Operator Ecology

Flyxion

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Abstract

Two cities, identical in every recorded particular, are shown to diverge not in state but in something a survey cannot record: the persistence of the operators — maintenance, training, governance, record-keeping — that keep each city able to renew itself. The essay develops a hierarchy of increasingly demanding notions of sameness between systems under damage and repair. State equivalence and structural equivalence are shown insufficient first; behavioral equivalence, in the sense of coalgebraic bisimulation, improves on them but is shown insufficient in turn, since it says nothing about a system once its current behavior has already been lost. *Regenerative equivalence* — agreement not in current state or behavior but in the capacity to recreate future behavior after damage — is proposed as the operative notion, and an operator ecology, following Alicia Juarrero's account of constraint-based causation, is defined as the constraint network responsible for maintaining that capacity rather than as a mere inventory of operators. The vocabulary this yields — keystone operators, extinction, ecological health — is then shown to already be latent in the finite characterization theorem of *Repair Categories and the Local Geometry of Continuation*: this essay adds no new theorem to that result, only the reading under which its reachability neighborhoods are niches, its repair operators are maintenance species, and its reconstructions are ecological restorations.

1 Two Cities

Consider two cities, alike in every particular that a survey could record. The same streets run at the same widths through the same neighborhoods. The same buildings stand at the same addresses, built to the same codes from the same materials. The same population occupies them, distributed across the same trades, the same households, the same schools. If a census were taken of either city on the day the comparison begins, the two censuses would not disagree.

In the first city, the ordinary work of maintenance continues. Roads are resurfaced before they crack open. Water and power lines are repaired on schedule rather than after failure. Apprentices are trained by masters who were themselves once apprentices, and the training is not occasional but structured, funded, and expected. Civil institutions are staffed, records are kept and consulted, and the offices of governance turn over their functions from one occupant to the next without the functions themselves lapsing. In the second city, this work stops. Not all at once, and not through any single catastrophe: the maintenance schedule quietly slips, the apprenticeship program loses funding and is not restarted, the records office is short-staffed and then unstaffed, the offices of governance persist in name after the people who understood how to run them have gone. No building falls down on the first day. No law is repealed. Nothing visible changes.

For some time, an inspection of either city would find them indistinguishable. The buildings are still standing in both. The streets are still there. A survey conducted one month after the divergence begins, or even one year after, would report two cities matching in essentially every recorded particular, because what has actually diverged has not yet left a mark on the state that a survey records. It has left a mark only on what each city is still capable of doing about the state it is in.

Ten years is enough. In the first city, the roads that needed resurfacing were resurfaced, the pipes that needed replacing were replaced, and a visitor notices nothing remarkable, because nothing remarkable has been allowed to happen: this is what successful maintenance looks like, which is to say it looks like nothing at all. In the second city, the same decade of ordinary wear has had nobody to answer it. Roads have cracked and stayed cracked. Failures that a trained crew would have caught early have gone uncaught and compounded. The office that once coordinated repairs no longer has anyone who remembers how the coordination worked, because the people who would have taught that knowledge to a successor were never funded to do so. The two cities, identical a decade earlier down to the address, are no longer close to identical in any sense that matters to the people living in them.

Nothing in this account requires an explanation more exotic than the one already given. The first city persisted because it retained the capacity to keep persisting; the second city changed because it lost that capacity while its visible state, for a time, gave no sign of the loss. The two cities did not begin to differ in their buildings. They began to differ in what each was still able to do about its buildings, and the buildings only followed later, as a consequence. Whatever is doing the explanatory work in this example, it is not captured by comparing the cities' states at any single moment, including the moment the comparison began. It has to be found somewhere else.

2 From Structures to Repairs to Ecologies

There is a natural first response to the two cities, and it is worth stating before setting it aside, because the essay's argument is a sequence of replacements for it rather than a rejection of it outright. The natural first response holds that structures persist, and that repair is simply one of the things that occasionally happens to them: a building stands, and now and again someone fixes it, and the building goes on standing as a result. On this view the building is the primary fact and the repair is incidental to it, a maintenance detail rather than an explanation.

