Quadratic Gravity as a Derived Interface Theory: Admissibility Manifolds, Projection Geometry, and the Emergence of Observable Physics

Flyxion

Independent Researcher

June 23, 2026

Abstract

We argue that the standard rejection of higher-derivative quantum gravity on grounds of ghost instability rests on an assumption stronger than experiment requires: that the internal state space of a theory is identical to its observable sector. Relaxing this identification, we develop a framework of admissibility manifolds in which physical constraints are imposed on projected observable histories rather than on the coordinates of the ambient state space. Within this framework, negative-norm sectors need not constitute observational pathologies; they may instead correspond to hidden admissibility directions that survive internally while canceling or decoupling under projection to observable quantities. We formulate the Observable Admissibility Theorem and the Projection Sufficiency Principle, and show that Krein-space quantum mechanics—independently proposed by Turok and Bateman for quadratic gravity—arises as a special case of admissibility-flow dynamics. The modified Born rule of Turok and Bateman, expressed via projection operators and a ghost-parity symmetry, is derived from the general principle that probabilities are computed from projected quantities on M_{obs} rather than from inner products on M_{adm}. We further argue that the Born rule itself admits a coarse-graining derivation: following Needham's geometric reading of complex multiplication as rotation-scaling transport and Barandes's reconstruction of quantum theory from stochastic processes admitting a unitary lift, we show that probabilities arise from projection of field-valued rotation-scaling transport on M_{adm} onto observable transition structure on M_{obs}; the resulting transition kernel is unistochastic when the fiber of Π admits a unitary action preserved under the dynamics. We further connect this framework to Observational–Interventional Separation, the critique of latent fundamentalism, and the Derived Interface Conjecture: that observable spacetime theories are effective interfaces generated by projection from a larger admissibility manifold, with gauge symmetry, Hilbert structure, metric positivity, and the Born rule itself appearing as interface properties rather than ontological necessities.

Contents

Prefatory Note

The arguments developed here belong to a research program centered on the conviction that reachability, admissibility, and constraint closure are the correct ontological primitives—prior to objects, representations, prediction, and optimization. The present paper applies that orientation to a problem in the foundations of physics that ramifies far beyond its motivating example: when is a pathology in the representational machinery of a physical theory a pathology of the world the theory describes?

The motivating example is provided by Neil Turok and Sam Bateman's proposal to rehabilitate quadratic gravity using a Krein-space formulation of quantum mechanics with a modified Born rule based on ghost-parity symmetry. Their proposal attacks a specific assumption—that the state space of a physical theory must be a positive-definite Hilbert space—and argues that this assumption is stronger than observational requirements demand. But the deeper claim of the present paper is that the Turok–Bateman move is one instance of a structural distinction that recurs throughout physics and philosophy of science: the distinction between representation and observation.

That distinction appears in many guises, and the paper traces them deliberately in parallel. The identification of a gauge orbit with a physical state, the identification of a wave function with a measurement outcome, the identification of a Markov blanket with an agent boundary, the identification of a model coordinate with an ontological object—all are instances of the same error, which we call latent fundamentalism. The admissibility framework is a systematic attempt to replace that error with a principled criterion: physical constraints belong on the projected observable sector, not on the internal representational machinery.

The paper may therefore be read in two ways. As a contribution to quantum gravity, it argues that higher-derivative theories with ghost sectors are not automatically inadmissible, and that quadratic gravity in particular deserves rehabilitation. As a contribution to the theory of physical representation, it argues that many historical disputes—about ghosts, gauge redundancy, wave function realism, and the multiverse—arise from confusing the geometry of representation with the geometry of observation. The quadratic gravity discussion provides the most technically demanding test case; the Observational–Interventional Separation theorem and the critique of latent fundamentalism are intended to survive it.

The paper proceeds in nine main sections and four appendices. Sections 1–3 establish the framework of admissibility manifolds and prove the two principal theorems. Sections 4–5 analyze the Ostrogradsky instability and Krein-space formalism, recovering the Turok–Bateman construction as a derived consequence. Section 6 addresses the emergence of quantum probability from projection. Section 7 interprets quadratic gravity as effective admissibility-flow geometry, with Einstein gravity and quadratic gravity as successive terms in a curvature expansion. Section 8 develops the epistemological foundations through Observational–Interventional Separation and the critique of latent fundamentalism. Section 9 states and discusses the Derived Interface Conjecture.

1. The Hilbert Assumption and Its Hidden Content

1.1. The Orthodox Framework

Standard quantum mechanics rests on two closely related commitments. The first is that the state space of a physical system is a complex Hilbert space $\mathcal{H}$, equipped with a positive-definite inner product satisfying $\langle \psi \mid \psi \rangle > 0$ for all nonzero ψ. The second, seldom made explicit, is that this space *is* the physical space: every element of $\mathcal{H}$ is a candidate physical state, and every self-adjoint operator on $\mathcal{H}$ is a candidate observable. The Born rule connects them by assigning to each measurement outcome F the probability

$$P(F \mid I) = |\langle F \mid S \mid I \rangle|^2, \tag{1}$$

where $|I\rangle$ and $|F\rangle$ are normalized initial and final states and S is the S-matrix.

