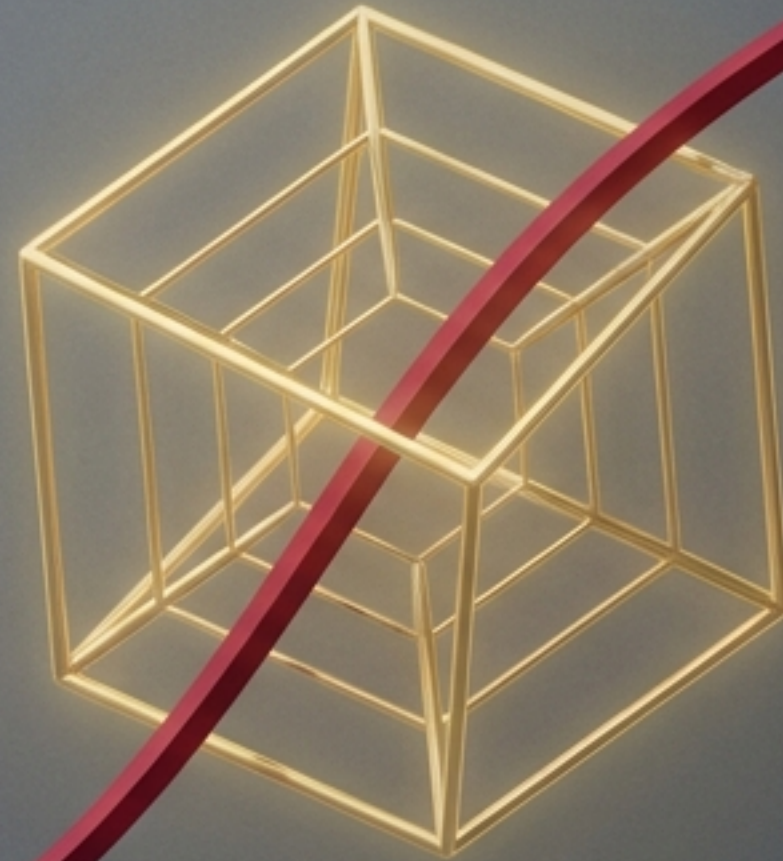


# Constraint, Projection, and Reachability

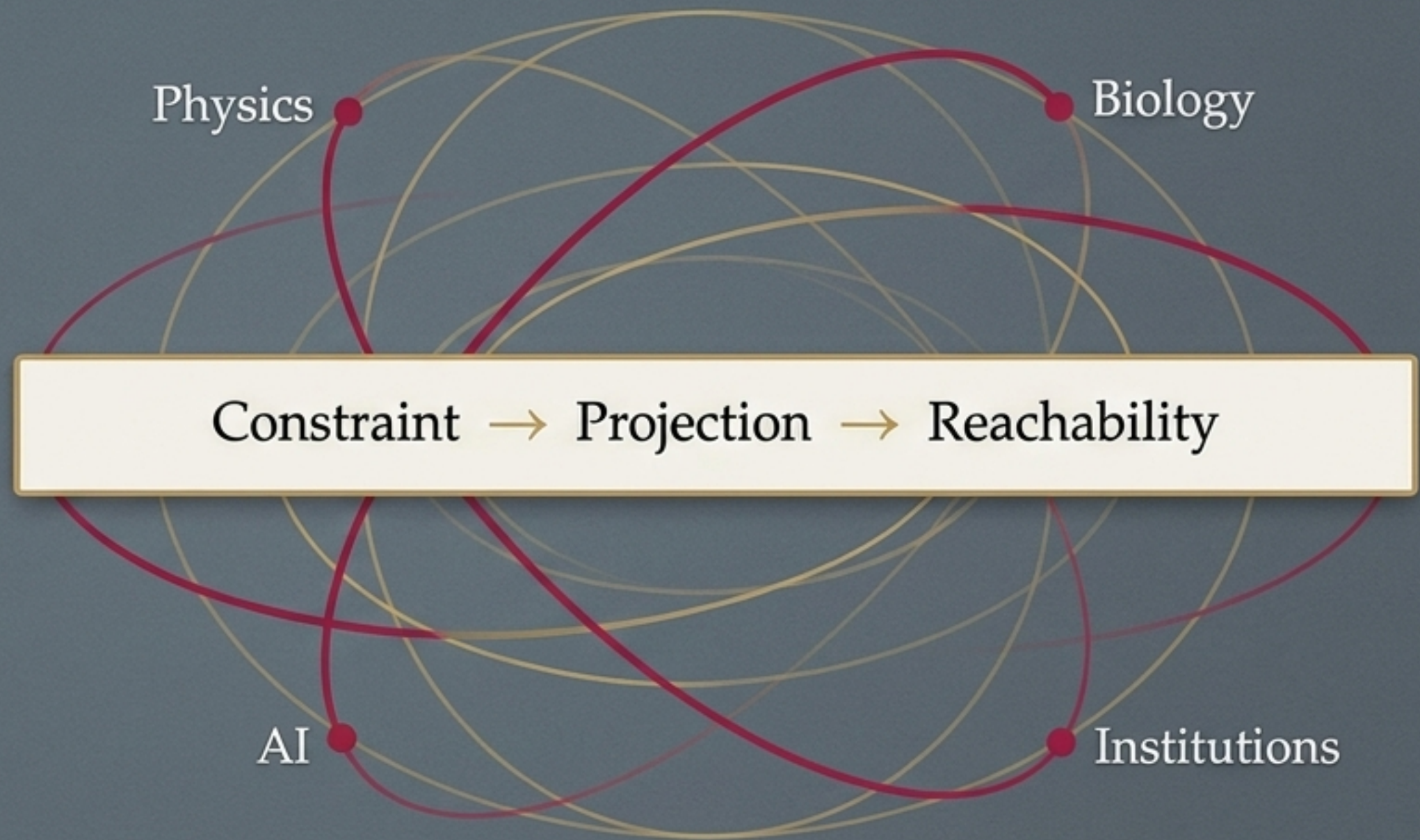
A Geometric Theory of Cognition,  
Meaning, and Complex Systems.



Based on the 2026 mathematical  
monograph by Flyxion.

# The Geometry of Meaning

Reachability geometry is the common structure underlying cognition, meaning, computation, biological organization, social institutions, and cosmological dynamics.