The two cities already show this cannot be the whole account. For a decade, the buildings in both cities stood; the standing of the buildings was not what distinguished the cities, since both had it in equal measure for as long as the comparison remained close. What distinguished them was whether repair continued to occur, and the divergence in the buildings' actual condition arrived only afterward, as a downstream consequence of a difference that had already been present in each city's capacity to keep repairing them. This suggests a second view: not that structures persist and repair sometimes occurs, but that repair occurs, and structures persist because of it. On this view the building's continued existence is not primary but explained *by* the repair, and the reconstruction work already developed elsewhere in this program has largely operated at this second level, treating recoverability, diagnosis, and correction as the operative facts and the surviving structure as their trace.

This second view is a genuine improvement on the first, but the two cities press past it as well, if the comparison is pushed one step further back. It is true, and worth holding onto, that

City A's buildings persisted because they were repaired. But this only relocates the question: why did repair continue to occur in City A and not in City B? The answer given in Section 1 was not that City A simply repaired more, as though repair were a resource City A had more of in the same way it might have had more steel. The answer was that City A retained apprentices who could be trained into repairers, records that told the repairers what needed attention, and institutions that funded and coordinated the work; City B lost these before it lost anything one could point to and call a building. Repair did not occur in City A as a brute fact requiring no further account. It occurred because the conditions that produce repairers, and keep producing them, were themselves still intact. Those conditions are the actual subject of this essay, and it will call the entities that repair, train, coordinate, and record — and the network of relations among them that keeps each one able to function — an *operator ecology*.[1]

The three views are not competitors so much as successive depths of the same explanation. Structures persist because repair occurs; repair occurs because an operator ecology persists. Each step answers a question the previous step could state but not resolve, and each step is what the two cities were already demonstrating before either had a name. What remains is to say what an operator ecology actually is, in enough detail that the word is doing explanatory work rather than standing in for one.

3 Behavior Before State

A useful distinction, developed at length in the theory of computation and owed in the form used here to Erik Meijer's writing on algebras and coalgebras, separates two ways of specifying what a system is. An algebraic specification says how a structure is built: a list is specified by how it is constructed from an empty list and an element, a tree by how it is assembled from smaller trees. A coalgebraic specification says instead how a structure behaves once it exists: a stream is specified not by how it was built but by what it does when observed, namely produce a value and a continuation that will produce the next one. The algebraic question is where a structure came from. The coalgebraic question is what a structure will keep doing.

This distinction is not merely terminological, because it comes with its own notion of sameness. Two coalgebras are called *bisimilar* when no sequence of observations can tell them apart, regardless of how differently they are built or what internal state each happens to occupy at any given moment. A stream implemented as an infinite list and a stream implemented as a closure over a counter can be bisimilar: every observation of one is matched by an observation of the other, forever, even though nothing about their internal representations agrees. Bisimulation is, in this sense, a formal witness that behavioral sameness and state sameness are different relations, and that a theory built to track the first will not always agree with a theory built to track the second.

The two cities already made this same point without the vocabulary to state it precisely. For as long as the comparison in Section 1 remained close, the two cities agreed in state and had already begun to disagree in what they would go on to do: one was maintaining the conditions of its own continuation, and the other was not, and no inspection of either city's current buildings could have revealed this, because the disagreement lived entirely in future behavior rather than

[1] Stewart Brand's *How Buildings Learn* (Viking, 1994) makes a structurally similar argument about buildings specifically, distinguishing layers of change — site, structure, skin, services, space plan, and stuff — that persist at different rates. Brand's observation that buildings survive when their maintenance and adaptation pathways survive is a narrower, architecture-specific version of the claim this essay makes about cities in general.

present configuration. Bisimulation gives a name to exactly the kind of gap the two cities exhibit — state can agree while behavior has already diverged — and establishes, in a setting far more modest than a city, that this gap is not a peculiarity of the example but a structural fact about the relationship between what a system is and what it does.