When quantum mechanics is extended to incorporate gravity, the second commitment creates difficulties. The Einstein–Hilbert action

$$S_{\mathrm{EH}} = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\, R \tag{2}$$

is not perturbatively renormalizable. Achieving renormalizability requires adding quadratic curvature terms, as first established by Stelle in 1977:

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G} + \alpha R^2 + \beta C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \right), \tag{3}$$

where $C_{\mu\nu\rho\sigma}$ is the Weyl tensor. The resulting propagator introduces additional poles corresponding to massive spin-2 degrees of freedom with opposite-sign kinetic term. In the canonical quantization of such theories, these states carry negative inner product: they are *ghosts*. The standard inference follows: negative inner product implies negative probability via the Born rule; negative probability is unphysical; ghost-containing theories are unphysical.

Turok has described this inference as appearing "everywhere in the literature"—and then argued that it is wrong.

1.2. The Hidden Assumption Identified

The standard inference contains two steps. The first is mathematical and unimpeachable: in a positive-definite Hilbert space, states of negative inner product do not exist by definition. The second step is physical and non-trivial: it identifies the inner product on the state space with the empirical probability function. That identification is an assumption.

We call it the *Hilbert Assumption*:

$$\text{Observable Physics} = \text{Entire State Space}. \tag{4}$$

The Hilbert Assumption is what makes the inner product on the state space directly interpretable as a probability amplitude. It is satisfied in every physical situation so far tested. But the question is whether it is a *necessary* condition for a consistent physical theory or merely a sufficient one that has been promoted to necessity by historical habit.

Turok's observation is pointed: "A quantum state is nothing but a label for a system. Its norm is neither here nor there. You can't observe the norm of a quantum state." What you observe are transition probabilities between states, and those are constrained to be non-negative and to sum to unity—a weaker condition than requiring positive norm on every element of the state space. The present paper takes this observation and embeds it in a general framework: the Hilbert Assumption is a special case; the general case allows the state space to be larger than the observable sector.

1.3. Historical Precedents for a Larger State Space

The move of allowing a larger internal space than the observable space is not unprecedented in physics. Two examples are immediately instructive.

Quantum electrodynamics in Lorenz gauge employs an indefinite-metric state space containing negative-norm longitudinal photon states (the Gupta–Bleuler construction). Physical states are selected by a subsidiary condition that projects onto the positive-norm sector. The negative-norm states are present in the formalism because working in the larger space respects the manifest Lorentz invariance of the theory; they are absent from any physical prediction.

The BRST formalism for Yang–Mills theories extends this pattern: one works in a state space that includes Faddeev–Popov ghost fields obeying the wrong statistics. Physical states are the cohomology classes of the BRST operator Q: states annihilated by Q modulo states of the form $Q|\chi\rangle$. Again, the ghost fields exist in M_{adm} but are absent from M_{obs}.

Turok acknowledges this precedent: physicists are "actually very used to working in pseudo-Hilbert spaces. The usual prescription is you think you're in this big space which has both positive and negative and null states, and then you perform a projection and you say there's a physical subspace where everything is positive." The novelty of the Krein-space proposal is not the existence of the larger space but a modification of what counts as the projection: instead of projecting onto a positive-norm subspace, one traces over the entire space using a modified probability functional. From the admissibility perspective, both procedures are instances of the same principle—constraints on observable quantities rather than on internal coordinates.

2. The Framework of Admissibility Manifolds

2.1. The Three-Level Hierarchy

Let M_{adm} denote the *admissibility manifold*: the space of all field histories consistent with the constraint closure conditions governing the fundamental dynamics.

Remark 2.1 (Mathematical status of M_{adm}). The admissibility manifold is a space of *histories*, not states. An element $h \in M_{\mathrm{adm}}$ is a field trajectory over a spacetime region, not a configuration at a moment of time. This trajectory-centric choice is deliberate and consequential: it permits the framework to address Ostrogradsky-type pathologies, which are properties of phase-space directions rather than of trajectories, and it connects naturally to the path-integral formulation of quantum field theory in which histories are the fundamental objects.

The appropriate category for M_{adm} varies by application. For perturbative quantum field theory, it is a space of distributional field configurations, and the relevant topology is that of tempered distributions. For the Krein-space application of Sections 3–5, it is a Krein space (an indefinite inner-product space with a fundamental decomposition $\mathcal{K} = \mathcal{K}_+ \oplus \mathcal{K}_-$). For the RSVP field-theoretic application of Section 6, it is a space of field-valued transport operators equipped with a fiber bundle structure over M_{obs}. The framework does not presuppose any single category; it specifies instead the properties any admissibility manifold must satisfy: the existence of the surjective projection Π and the well-definedness of the admissibility conditions of Definition 2.1 on its image. Readers seeking a canonical formal setting may take M_{adm} to be a Hilbert manifold modeled on a separable Hilbert space, with Π a smooth surjective submersion; all results in Sections 2–3 hold in this setting.

The geometry of M_{adm} is not postulated; it is whatever structure is imposed by the constraint closure conditions of the theory.

Above M_{adm} sits the *projection manifold* M_{proj}: equivalence classes of admissible histories under the symmetries generating non-observable degrees of freedom. Gauge transformations, diffeomorphisms, BRST-exact states, and ghost-parity equivalences all generate such classes. Above that sits the *observable manifold* M_{obs}: empirically distinguishable outcomes. These spaces are connected by surjective projections:

$$M_{\mathrm{adm}} \xrightarrow{\Pi_1} M_{\mathrm{proj}} \xrightarrow{\Pi_2} M_{\mathrm{obs}}, \tag{5}$$

with composite projection $\Pi = \Pi_2 \circ \Pi_1 : M_{\mathrm{adm}} \to M_{\mathrm{obs}}$.

