition is exactly the isotropic, conformal, two-dimensional special case of polar decomposition.

Proof. From the proposition above, $A^T A = \rho^2 I$ for $\rho = |f'(z_0)|$. Therefore $S = (A^T A)^{1/2} = \rho I$. The orthogonal factor $R = AS^{-1}$ is then an orientation-preserving orthogonal matrix in two dimensions, hence a planar rotation. □

Interpretive summary

The general polar decomposition says that every invertible linear map can be written as an orthogonal part and a positive stretching part. The holomorphic case is special because the stretching part is isotropic rather than anisotropic: every direction is scaled equally. The derivative of a holomorphic map does not merely stretch and rotate; it stretches uniformly and rotates, which is why it preserves angles.

For a general real linear map A, the symmetric factor S may scale different directions by different amounts, introducing anisotropic stretching equivalent to a shear after composition with R. This is precisely what fails in the non-holomorphic case: the violation of the Cauchy–Riemann equations is equivalent to the introduction of anisotropy into the polar factor S. Conformality and isotropic scaling are equivalent conditions, both captured by the requirement that $A^T A = \rho^2 I$.

This is the mathematical sense in which the amplitwist acts as a prototype for the broader operator framework of the essay. Whenever a system can be modeled locally by operators of the form rotation $\times$ scaling, it inherits the same basic geometry as the derivative of a holomorphic map, with all the properties— conformality, universality of composition, and the compact parameterization by just two real numbers ρ and θ—that follow from it.

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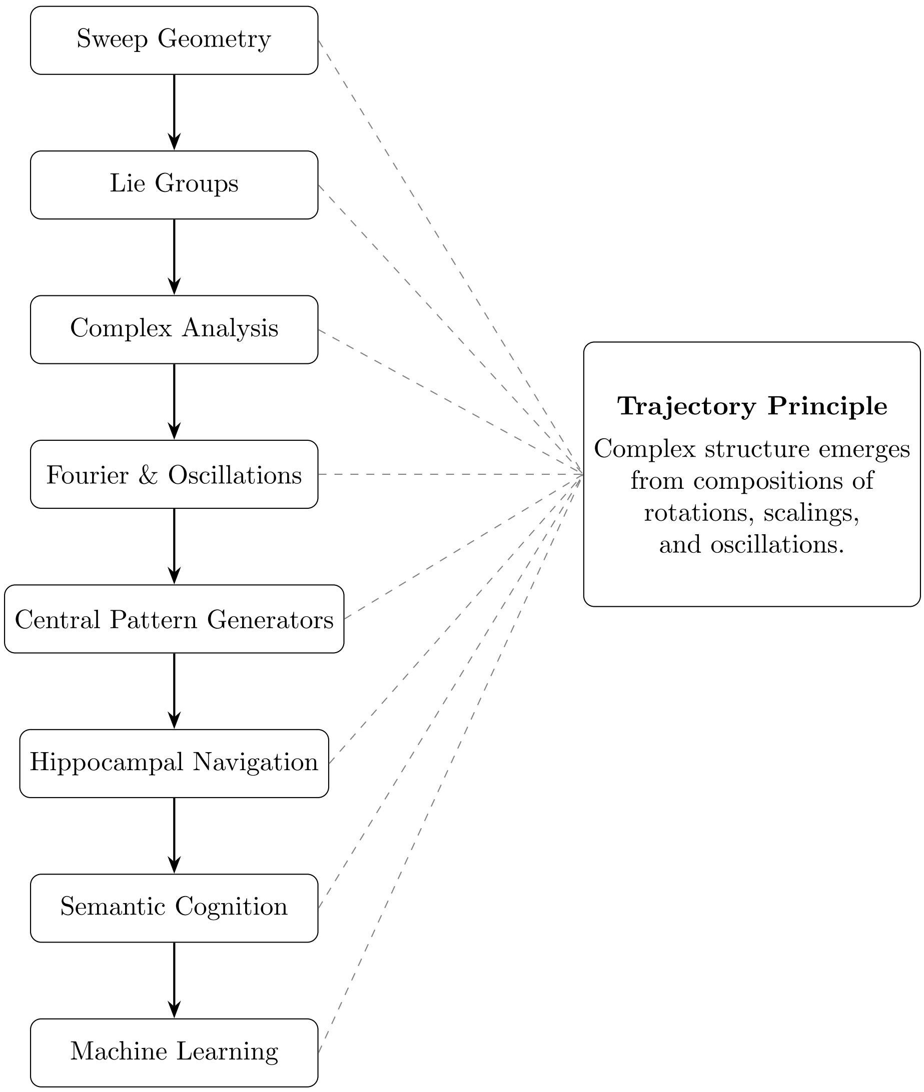

Figure 12: The trajectory stack. Each field independently discovers that complex structure emerges from compositions of simple geometric transformations. The dashed arrows indicate that all levels instantiate the same underlying trajectory principle.