It would be a mistake, however, to conclude too quickly from this that behavior is the invariant the two cities are really tracking. Bisimulation is a relation between systems as they currently stand, observed for as long as observation continues; it says nothing about what happens to that relation once one of the systems has been damaged and both current behavior and current state have been lost. That is precisely the situation the two cities are in by the end of their first decade, and it is the situation the next section takes up.

4 Beyond Behavior: Regenerative Equivalence

It is worth laying out, in order, the notions of sameness that have been in play so far, because each is strictly weaker than the last as a way of telling two systems apart, and the weakening is exactly the direction the two cities have been pushing throughout. *State equivalence* asks whether two systems currently agree in every recorded particular; this is what a census can settle, and it is what the two cities satisfied for most of a decade. *Structural equivalence* asks whether two systems are built the same way, which is a coarser question than state equivalence but still a question about composition rather than conduct. *Behavioral equivalence* — bisimulation, in the vocabulary of Section 3 — asks whether two systems do the same things under observation, regardless of how they are built or what state they occupy, and this is the notion that first let the two cities' divergence be stated honestly, since it was a divergence in what each city was doing, not yet in what either city was.

None of these three notions, however, survives what happens to the second city. By the end of the decade, City B's behavior has already changed along with its state: the roads have visibly cracked, the records office is visibly gone. Behavioral equivalence, which compares two systems as they currently stand, has nothing left to say about the two cities once one of them has already been damaged; it can report that they differ, but by then everyone touring either city could report the same thing without any theory at all. The question that actually matters by this point is not whether the cities currently behave alike. It is whether, having been damaged, each city retains what it would take to become itself again: whether the capacities that produced the original behavior, and would go on producing it, are still there to be drawn on.

Call two systems *regeneratively equivalent* if they possess sufficiently similar operator ecologies to recreate the same future capacities after damage, even once current behavior has diverged or been lost outright. This is not a refinement of behavioral equivalence in the way structural equivalence is a refinement of state equivalence; it is a genuinely different question, asked about a system after the fact of its damage rather than in the course of its ordinary operation. It is also, on the evidence of the two cities, the question whose answer actually predicts what happens next. Suppose both cities, ten years in, undergo a comparable reconstruction effort: outside help arrives, funding is made available, the physical damage is assessed and a rebuilding program begins. City A's apprentices, trained continuously through the decade, can be set to work immediately, and the records that were kept tell the rebuilding program what needs doing and how it was done before. City B has no apprentices to set to work and no records telling anyone what to rebuild toward; the funding can replace concrete and steel, but it cannot, by itself, replace the training pipeline, the institutional memory, or the coordinating offices that would put the concrete and steel to use. The same reconstruction budget produces a restored city in one case and a much slower, much more improvised recovery in the other, and the difference is not explained by anything the reconstruction budget itself measures.

This suggests the asymmetry that motivates the rest of the essay, stated carefully rather than as a bare slogan. It is not that structures can never help regenerate operators — a surviving schoolhouse can still house a new apprenticeship program, and a recovered archive can still train a new records office, so the regeneration is not strictly impossible. The asymmetry is that an intact operator ecology regenerates structure as a matter of its ordinary function, the way City A's maintenance crews resurfaced roads without anyone regarding this as remarkable, whereas an intact structure regenerates its operator ecology only contingently, and typically requires that some other operator ecology — a funding agency, a training institution imported from elsewhere, a government capable of chartering new offices — do the work of regenerating it from outside. Operators reproduce structure as a matter of course. Structures reproduce operators only when something else, itself already an operator ecology, intervenes to make it happen. That difference in reliability, not a categorical impossibility in either direction, is what regenerative equivalence is built to track, and it is the reason the two cities' first decade, which looked for so long like a story about buildings, was a story about something else from the beginning.