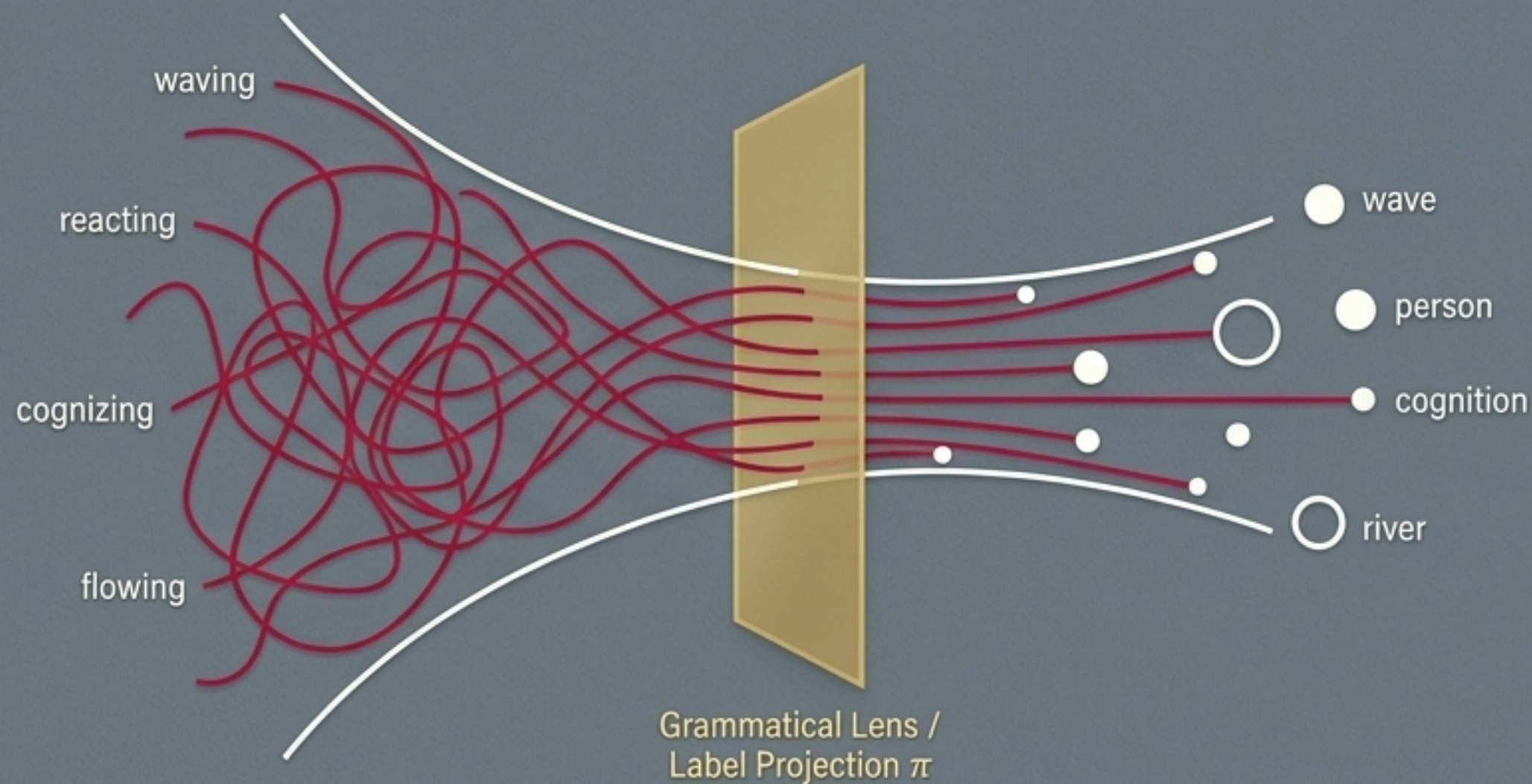


*A theory is the more impressive the greater the simplicity of its premises, the more different kinds of things it relates, and the more extended its area of applicability. — Albert Einstein*

# The Foundational Error

Language is a machine that generates ontology. We mistake the projected label for the fundamental reality.

## The Noun Fallacy Funnel

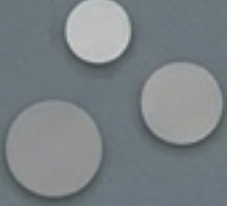


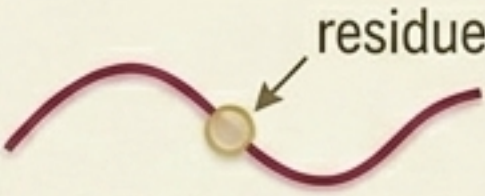

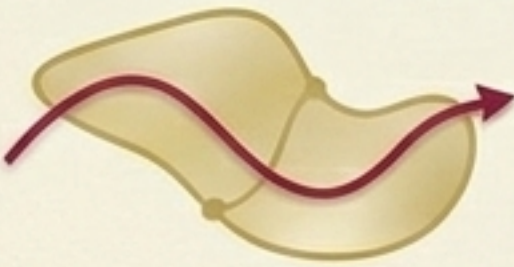

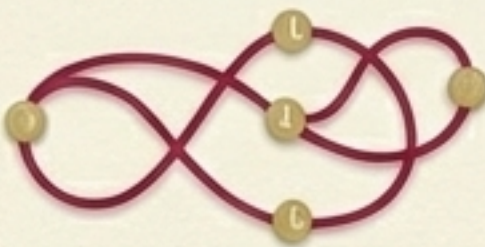


## The Noun Fallacy:

Treating objects as ontologically primary is a cognitive convenience, a residue of perceptual systems tracking persistent entities.