The Hilbert Assumption corresponds to Π being an isomorphism. The BRST approach treats Π as a proper surjection with nontrivial kernel but requires M_{adm} to be a Hilbert space. The Krein-space approach relaxes the latter: M_{adm} need not be a Hilbert space, provided Π maps it to a space of admissible probability distributions.

Definition 2.1 (Observable Admissibility). *A history $h \in M_{\mathrm{adm}}$ is* observable-admissible *if its projection $\Pi(h) \in M_{\mathrm{obs}}$ satisfies:*

(i) probabilities are non-negative: $P(O) \geq 0$ for all observables O;

(ii) probabilities for any exhaustive partition sum to unity: $\sum_i P(O_i) = 1$;

(iii) the causal order on M_{obs} is preserved under dynamical evolution.

We write $\mathcal{A}_O(\Pi(h)) = 1$ when these hold. The admissibility set in M_{adm} is

$$\mathcal{A}_H = \Pi^{-1}\big(\{\Pi(h) : \mathcal{A}_O(\Pi(h)) = 1\}\big). \tag{6}$$

This definition is deliberately minimal. It does not require the inner product on M_{adm} to be positive definite. It does not require a canonical symplectic structure. It requires only that the image of h under Π support a well-defined causal probability measure over observable outcomes.

Principle 2.1 (Projection Sufficiency). Physical constraints are conditions on projected observable histories. They need not be conditions on the internal coordinates of M_{adm}.

Hilbert-space quantum mechanics is recovered as the special case where $M_{\mathrm{adm}} = \mathcal{H}$ with positive inner product and Π is the identity. The principle allows M_{adm} to carry richer structure—including negative-norm directions—provided those directions do not compromise admissibility under Π.

2.2. The Coordinate–Observable Distinction

A central distinction runs through all that follows. A *coordinate pathology* is a feature of the internal representation in M_{adm} with no direct bearing on observable quantities:

negative inner product, gauge redundancy, coordinate singularities, Ostrogradsky's unbounded phase-space directions. An *observable pathology* is a violation of one of the three conditions in Definition 2.1: negative probabilities for measurement outcomes, failure of probability conservation, acausal signaling.

The failure to maintain this distinction accounts for much of the resistance to higher-derivative theories: coordinate pathologies have been treated as automatically implying observable pathologies. The following theorem shows that this is not a general principle.

Theorem 2.2 (Projection Pathology Separation). *A coordinate pathology in M_{adm} does not in general imply an observable pathology in M_{obs}.*

Remark 2.2 (Logical status of this theorem). The theorem is a structural consequence of the definitions rather than a deep result: admissibility is placed on the image of Π, so whether a property of the domain induces a property of the image depends on Π alone. The theorem's importance is therefore diagnostic rather than constructive. It identifies the *correct question*—does Π map this pathology to M_{obs} or to the fiber?—and establishes that this question cannot be answered by inspecting the pathology alone. Concretely: showing that a theory has a ghost in M_{adm} does not constitute a proof that the theory has an observable instability. An additional argument about the structure of Π is always required. The historical errors the theorem diagnoses arose from omitting that step.

Proof. A coordinate pathology is a property P of some $h \in M_{\mathrm{adm}}$. Observable admissibility is determined entirely by $\mathcal{A}_O(\Pi(h))$. Since Π is many-to-one, elements of M_{adm} carrying P may map to elements of M_{obs} satisfying all three admissibility conditions. Whether they do so depends on the structure of Π, not on P alone. □

The inference from "this theory has ghost states" to "this theory predicts negative probabilities" requires an additional premise: that the projection Π does not cancel or absorb the ghost contribution before reaching M_{obs}. The history of gauge theories establishes that this premise is routinely false. The Turok–Bateman proposal is asking whether it is also false for the massive spin-2 ghost of quadratic gravity.

3. Ghosts as Hidden Admissibility Directions

3.1. The Origin of Ghosts in Higher-Derivative Theories

In quantum field theory, a ghost is a propagating degree of freedom whose kinetic term has the wrong sign. For a higher-derivative theory, this arises from partial-fraction decomposition of the propagator. For quadratic gravity, the spin-2 part of the propagator takes the schematic form

$$\Delta^{(2)}(k) \sim \frac{1}{k^2} - \frac{1}{k^2 + m_2^2}, \tag{7}$$

where $m_2 = M_{\rm Pl}/\sqrt{2\beta}$. The negative sign on the second term indicates that the corresponding state has negative inner product with itself. In canonical quantization, the creation and annihilation operators for this mode satisfy commutation relations with opposite sign, yielding a state space with indefinite inner product: the Krein space.

The standard response—declare such states unphysical and seek a projection onto a positive Hilbert subspace—is unavailable here because no such projection preserving all the symmetries of quadratic gravity has been found. Turok and Bateman's response is different: rather than projecting out the ghost states before computing probabilities, they include them in a modified probability functional that is engineered to produce non-negative results regardless of the sign of individual state norms.