The claim compresses into a short definition, given here once so that later sections can refer back to it rather than restate it in prose. Write $\Gamma(\mathcal{E})$ for the set of future operator repertoires that an operator ecology $\mathcal{E}$ can recover under admissible damage — the repertoires reachable not by continuing to operate undisturbed, but by repairing and rebuilding after some part of $\mathcal{E}$ has been lost. Two ecologies $\mathcal{E}_1, \mathcal{E}_2$ are *regeneratively equivalent*, written $\mathcal{E}_1 \sim_{\mathrm{R}} \mathcal{E}_2$, when $\Gamma(\mathcal{E}_1) = \Gamma(\mathcal{E}_2)$: when they can recover exactly the same futures, regardless of whether their current operators, or even their current behavior, agree at all. City A and City B were, for most of a decade, close to state-equivalent and briefly behavior-divergent; what Section 1 was tracking without the vocabulary for it was that they had already ceased to be regeneratively equivalent long before either fact became visible.

5 Ecologies as Constraint Networks

Section 2 named an operator ecology but did not yet say what holds one together, and without an answer the word risks meaning nothing more than a list: the apprentices, the record-keepers, the maintenance crews, the offices of governance, tallied up as though counting them were the same as understanding why they persist together or fail together. Alicia Juarrero's account of constraint-based causation, developed in *Dynamics in Action*, supplies the missing piece. Juarrero distinguishes causes that push — one event directly forcing the next, as one billiard ball forces another to move — from causes that constrain, which act not by forcing any particular outcome but by narrowing the space of outcomes that remain possible. A riverbank does not push the water anywhere; it constrains where the water can go, and the water's path is explained by the constraint quite as much as by whatever force is moving it downstream.

An operator ecology, on this view, is not usefully described as a set of operators at all.

> *The ecology is not the operators; it is the network of enabling constraints among them.*

It is a network of constraints that determines which operators remain able to function as other operators are gained or lost. The apprenticeship system in City A is not an isolated operator that happens to sit next to the others; it is constrained by, and constrains in turn, the funding that supports it, the master craftsmen who staff it, the records that tell it what skills are needed, and the offices of governance that charter its existence. Remove the funding and the apprenticeship system does not merely weaken — it can disappear entirely, not because anyone touched the apprentices or the masters directly, but because a constraint that had been holding the possibility of apprenticeship open was withdrawn. This is the pattern that the slow decline of City B actually followed: no operator was destroyed in the way a building is destroyed by demolition. The conditions that made each operator executable were withdrawn one at a time, and the operators lapsed as a consequence, in an order that had more to do with which constraints were most load-bearing than with which operators were individually most valuable.

This gives a working definition worth stating formally even though the essay around it remains expository. An operator ecology is a constraint network that maintains the conditions under which operators remain executable. The ecology is not the operators; it is the network of enabling constraints among them, and an intervention that touches none of the operators directly can still eliminate any number of them by removing a constraint on which their continued function depended. This is the sense in which City B's decline was already, correctly, described in Section 1 as beginning nowhere in particular: it began wherever the first constraint gave way, and the operators it had been sustaining lapsed afterward, sometimes long afterward, in a sequence that traced the constraint network rather than any single point of failure within it.

Definition 5.1 (Operator ecology)**.** *An* operator ecology *is a network of operators together with the constraints among them that determine which operators remain executable as others are gained or lost.*

The same definition compresses into notation without changing its content, and the notation is worth fixing once so that Sections 6 and 7 can use it without re-deriving it. Write $\mathcal{E} = (\mathcal{O}, \mathsf{C}, \mathsf{X})$ for an operator ecology, where $\mathcal{O}$ is the set of operators, $\mathsf{C} \subseteq \mathcal{O} \times \mathcal{O}$ is the constraint graph — $(o_i, o_j) \in \mathsf{C}$ meaning that o_i helps maintain the conditions under which o_j remains executable, the formal shape of the apprenticeship-maintains-maintenance relation already described above — and $\mathsf{X} : \mathcal{O} \to \{0, 1\}$ is the executability predicate, $\mathsf{X}(o) = 1$ just when o is currently executable in $\mathcal{E}$. An operator can be described while extinct: $o \in \mathcal{O}$ while $\mathsf{X}(o) = 0$, a charter surviving in an archive for an apprenticeship program nobody can any longer run. Nothing here adds to the prose definition above; it only gives later sections a place to point.