**Theorem 3.1 (Noun Fallacy).** *Let  $\gamma_1, \gamma_2 \in \Gamma$  be distinct trajectories ( $\gamma_1 \neq \gamma_2$ ) and let  $\pi : \Gamma \rightarrow \mathcal{O}$  be a label projection such that  $\pi(\gamma_1) = \pi(\gamma_2) = o$ . Let  $f : \Gamma \rightarrow \mathbb{R}$  be a causal functional (i.e., one that depends on the full trajectory, not just its label). Then the projected functional  $\hat{f} : \mathcal{O} \rightarrow \mathbb{R}$ , defined by  $\hat{f}(o) = f(\gamma)$  for any  $\gamma \in \pi^{-1}(o)$ , is not well-defined in general:  $f(\gamma_1) \neq f(\gamma_2)$  while  $\pi(\gamma_1) = \pi(\gamma_2)$ . Consequently, any causal reasoning performed at the label level  $\mathcal{O}$  is subject to systematic indeterminacy.*

# Trajectories Over Objects

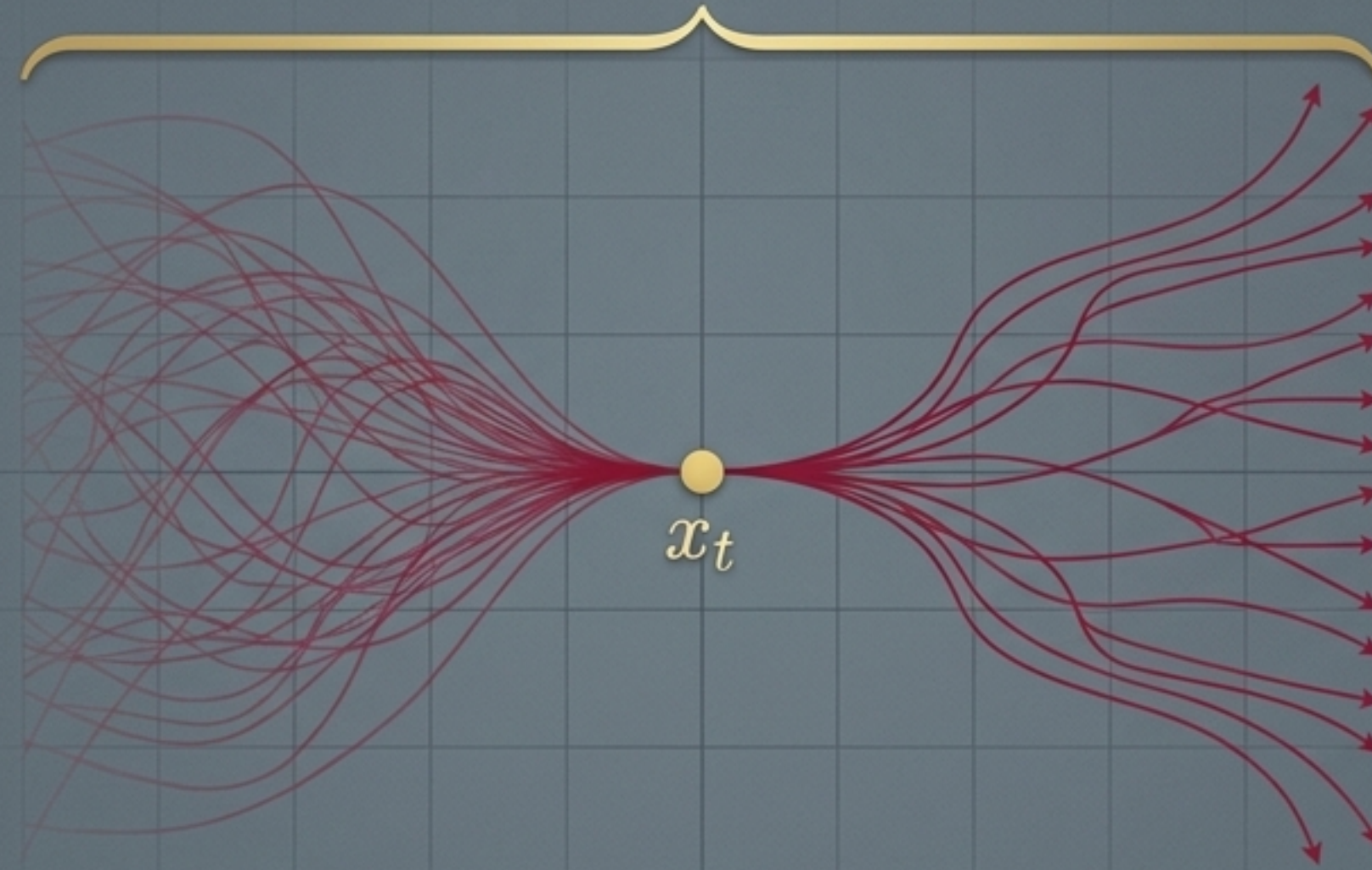
|                 | Standard Picture  | CPR Picture   |
|-----------------|---|---|
| Ontology        |  Objects are primary                                     |  Trajectories are primary  |
| State / Reality |  Instantaneous states dictate reality                   |  States are mere projections (residues) of stable trajectories       |
| Structure       |  Classical science builds objects then constrains them |  Constraint fields exist first; objects are what the field permits |
| Time            |  A container in which events occur                     |  The structure of event-succession itself                          |

*There is no escape from the primacy of process. — A.N. Whitehead*

# The Instantaneous State Illusion

State recovers from trajectory; trajectory does not recover from state.