Definition 3.1 (Hidden Admissibility Direction). *An element $g \in M_{\rm adm}$ is a* hidden admissibility direction *if $[g, g]_{\mathcal{K}} < 0$ (a coordinate pathology) and yet $\mathcal{A}_O(\Pi(g)) = 1$ (no observable pathology).*

3.2. The Krein Space and Ghost Parity

A Krein space $\mathcal{K}$ is an indefinite inner product space with decomposition

$$\mathcal{K} = \mathcal{K}_+ \oplus \mathcal{K}_-, \tag{8}$$

where $\mathcal{K}_+$ carries a positive-definite inner product and $\mathcal{K}_-$ carries a negative-definite one. The Krein inner product is

$$[u, v]_{\mathcal{K}} = \langle u_+, v_+\rangle_+ - \langle u_-, v_-\rangle_-. \tag{9}$$

This inner product is non-degenerate but indefinite. Turok's Minkowski analogy is exact: positive-norm states correspond to timelike vectors, negative-norm states to spacelike vectors, null-norm states to lightlike vectors. Minkowski spacetime is not deficient as a geometry because some vectors have negative norm; a Krein space is not deficient as an algebra because some states have negative inner product.

The *ghost-parity operator* is

$$G = \Pi_+ - \Pi_-, \tag{10}$$

with $\Pi_\pm$ orthogonal projectors onto $\mathcal{K}_\pm$, satisfying $G^2 = I$ and $G^\dagger = G$. When G commutes with the Hamiltonian and S-matrix, positive-norm and negative-norm sectors evolve independently. This selection rule is the key to the modified Born rule.

3.3. The Modified Born Rule and its Derivation from Projection

In standard quantum mechanics, the Born rule $P(F \mid I) = |\langle F \mid S \mid I\rangle|^2$ can be rewritten without explicit normalization as

$$P(F \mid I) = \mathrm{tr}(\Pi_F \, S \, \Pi_I \, S^\dagger), \tag{11}$$

where $\Pi_I = |I\rangle\langle I|/\langle I \mid I\rangle$ and $\Pi_F = |F\rangle\langle F|/\langle F \mid F\rangle$ are projectors and the trace is over the Hilbert space. This form is equivalent to the standard Born rule when all state norms are positive but is well-defined as a functional of projectors even when the inner product on the full state space is indefinite.

Turok and Bateman adopt exactly this form: defining $A = \Pi_F \cdot S \cdot \Pi_I$, the probability is $P(F \mid I) = \mathrm{tr}(A^\dagger A)$, where the trace runs over all states in $\mathcal{K}$, including ghost states. Turok describes this as "more economical" than the traditional BRST projection: there is no need to construct a physical subspace; one simply sums over everything and the ghost-parity symmetry ensures the result is positive.

Theorem 3.2 (Observable Admissibility)**.** *Let $\mathcal{K} = \mathcal{K}_+ \oplus \mathcal{K}_-$ be a Krein space with ghost-parity operator G. Let the probability functional be*

$$P(F \mid I) = \mathrm{tr}_{\mathcal{K}}(A^\dagger A), \quad A = \Pi_F \cdot S \cdot \Pi_I. \tag{12}$$

If G is a symmetry of S (i.e., $[G, S] = 0$), then $P(F \mid I) \geq 0$ for all I, F, and $\sum_F P(F \mid I) = 1$.

Proof. We proceed carefully to avoid importing Hilbert-space assumptions into the Krein-space setting.

Positivity. The key subtlety is that the adjoint $A^\dagger$ and the norm $\|A|\psi\rangle\|^2$ must be interpreted with respect to the *positive* Hilbert space structure on $\mathcal{K}_+$, not the indefinite Krein inner product. Equip $\mathcal{K}$ with the positive definite inner product $\langle u, v\rangle_+ = \langle u_+, v_+\rangle + \langle u_-, v_-\rangle$, where $u = u_+ + u_-$ with $u_\pm \in \mathcal{K}_\pm$. With respect to $\langle\cdot,\cdot\rangle_+$, the operator $A_+^\dagger A$ is positive semidefinite, and its trace satisfies $\mathrm{tr}_+(A_+^\dagger A) = \sum_n \langle e_n | A_+^\dagger A | e_n\rangle_+ \geq 0$ for any orthonormal basis $\{|e_n\rangle\}$ of $(\mathcal{K}, \langle\cdot,\cdot\rangle_+)$. The probability functional is then $P(F \mid I) = \mathrm{tr}_+(A_+^\dagger A) \geq 0$.

Normalisation. We must show $\sum_F \mathrm{tr}_+(A_F^\dagger A_F) = 1$, where $A_F = \Pi_F S \Pi_I$ and the sum runs over a complete set of orthogonal final projectors $\{\Pi_F\}$ summing to the identity $I_{\mathcal{K}}$. Using ghost-parity symmetry $[G, S] = 0$, the S-matrix block-diagonalises with respect to the $\mathcal{K}_\pm$ decomposition: $S = S_+ \oplus S_-$, where $S_\pm : \mathcal{K}_\pm \to \mathcal{K}_\pm$ are unitary on their respective sectors. The completeness sum then separates:

$$\sum_F \mathrm{tr}_+(A_F^\dagger A_F) = \mathrm{tr}_+\Big(\Pi_I S^\dagger \sum_F \Pi_F S \Pi_I\Big) = \mathrm{tr}_+\big(\Pi_I S^\dagger S \Pi_I\big).$$

Since S_+ is unitary on $\mathcal{K}_+$ and S_- is unitary on $\mathcal{K}_-$, we have $S^\dagger S = I_\mathcal{K}$, and the expression reduces to $\mathrm{tr}_+(\Pi_I) = 1$ (the projector Π_I projects onto a single normalized state). This completes the proof.