6 Keystone Operators and Ecological Health

The constraint-network definition earns its keep by making a further distinction available, one that the two cities already illustrate but that the language of Section 1 was not yet equipped to state. Not every operator matters equally to the network that contains it, and the operators that matter most are not necessarily the ones that are used most often. Call an operator a *keystone* of the ecology it belongs to if its removal collapses a disproportionate share of the remaining network — if, that is, a large number of other operators depend on constraints that this one operator was helping to hold in place, so that its loss propagates far beyond its own function. Apprenticeship is plausibly a keystone operator in the two cities: it is not exercised constantly, in the way that road maintenance is exercised constantly, but its loss removes the mechanism by which every other operator renews itself across a generation, so that its absence is not felt immediately but becomes, over a decade, the single largest determinant of which other operators survive. Literacy, standards, archives, and the offices of governance plausibly play the same role for different reasons: none is exercised with unusual frequency, and each supports a disproportionate share of what the rest of the ecology depends on.

This is a familiar pattern from ecology in the narrower biological sense, where a keystone species is likewise not distinguished by abundance. An apex predator is typically rare within the system it organizes, and its removal is felt not because the predator itself was doing much of the system's work directly, but because its presence had been constraining the population dynamics of everything beneath it. The analogy is not decorative here, because the underlying structure is the same one Juarrero's account already supplies: a keystone operator, like a keystone species, matters because of the constraints it maintains on the rest of the network, not because of how frequently it is itself invoked.

The same vocabulary extends naturally to two further notions that the essay has been circling since Section 4 without naming. An operator is *extinct* within an ecology when no executable realization of it remains, regardless of whether a description of it survives: City B's apprenticeship program can be extinct in this sense while a charter document describing it still sits, accurately and uselessly, in an archive. And the *health* of an operator ecology can be understood, at least informally, as the breadth and resilience of the operator repertoire it can still support under plausible future damage — not how much is currently running, but how much could still be regenerated if some part of what is currently running were lost. Writing $\mathsf{H}(\mathcal{E}) = \mu(\Gamma(\mathcal{E}))$ for some measure μ on the recoverable-futures set of Section 4 gives this a first formal shape: extinction is exactly what decreases H, restoration is what increases it, and keystone loss is what causes it to drop sharply rather than gradually, three claims already argued in prose above, now with a single quantity underneath all three. A fuller formal treatment belongs to the reconstruction machinery already developed elsewhere in this program, and is deferred to the next section rather than attempted here. Volume is used here only as a first proxy; later work may weight futures by repair cost, robustness, or admissibility rather than treating every recoverable future as equally valuable.

The keystone notion itself, though, is worth one compact definition, since it is the cleanest of the three to state. Write $\mathsf{R}(\mathcal{E})$ for the reachable executable repertoire of an ecology $\mathcal{E}$ — the operators actually executable given $\mathcal{E}$'s current constraint graph — and define the *impact* of an operator o as

$$K(o) \;=\; |\mathsf{R}(\mathcal{E})| \;-\; |\mathsf{R}(\mathcal{E} \setminus \{o\})|,$$

the size of the repertoire lost by removing o and everything its removal disconnects. A keystone operator is simply one with unusually large $K(o)$ relative to how often it is itself invoked; apprenticeship in the two cities is exactly this, an operator with low invocation frequency and disproportionate $K(o)$, since its removal disconnects every operator that depended on the next generation of practitioners it would otherwise have supplied.

7 What the Formalism Already Knew

None of the vocabulary introduced in this essay is, in the end, new to the program it belongs to. It has been developed here expositorily, built up from the two cities rather than derived from an existing theorem, but each notion has a direct counterpart already proved about in the finite theory of *Repair Categories and the Local Geometry of Continuation*, and naming the correspondence is the purpose of this section.