$$E_t(\gamma) = \gamma(t)$$



## The Problem:

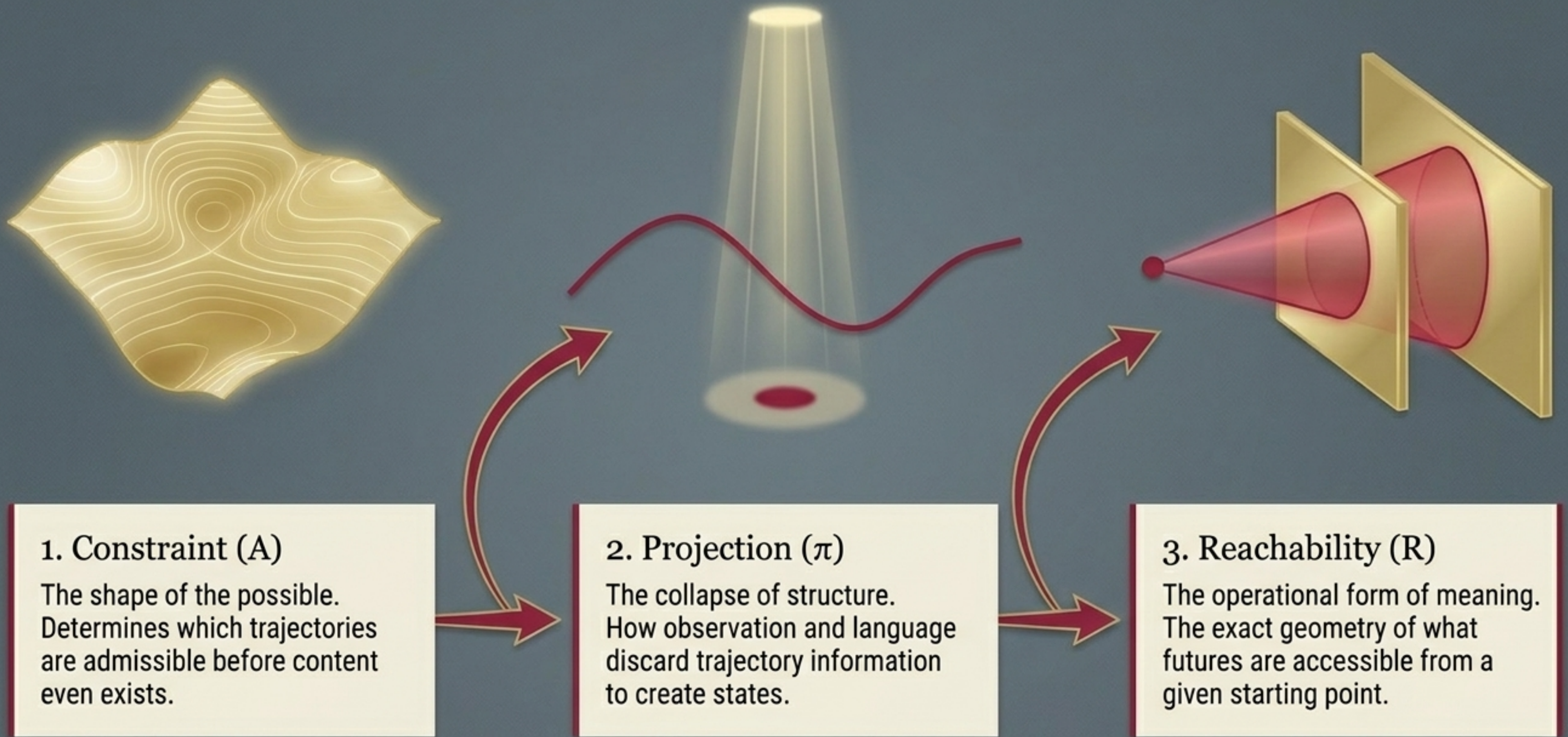
The instantaneous evaluation map  $E_t$  is non-injective (many-to-one). A state tells you where a system is, but destroys information about how it got there and its higher-order structure.

## The Consequence:

To recover the trajectory from a state, you must know the constraint field governing it.

**Proposition 2.1 (Trajectory Primacy).** *The instantaneous evaluation map  $E_t$  is non-injective (many-to-one). Specifically, the preimage  $E_t^{-1}(x) = \{\gamma \in \Gamma : \gamma(t) = x\}$  contains uncountably many trajectories with distinct past velocities and higher-order derivatives at every  $x \in X$ .*