The ghost-parity assumption $[G, S] = 0$ is precisely the condition that keeps $\mathcal{K}_+$ and $\mathcal{K}_-$ from mixing under time evolution. If this condition fails—if the dynamics couple positive- and negative-norm sectors—then S does not block-diagonalise, $S^\dagger S \neq I_\mathcal{K}$ in the relevant sense, and normalisation can fail. The ghost-parity condition is therefore not merely a technical convenience but the essential physical constraint whose verification in the full spin-2 sector of quadratic gravity remains open. □

The ghost states in $\mathcal{K}_-$ are hidden admissibility directions: they participate in the computation of $P(F \mid I)$ through the trace but do not appear in the final observable probability as negative contributions. The admissibility interpretation is direct: the trace over $\mathcal{K}$ is a computation on M_{adm} that produces a quantity on M_{obs}.

Turok notes that in this construction, the amplitude A itself is not a physical quantity—"it's not in quantum mechanics, you've got to square it." This observation is significant: it separates the ontological status of amplitudes (elements of M_{adm}) from that of probabilities (elements of M_{obs}). The admissibility framework makes this separation explicit.

4. The Ostrogradsky Theorem Revisited

4.1. The Classical Statement

Ostrogradsky's 1850 theorem establishes that a Lagrangian with irreducible dependence on derivatives of order higher than the first possesses a Hamiltonian unbounded below. For a system with Lagrangian $L(q, \dot{q}, \ddot{q})$ in which $\ddot{q}$ appears non-degenerately, the Ostrogradsky construction introduces canonical coordinates

$$Q_1 = q, \quad P_1 = \frac{\partial L}{\partial \dot{q}} - \frac{d}{dt}\frac{\partial L}{\partial \ddot{q}}, \tag{13}$$

$$Q_2 = \dot{q}, \quad P_2 = \frac{\partial L}{\partial \ddot{q}}, \tag{14}$$

yielding the Hamiltonian

$$H = P_1 Q_2 + P_2\, \ddot{q}(Q_2, P_2) - L, \tag{15}$$

which is linear in P_1 and hence unbounded below. The standard inference is that coupling such a system to any conventional positive-energy system produces runaway energy exchange: the higher-derivative system lowers its energy without bound while the coupled system raises its energy without bound. The theorem is mathematically correct as a statement about the extended phase space $M^{\text{ext}}_{\text{adm}}$.

4.2. Gravity's Native Negative Energy and the Trajectory Shift

Turok's central observation is that gravity already involves negative energy in a way that no one regards as problematic: the gravitational potential energy of a bound system is negative, and the total energy of an isolated gravitating system can be less than that of its widely separated constituents. More strikingly, observations show that the universe is expanding exponentially—a process that, out of context, resembles an instability. "That sounds awfully like an instability," Turok remarks, "but it is absolutely stable" when analyzed as a cosmological solution.

When Turok and Bateman analyze the four-derivative gravitational theory, they find that the Ostrogradsky runaway direction in the extended phase space corresponds precisely to normal exponential cosmological expansion. The instability is not an observable pathology; it is a coordinate description of ordinary cosmological behavior.

This is the trajectory shift. State-based reasoning asks: what is the sign of the energy of this configuration? Trajectory-based reasoning asks: what are the actual histories generated by the dynamics, and are they admissible in M_{obs}? The Ostrogradsky theorem establishes a property of the state space; it does not establish a property of the generated histories.

Principle 4.1 (Observable Stability)*.* Observable stability is determined by the admissibility of projected trajectories, not by the sign of the Hamiltonian on M_{adm}.

The Pais–Uhlenbeck oscillator provides a canonical illustration. Its Hamiltonian is linear in one canonical momentum and hence unbounded below. Yet its equations of motion reduce to those of two coupled simple harmonic oscillators with frequencies ω_1 and ω_2: all solutions are bounded and periodic. The unbounded directions in the extended phase space are genuine, but no admissible trajectory reaches them. This is exactly the Projection Pathology Separation Theorem in concrete form: the coordinate pathology (unbounded Hamiltonian) does not propagate to an observable pathology (actual runaway behavior) because the dynamics confines admissible trajectories to the bounded sector of $M^{\text{ext}}_{\text{adm}}$.

4.3. From State Properties to History Properties

A recurrent pattern in the admissibility program is the replacement of state-property reasoning with history-property reasoning. This replacement arises because observable physics is inherently historical: what we measure are outcomes of processes extended in time, not instantaneous snapshots of a state space.

For the Ostrogradsky instability, the frozen distinction is between the sign of the Hamiltonian (a state property) and the boundedness of the actual trajectories (a history property). Treating the former as a certificate of the latter is precisely the conflation of coordinate and observable pathologies that the Projection Pathology Separation Theorem forbids.

Turok's resolution of the Ostrogradsky instability in the gravitational context is a concrete instance of this replacement: he does not modify the Hamiltonian, nor does he add a constraint to restrict the phase space. He instead analyzes the actual trajectories of the theory and finds that they correspond to known, stable cosmological solutions. The extended phase space $M_{\rm adm}^{\rm ext}$ has unbounded directions; the admissible trajectories do not reach them.

5. Asymptotic Freedom, the Hierarchy Problem, and UV Completeness

5.1. The Renormalizability and Asymptotic Freedom of Quadratic Gravity

Two properties of the quadratic gravity theory make it particularly attractive as a candidate quantum theory of gravity, and both are consequences of the four-derivative structure of the action.

The first is *perturbative renormalizability.* The four-derivative kinetic terms suppress high-frequency fluctuations more strongly than the two-derivative Einstein–Hilbert term, and all ultraviolet divergences in perturbation theory can be absorbed into redefinitions of the coupling constants α and β. This was established by Stelle in 1977 and remains uncontroversial.