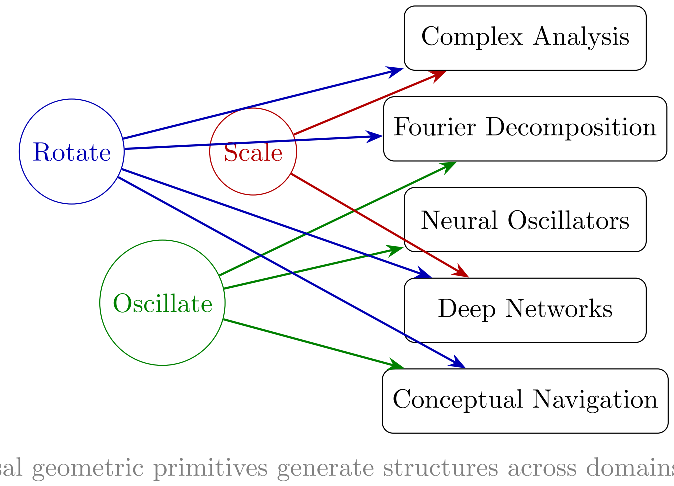

Figure 13: Rotation, scaling, and oscillation function as universal operators generating complex structures in mathematics, neural systems, and cognition. Each field draws on a different subset of the three primitives; together they cover all domains discussed in this essay.

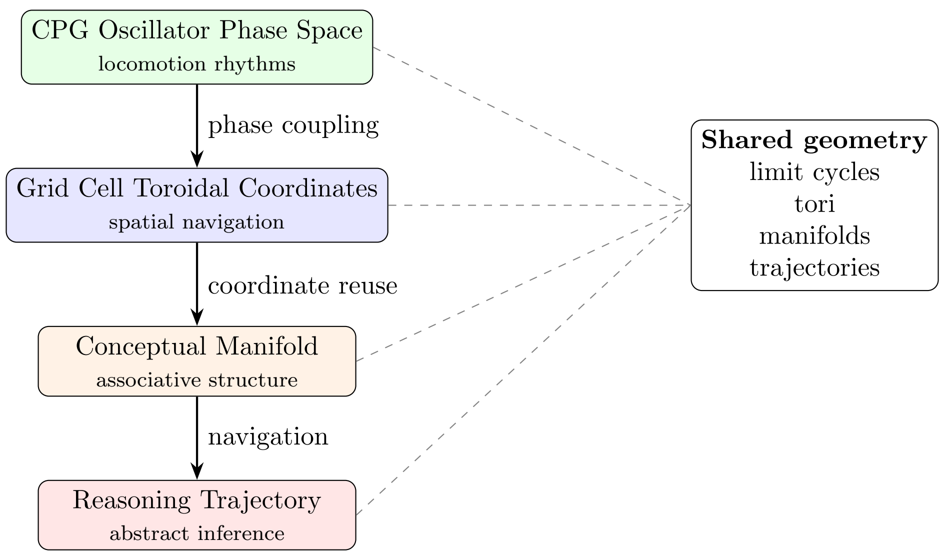

Figure 14: The cognitive phase-space hierarchy. The same phase-space geometry appears across evolutionary timescales and levels of organization: motor oscillators generate toroidal dynamics, grid cells reuse that structure for spatial coordinates, conceptual spaces inherit the same manifold geometry, and reasoning corresponds to trajectories through that manifold.

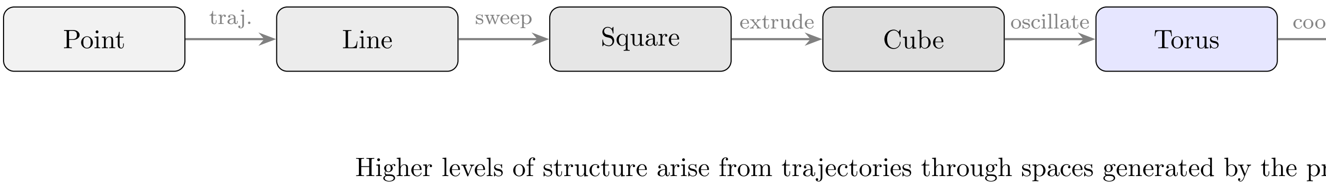

Figure 15: The trajectory ladder of dimension and cognition. Geometric objects arise from swept trajectories of lower-dimensional predecessors; conceptual and cognitive structures arise from trajectories through representational manifolds generated by neural dynamics. Each step adds a new independent direction of generative freedom.

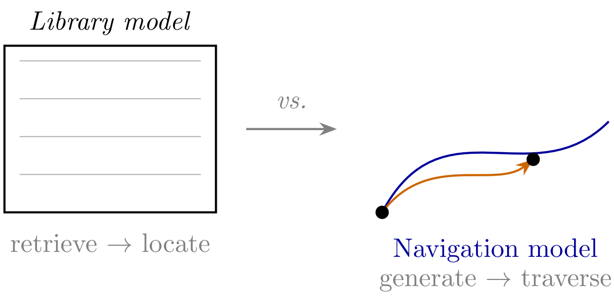

Figure 16: Knowledge as storage versus knowledge as navigation. The library model treats cognition as retrieval of fixed items from fixed locations. The navigation model treats cognition as the generation of trajectories through a structured terrain. The same points appear in both views, but only the navigation model captures the dynamical process by which they are produced.

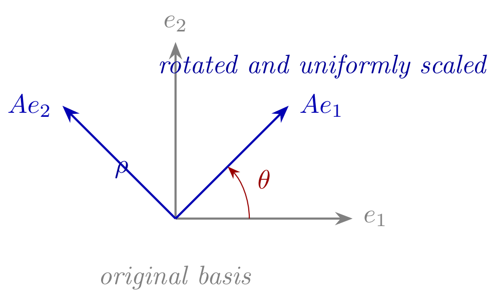

Figure 17: Local amplitwist action on the standard basis. A holomorphic derivative sends the orthogonal basis $\{e_1, e_2\}$ to another orthogonal basis, rotated by θ and uniformly scaled by ρ. No shear or anisotropic stretching occurs; this is the distinctive property of conformal maps.