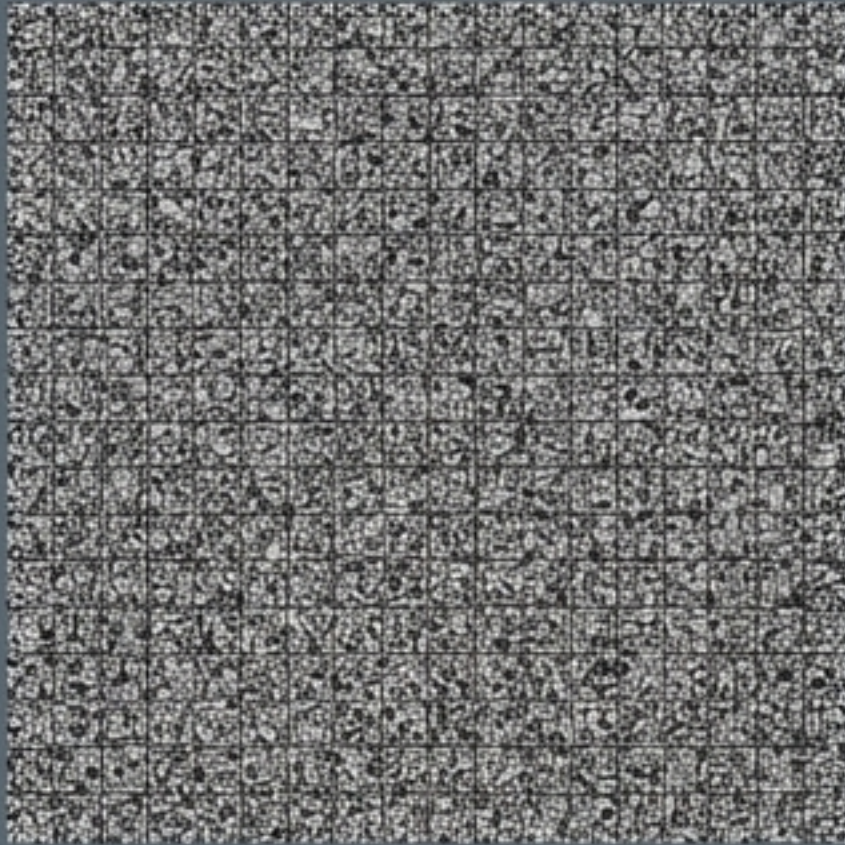
# The Architecture of the Possible



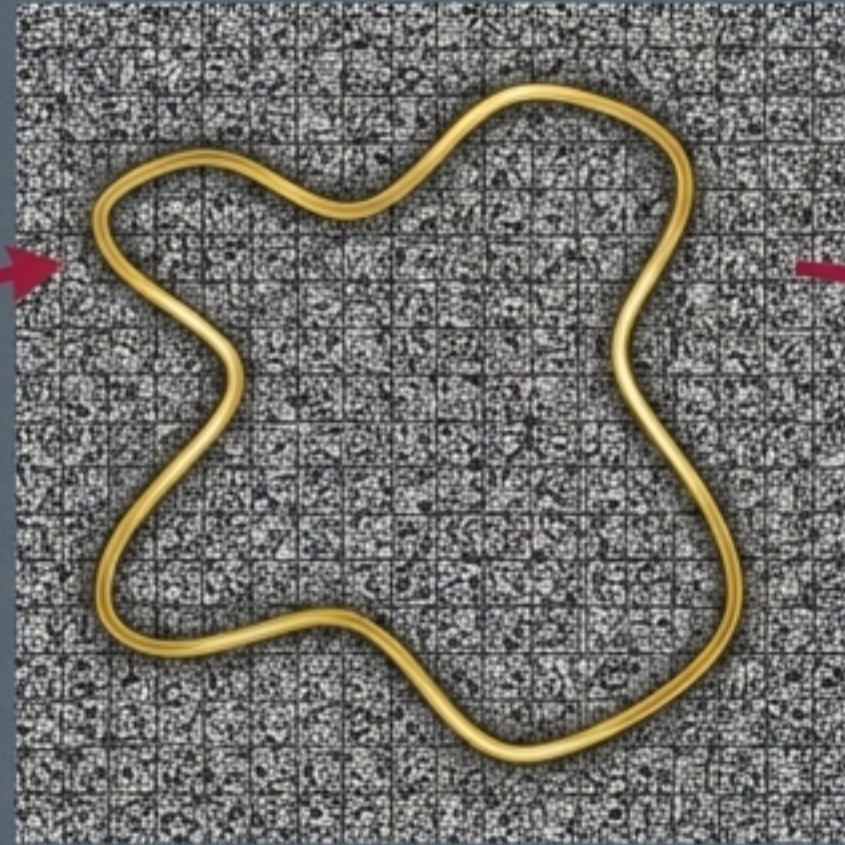
# Pillar 1: Constraints Before Contents

**Lemma 4.1 (Constraint Priority).** *Let  $\mu$  be a measure on  $U$ . The geometric and topological structure of the content space  $X = X(A)$  — including its boundary  $\partial X$ , its interior  $\text{int}(X)$ , its connected components, and its induced measure — is determined entirely by the spatial variation of  $A$ . No content can exist without a prior constraint field.*