The second is *asymptotic freedom.* As shown by Avramidi in the 1980s, the renormalization group running of the couplings causes them to flow to zero at high energies, analogously to the strong coupling constant of QCD. At short distances—above the Planck scale—the theory becomes arbitrarily weakly coupled, reducing to "just waves which don't interact," as Turok describes it. The theory is therefore UV-complete in the same sense that QCD is UV-complete: it has a well-defined continuum limit.

The analogy with QCD is structurally deep. QCD is asymptotically free in the UV and strongly coupled in the IR, with confinement at low energies ensuring that the only observable excitations are color-neutral hadrons rather than individual gluons. Quadratic gravity may have a similar infrared structure: weakly coupled at the Planck scale, strongly coupled at some lower scale, with a non-trivial spectrum of bound states. Turok notes that a student is currently investigating this question by studying the theory on the lattice—the same technique used to study the non-perturbative IR behavior of QCD.

5.2. The Spectrum of Excitations

The physical spectrum of quadratic gravity in the Weyl sector can be analyzed by decomposing metric perturbations around flat spacetime. The theory contains four classes of excitations:

(i) the standard massless graviton (spin-2, positive norm);

(ii) a massive ghost graviton with mass $m_2 = M_{\rm Pl}/\sqrt{2\beta}$ (spin-2, negative norm in the Krein space);

(iii) a massive vector mode;

(iv) a scalar conformal mode governed by the Ricci scalar term.

The scalar mode is the subject of Turok and Bateman's current results: in the limit $\beta \to 0$ (Weyl coupling goes to zero), the spin-2 ghost and vector modes decouple and the remaining theory is the scalar conformal sector. They show that this sector admits the ghost-parity symmetry and that the modified Born rule produces positive probabilities. The scalar sector is, in Turok's words, "a kind of toy model for quantum gravity"— not the full theory, but a limit of it in which the construction can be verified rigorously.

The open question is whether the ghost-parity symmetry extends to the full theory with the spin-2 sector included. Turok acknowledges: "The trick we used to make sense of it may or may not apply to the full thing. We will have to search in the full theory: is there a similar discrete symmetry? If there is, then this will be a complete theory of quantum gravity."

5.3. The Hierarchy Problem and Dimensional Transmutation

One of the deepest puzzles in theoretical physics is the hierarchy of mass scales: the Planck mass at 10^{19} GeV, the weak scale at ~ 100 GeV, the QCD scale at ~ 1 GeV, and the cosmological constant at $\sim 10^{-3}$ eV. Why are these scales so widely separated? In the standard model, the Higgs mass must be tuned to prevent radiative corrections from pulling it up to the Planck scale.

Turok notes that quadratic gravity may resolve this through dimensional transmutation. An asymptotically free theory defined at the Planck scale with a small coupling constant will develop a strongly-coupled scale exponentially below the Planck scale, by the same mechanism that makes the QCD scale exponentially below the Planck scale without any fine-tuning. If the Higgs is a composite of the scalar conformal mode of quadratic gravity, its mass is naturally set by this dynamically generated scale rather than by a direct coupling to the Planck scale.

From the admissibility perspective, dimensional transmutation is a statement about the structure of M_{adm} in the infrared: the admissible trajectory space of the theory has a non-trivial structure at the dynamically generated scale that is invisible in the UV description.

6. Quantum Probability as Projection

6.1. The Unistochastic Structure

The Turok–Bateman reformulation of the Born rule connects to a picture in which quantum probability is not a foundational axiom but an emergent structure arising from coarse-graining over unobserved degrees of freedom.

Consider a field-theoretic dynamics on M_{adm} in which the fundamental degrees of freedom are field configurations $(\Phi, \mathbf{v}, S)$ subject to coupled constraint equations. Observable transition probabilities arise from integrating over all trajectories connecting initial to final configurations, weighted by the squared modulus of a field amplitude:

$$T_{ij} = \int_{\gamma: C_i \to C_j} |\Phi|^2 \, d^4x. \tag{16}$$

The matrix T with entries T_{ij} is doubly stochastic: non-negative entries (squares of moduli) summing to unity in each row and column. This is a *unistochastic* matrix.

The Born rule of standard quantum mechanics is the special case in which the integral over trajectories reduces to a sum over paths between eigenstates and Φ reduces to the quantum-mechanical amplitude ψ. The full field-theoretic expression is more general: it includes contributions from all degrees of freedom in M_{adm}, whether or not they are directly observable.

6.2. The Born Rule as Derived Quantity

Under this interpretation, the Born rule is a derived quantity. It is the leading-order approximation obtained when the coarse-graining projection Π is nearly trivial—when unobserved degrees of freedom decouple from observed ones so cleanly that the full integral over M_{adm} reduces to the modulus-squared of a Hilbert-space amplitude.

The Turok–Bateman modified Born rule $P(F \mid I) = \mathrm{tr}(A^\dagger A)$ is not a revision of quantum mechanics; it is a more faithful expression of the same underlying probability computation for the case in which the decoupling is less complete. The ghost sector cannot be simply discarded; it must be integrated over. The trace in $\mathrm{tr}(A^\dagger A)$ is exactly this integration: summing over the unobserved degrees of freedom—including the ghost sector—while producing an observable quantity. Ghost states in $\mathcal{K}_-$ are trajectories in M_{adm} that pass through negative-norm configurations; their contribution to T_{ij} is included, but their individual norms play no direct role in the observable probability.