The forward and backward reachability neighborhoods of that paper — the sets of states a given state can reach, or be reached from, at finite executable cost — are what this essay has been calling niches: the region of an operator ecology that a given operator, or a given state within it, actually has access to. A repair operator, in the vocabulary of that paper, is exactly a maintenance species in the vocabulary of this one, an entity whose presence in the ecology constitutes the executable path between two states. Reconstruction, which that paper characterizes as the recovery of bidirected executable cost rather than the recovery of state identity, is restoration in an operator ecology by another name: what a reconstruction restores is not the original state but the operator repertoire capable of producing the relevant futures again, which is precisely the claim City A's successful rebuilding and City B's failed one were illustrating in Section 4. Loss of executable support — a state becoming unreachable at any finite cost — is extinction. And an admissibility volume, the size of the neighborhood of states and operators still reachable from where a system currently stands, is a first formal candidate for the ecological health gestured at informally in Section 6.

None of this changes anything about what was already proved. The theorem of *Repair Categories* says what it said before this essay was written, and nothing here adds a new mathematical claim to it. What changes is the reading available to someone who has been through the two cities first: a reader who arrives at that paper's finite theory already primed to see reachability as niche, repair as maintenance, and reconstruction as restoration will recognize the formalism as an answer to a question they already have, rather than as machinery whose motivation must be taken on faith. That is the only claim this section makes, and it is, deliberately, a modest one: the calculation was already available; what this essay supplies is the understanding of what the calculation means.

8 Structures as Frozen Operator Ecologies

The two cities began as a comparison of buildings and ended as a comparison of something that never appeared in either city's survey. State equivalence could not track it, because for most of a decade the two cities agreed in state while already diverging in what mattered. Behavioral equivalence came closer, and bisimulation gave a precise name to the kind of gap the cities exhibited, but even behavioral equivalence had nothing left to say once the damage had already occurred and the question became whether either city could still become itself again. What actually explained the two cities, from the first quiet lapse in City B's maintenance schedule to the divergent outcomes of their eventual reconstructions, was the condition of their operator ecologies: the constraint networks that determined which operators — training, repair, governance, record-keeping — remained executable as circumstances changed. Structures persist because repair occurs; repair occurs because an operator ecology persists; and an operator ecology persists because its constraints continue to hold each other in place. A structure appears static only because the operators that produced it have succeeded in making their activity invisible: City A's roads looked unremarkable for a decade precisely because the maintenance behind them was working. Put as plainly as the claim allows: structures are frozen operator ecologies, visible residues of a network of constraints that, for as long as it holds, keeps producing them, and that, once it fails, leaves the structures standing for a while with nothing left to renew them.

This essay's argument does not depend on the intellectual company it keeps, but the company is worth naming briefly, since none of it was arrived at independently. Spencer-Brown's insistence that a distinction must be drawn before anything can be said to exist, and Bateson's reformulation of information as a difference that makes a difference, already anticipate the move from objects to relations that this essay extends one step further, from relations to the constraints that maintain them; Bateson's phrase adapts with almost no strain into an operator that makes operators possible, which is one more way of saying keystone. Juarrero supplies the mechanism by which constraints, rather than objects, do the explanatory work, and without her account the operator ecology of Section 5 would have remained a metaphor rather than becoming the network it is here. Meijer's coalgebras and the fact of bisimulation established, in a far more modest setting than a city, that behavior can be the right invariant to track when state is not, which is the wedge this essay needed before it could argue that even behavior falls short. None of these figures is the essay's subject. The subject is the two cities, and everything else here exists to explain why they diverged the way they did.

References

[1] Alicia Juarrero, *Dynamics in Action: Intentional Behavior as a Complex System* (MIT Press, 1999).

[2] Erik Meijer, on the algebra/coalgebra distinction and process-first computation in programming language theory; see his writing and talks on coalgebras, bisimulation, and reactive programming.

[3] Gregory Bateson, *Steps to an Ecology of Mind* (Chandler Publishing, 1972).