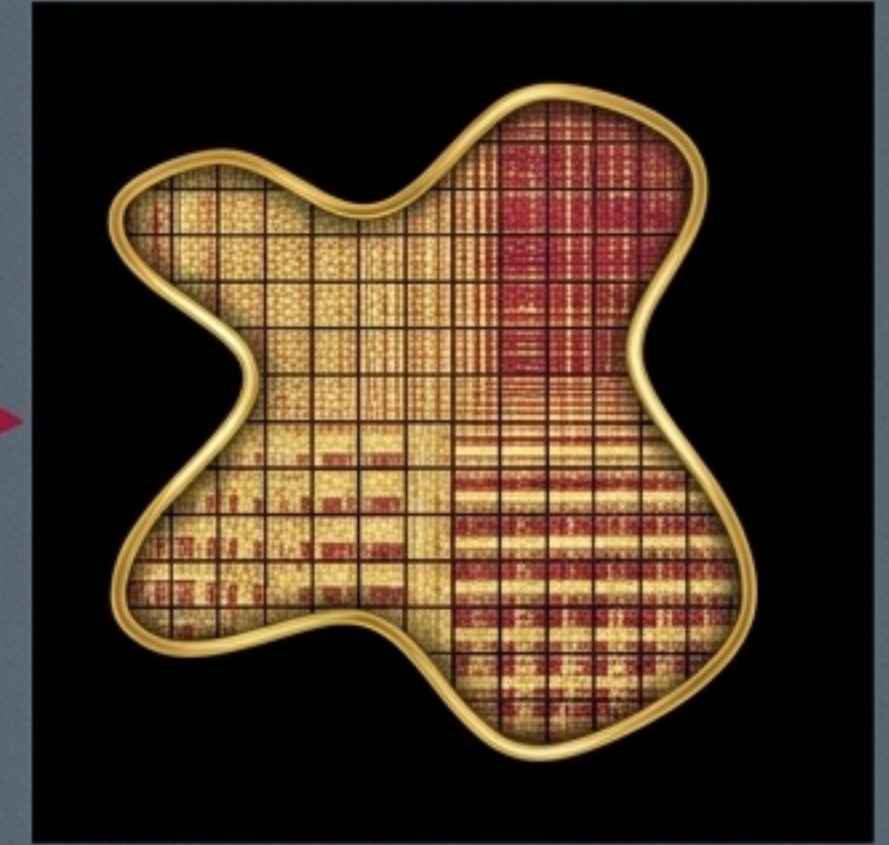
Pre-Semantic Space  $U$



Admissibility Field  $A$

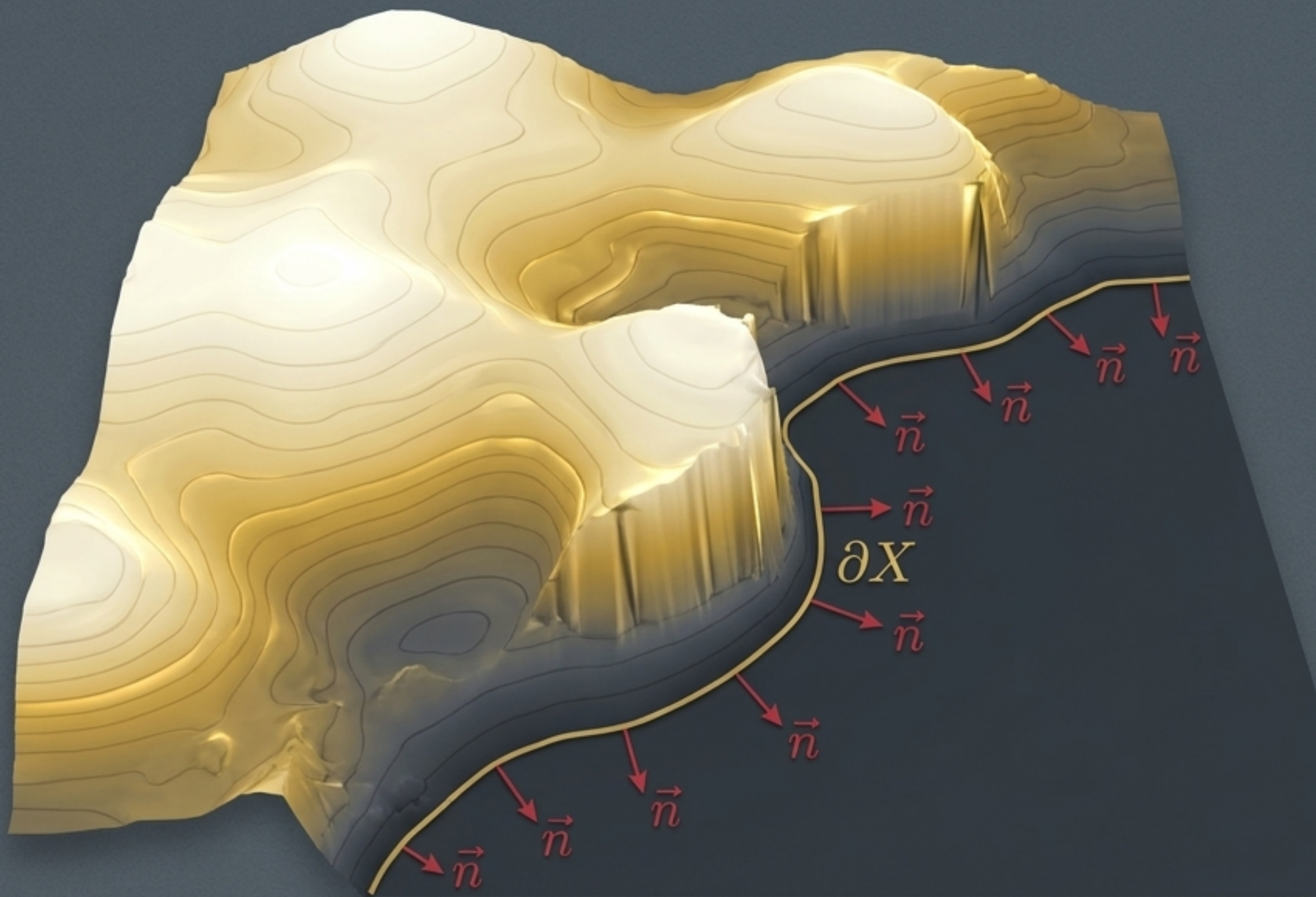


Content Space  $X$



No content can exist without a prior constraint field. The content space  $X$  is not fundamental; it is entirely determined by the spatial variation of the admissibility field  $A$ .

# The Geometry of Admissibility



## Graded Admissibility $\Phi(\mathbf{x})$

Possibility is not a binary switch, but a scalar capacity field measuring the degree of admissibility.

## The Level-Set Equation

The boundary separating viable from non-viable regions is the superlevel set

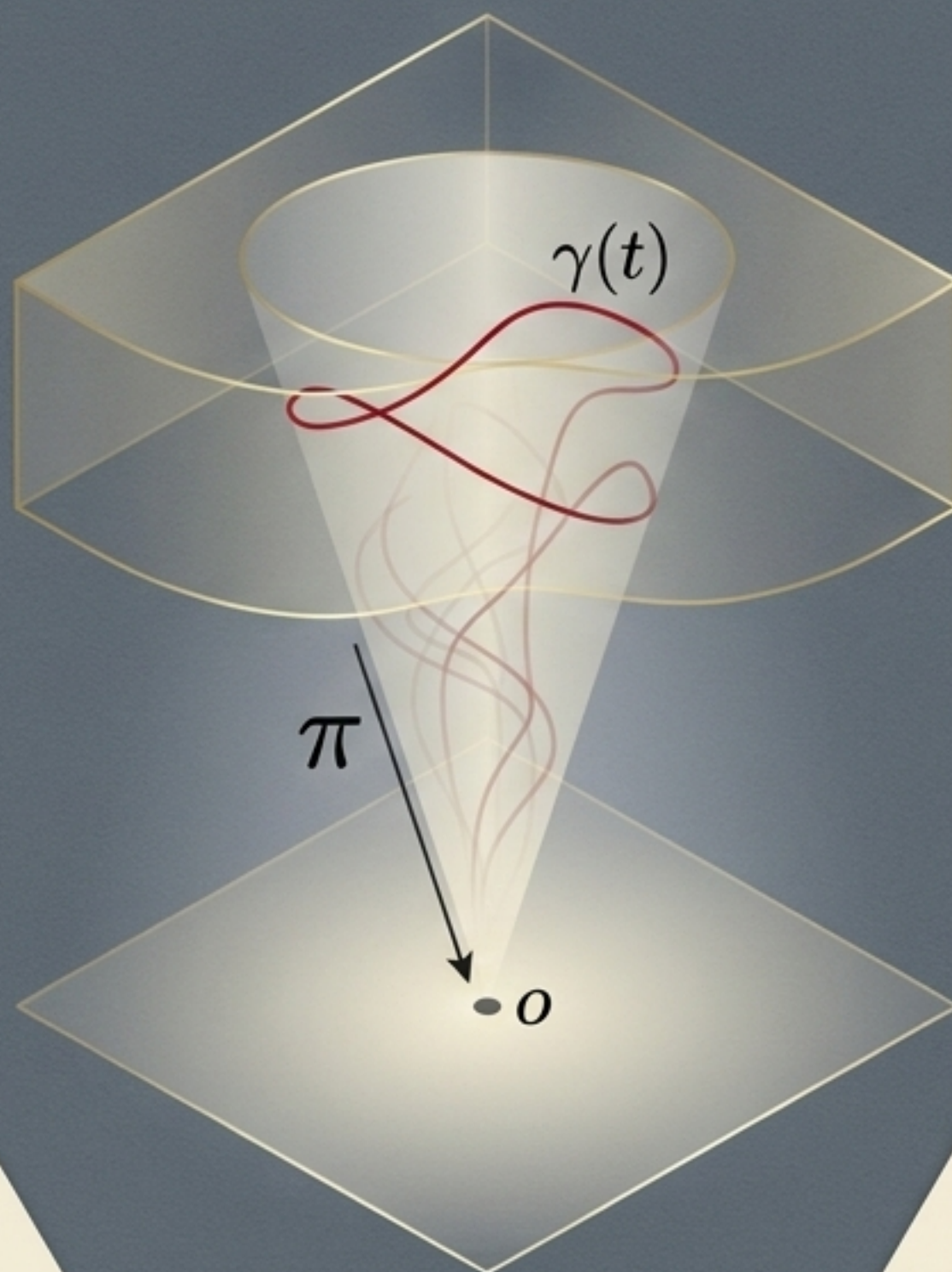
$$\partial X_\theta = \{\mathbf{x} : \Phi(\mathbf{x}) = \theta\}.$$

## The Motion of the Boundary

Determines evolution, morphological development, and thermodynamic flow.