Remark 6.1. Turok's observation that the amplitude A "is not a physical thing" connects to the admissibility distinction between M_{adm} and M_{obs}. Amplitudes are elements of M_{adm}; probabilities are elements of M_{obs}. The squaring operation in the Born rule is the projection Π.

6.3. Unsquared Numbers and Unistochastic Coarse-Graining

The argument so far has been algebraic: we have shown that probabilities arise from traces of positive semidefinite operators, and that ghost states contribute to the trace without violating positivity. But there is a more geometric way to see why probabilities are squares of something deeper, and why the "something deeper" includes directions—phases, rotations, negative norms—that are not themselves observable. This geometric picture connects the Krein-space construction to a broader principle about the structure of measurement.

The Unsquared Viewpoint

In Needham's geometric reading of complex analysis [20], a complex number $z = re^{i\theta}$ is not merely a number but a compound operator on the plane: it scales by r and rotates by θ. The imaginary unit i is therefore an "unsquared" direction, not because it is mysterious, but because it encodes a quarter-turn that the algebraic relation $i^2 = -1$ hides behind a sign. Multiplying two complex numbers compounds their scalings and adds their angles; taking the modulus squared discards the angle and retains only the scaling:

$$|z_1 z_2|^2 = |z_1|^2 |z_2|^2, \qquad \arg(z_1 z_2) = \arg z_1 + \arg z_2 \text{ [lost under } |\cdot|^2]. \tag{17}$$

The Born rule $P = |\langle F|S|I\rangle|^2$ is exactly this operation: it discards the phase accumulated during evolution and retains only the amplitude ratio. The probability is the squared modulus of an unsquared rotation-scaling process. Needham's observation that multiplication composes rotations and scalings makes the Born rule geometrically intuitive: squaring removes information about orientation while retaining information about magnitude [20]. What is squared away is the phase—the record of how much the state turned in Hilbert space during the transition.

The question the admissibility framework asks is: how much more can be squared away? If the internal space M_{adm} is larger than a Hilbert space—if it contains not just phases but also directions of indefinite norm, gauge redundancies, and ghost sectors—then the squaring operation (projection Π) discards correspondingly more. Positive probabilities can survive because they depend only on what Π retains, not on what it discards.

The RSVP Field Operator

In the RSVP framework, the analogue of the complex operator $z = re^{i\theta}$ is a field-valued operator

$$Z(x) = \Phi(x)\, e^{\widehat{L}(x)}, \qquad \widehat{L}(x) = \mathbf{v}(x)\cdot\nabla + \theta(x)\,\widehat{T}, \tag{18}$$

where $\Phi(x)$ is the scalar field supplying the scaling component, $\mathbf{v}(x)$ is the vector field supplying transport along field-space directions, and $\theta(x)\,\widehat{T}$ is a torsional or phase-like component encoding rotational structure in the field configuration space.

The operator $Z(x)$ acts on field configurations by simultaneously scaling their amplitude and transporting them along the vector flow with a torsional twist. It is the field-theoretic unsquared object: $Z(x)$ itself is not observable, but $|Z(x)|^2 = |\Phi(x)|^2$ is the local intensity that enters observable transition weights. The vector and torsional components are discarded when the modulus is taken—they are the field-space analogue of the phase in the complex number $re^{i\theta}$.

The trajectory integral over M_{adm} connecting macrostate C_i to macrostate C_j is

$$T_{ij}^{\text{RSVP}} = \int_{\gamma: C_i \to C_j} \big|Z[\gamma]\big|^2\, \mathcal{D}\gamma, \tag{19}$$

where $Z[\gamma]$ is the path-ordered product of $Z(x)$ along the field trajectory γ, and $\mathcal{D}\gamma$ is the appropriate measure on M_{adm}. The transport and torsion components accumulate and interfere along γ; their accumulated effect on $|Z[\gamma]|^2$ is the only thing that survives to M_{obs}.

Unistochasticity from Phase-Liftability

When the coarse-grained transition kernel T_{ij}^{RSVP} satisfies a *phase-liftability* condition—when the interference structure of the $Z[\gamma]$ contributions can be encoded by a unitary matrix—there exists a unitary U such that

$$T_{ij}^{\text{RSVP}} = |U_{ij}|^2. \tag{20}$$

A doubly stochastic matrix expressible in this form is called *unistochastic.* Birkhoff's theorem establishes that every doubly stochastic matrix is a convex combination of permutation matrices; the stronger condition of unistochasticity says it is also the element-wise square of a unitary matrix, which is a strictly smaller set.

Conjecture 6.1 (Projection-Induced Unistochasticity). Let $\Pi : M_{\text{adm}} \to M_{\text{obs}}$ be a coarse-graining map whose fibers admit a unitary action preserved under the dynamics generated by $Z(x) = \Phi(x)e^{\widehat{L}(x)}$. Then the induced transition kernel on M_{obs} is unistochastic.

Remark 6.2. This is stated as a conjecture rather than a theorem because the conditions on the fibers—$\mathrm{U}(n)$-homogeneity and Haar-measure regularity—are not guaranteed by the general admissibility framework but must be verified case by case. The proof sketch below establishes the result under these conditions; extending it to the full RSVP dynamics without homogeneity assumptions is an open problem. The conjecture is nevertheless the conceptual hinge of Section 6: it is the precise statement that the Born rule is a coarse-graining theorem rather than a foundational axiom.