## Pillar 2: Projection & Collapse

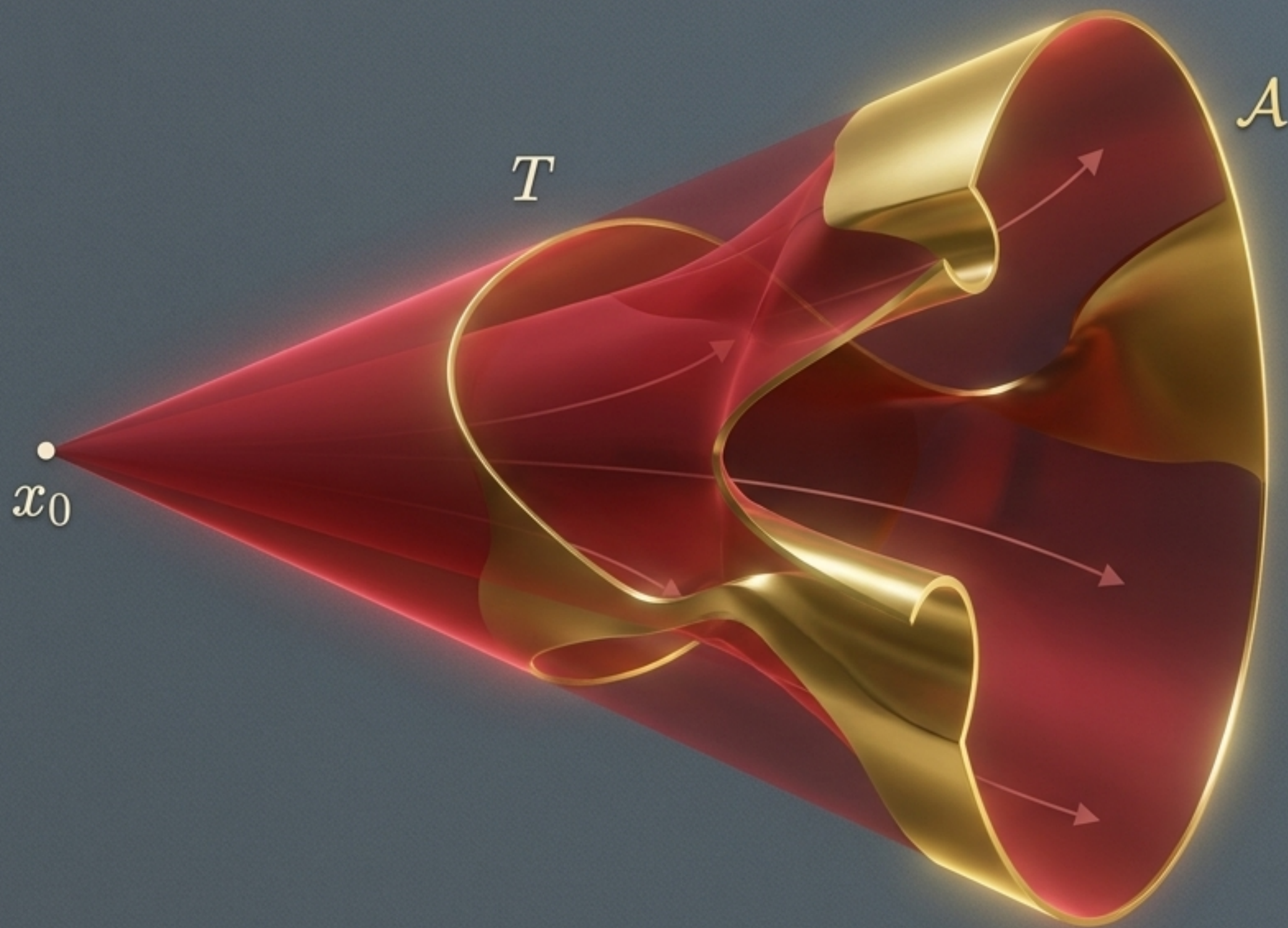
**Projection Collapse:** Distinct causal histories map to the same latent point. Causal distinguishability is lost.



**The Fiber  $\pi^{-1}(o)$ :** The set containing all trajectories that receive the same label.

**The Danger:** When distinct biological, physical, or semantic processes are projected into the same state, they become indistinguishable to the observer.

# Pillar 3: Reachability as Primitive



**Possibility is not an abstract predicate.** It is a geometric relationship between a starting point and the region accessible from it.

## Definitions Box

The Reachable Set  $R_A(x_0, T)$

The exact geometric volume of futures accessible from a given state, given the constraint system.

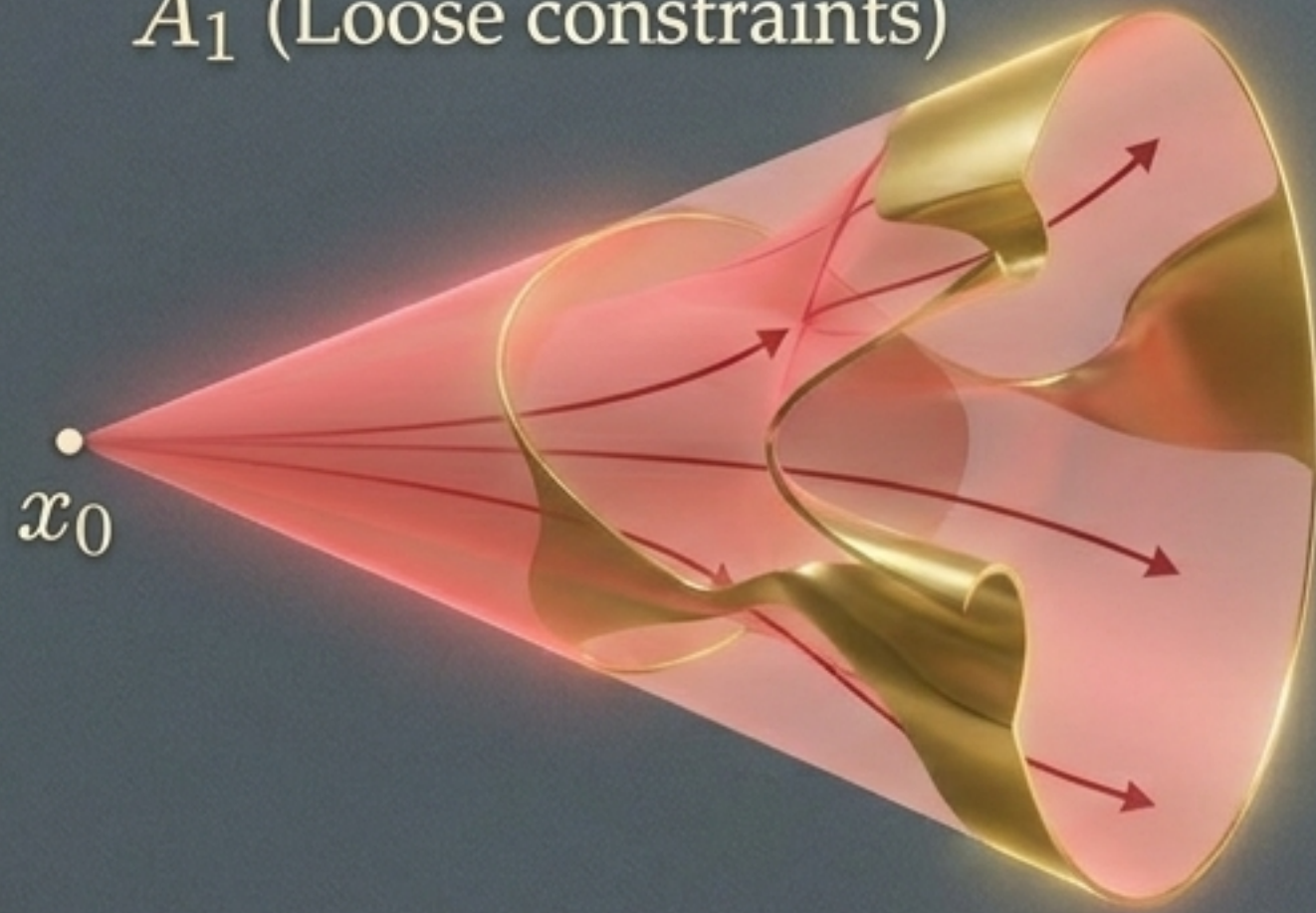
## The Illusion

You never see your own counterfactuals. Reachability is mostly invisible from the inside.

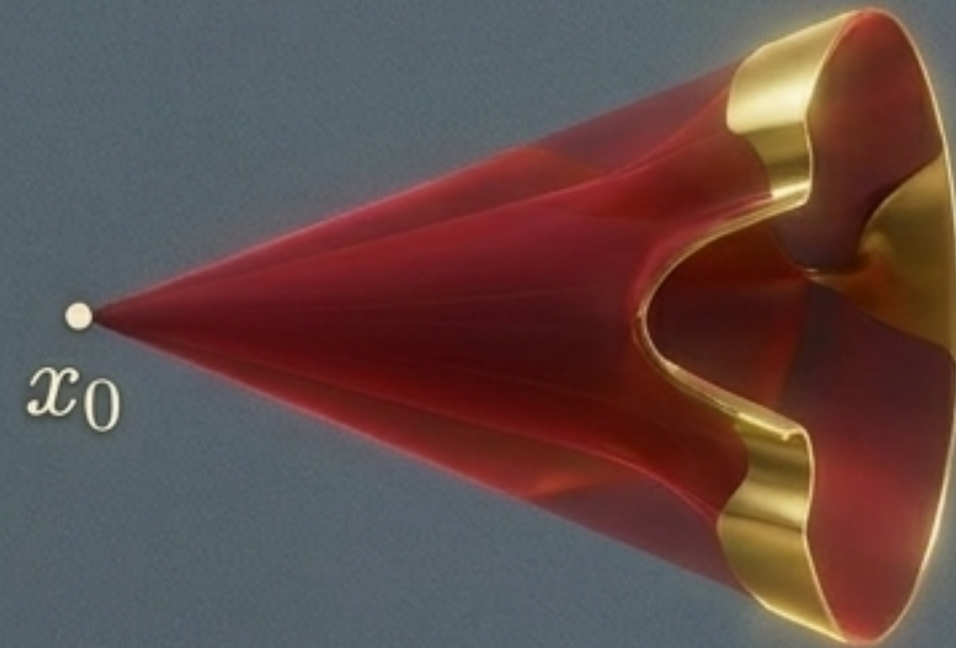
# The Reachability Monotonicity Theorem

$$A_1 \subseteq A_2 \Rightarrow R_{A_1}(x_0, T) \subseteq R_{A_2}(x_0, T)$$

$A_1$  (Loose constraints)



$A_2$  (Tight constraints)

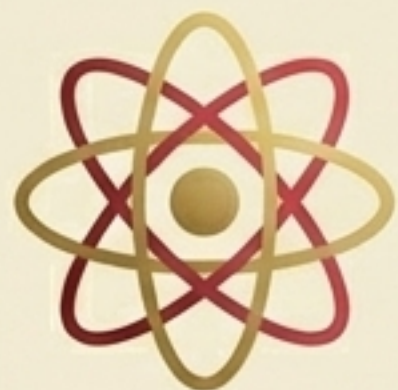


**Reachability** responds monotonically to constraint tightening.

As rules, physics, or norms become more constrained, the volume of possible futures mathematically shrinks.

Possibility is a **physically measurable** volume (Corollary 5.2).

# Cross-Domain Universality



## Physics

Thermodynamics

Admissibility fields dictate particle flow. The arrow of time is a feature of trajectory space, not state.



## Biology

Morphogenesis

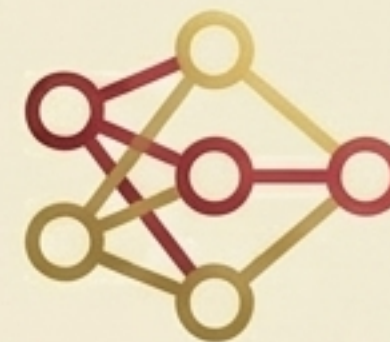
Waddington landscapes act as constraint fields. Adult phenotypes are projections of developmental trajectories.



## Institutions

Society

Laws/Norms are admissibility boundaries. Institutional memory is compressed causal history.



## AI Models

Computation

LLMs project token sequences into entity embeddings. Hallucination is a literal projection collapse (Noun Fallacy).

# The Grand Challenge: Inverse Constraint Recovery

Can we infer the invisible constraint field from the surviving trajectories?



## Open Problem 4.1

Given a distribution of observed trajectories  $\mathcal{P}$  within an unknown content space  $\mathcal{X}$ , how do we computationally recover the admissibility field  $\mathcal{A}$  that governs it?

## The Stakes

This is the central problem of all observational science — reconstructing hidden curvature from observed collapse.

# The Master Inversion

~~Objects → Processes → Reality~~

Constraints → Trajectories → Meaning

The task is not to see what no one has yet seen, but to think what nobody has yet thought about that which everybody sees. — Erwin Schrödinger

Abandon the balkanization of knowledge.  
Possibility is geometric. Trajectories are primary. Reachability is meaning.