Proof sketch (under homogeneity assumption). Let $\{C_i\}$ be a partition of M_{obs} into macrostates, and let $\mathcal{F}_i = \Pi^{-1}(C_i)$ be the corresponding fiber in M_{adm}. By hypothesis, each fiber $\mathcal{F}_i$ is a $\mathrm{U}(n)$-homogeneous space. The dynamics generated by $Z(x)$ transports an initial fiber $\mathcal{F}_i$ to a superposition of terminal fibers $\{\mathcal{F}_j\}$ via a unitary operator U on $\mathbb{C}^n$ satisfying $Z[\gamma] \mapsto U$ under the fiber action. The transition weight is

$$T_{ij}^{\text{RSVP}} = \int_{\gamma: C_i \to C_j} |Z[\gamma]|^2 \, \mathcal{D}\gamma = \int_{\mathcal{F}_i} \int_{\mathcal{F}_j} |\langle f_j | U | f_i \rangle|^2 \, d\mu(f_i) \, d\mu(f_j),$$

where μ is the $\mathrm{U}(n)$-invariant measure on the fibers. When the fibers are isomorphic as $\mathrm{U}(n)$-spaces and μ is the Haar measure, the double integral reduces to $|U_{ij}|^2$ by the Schur orthogonality relations, establishing unistochasticity. □

Phase-liftability holds when the RSVP dynamics is sufficiently regular: specifically, when the torsional components $\theta\widehat{T}$ generate a well-defined $\mathrm{U}(n)$ action on the coarse-grained state space and the coarse-graining Π respects this action in the sense that the fiber of Π over each macrostate C_i is a $\mathrm{U}(n)$-homogeneous space. The emergence of a unistochastic transition matrix from a deeper dynamical substrate parallels Barandes's reconstruction of quantum theory from stochastic processes admitting a unitary lift [21, 22]. Under these conditions, the path integral over $\gamma : C_i \to C_j$

factors into a modulus-squared of a sum of complex amplitudes, which is exactly the condition for (20).

The quantum-mechanical Born rule is recovered when n equals the dimension of the Hilbert space and U is the S-matrix restricted to the relevant sector.

The Interpretive Synthesis

The unistochastic matrix is therefore the observable shadow of hidden rotation-scaling transport in the RSVP field space, just as the squared modulus of a complex amplitude is the observable shadow of an unobserved phase rotation. This synthesis unifies several seemingly disparate elements:

Needham's complex geometry shows that $i^2 = -1$ is not an axiom but a geometric fact about quarter-turns; the imaginary unit is unsquared rotation.

Barandes's unistochastic framework shows that quantum transition probabilities need not be postulated but can emerge from any dynamics whose coarse-graining yields a phase-liftable stochastic kernel.

The RSVP field operator $Z = \Phi e^{\widehat{L}}$ provides a concrete field-theoretic realization in which the unsquared object is a compound of scaling and transport, and the Born rule emerges from discarding the transport component under coarse-graining.

The Krein-space ghost states are a further extension of the same idea. A ghost state in $\mathcal{K}_-$ is a direction in M_{adm} along which the internal geometry has a reflection—a direction whose squared norm is negative because the rotation it encodes is hyperbolic rather than elliptic. When the ghost-parity symmetry ensures that the hyperbolic contributions cancel in the trace $\mathrm{tr}(A^\dagger A)$, the ghost is squared away along with the phases, and the observable probability is non-negative.

The pattern is consistent across all cases: amplitudes, phases, negative norms, and ghost sectors are all internal geometry in M_{adm}. Probabilities are what survives after the projection Π discards this geometry—after the squaring, the modulus-taking, the trace. The present framework shares with Barandes's proposal the central structural claim that quantum probabilities need not be fundamental: they arise from projection of a richer underlying dynamics onto an observable transition structure [21, 22]. The two frameworks differ in ontology— Barandes grounds the derivation in classical stochastic processes, while the admissibility framework grounds it in field-theoretic admissibility flow— but both conclude that the Born rule is a theorem about coarse-graining rather than an axiom about measurement. The Born rule is not a fundamental postulate about the relationship between wave functions and measurement outcomes; it is a theorem about what coarse-graining does to rotation-scaling transport in a space that is generically larger, and structurally richer, than a Hilbert space.

6.4. Strings and the Assumption of Hilbert-Space Quantum Mechanics

Turok draws a sharp conclusion from the Krein-space proposal: the entire string theory programme rested on assumptions that may be unnecessarily strong. He identifies two key assumptions behind the conclusion that quantum gravity requires extra dimensions: first, that only theories with at most two derivatives in the action are allowed; second, and more fundamentally, that "the theory lives in a Hilbert space, which means that the norms of all quantum states are positive."

String theory satisfies the Hilbert Assumption by construction. Its consistency conditions—Weyl anomaly cancellation, modular invariance, the absence of negative-norm physical states—are all requirements that the theory project cleanly onto a positive-definite Hilbert space. The dimension of spacetime (10 or 26) and the existence of supersymmetry are consequences of these requirements. If the Hilbert Assumption is relaxed, the entire edifice of constraints changes.

Turok's conclusion is direct: "Now that we've seen that you don't need that assumption, the whole thing has no basis." This may be too strong; string theory may be correct for independent reasons. But the existence of a UV-complete four-dimensional quantum gravity theory without strings or extra dimensions—even in a restricted sector—removes the *necessity* argument for strings. The admissibility framework generalizes this: the string landscape and the Krein-space quadratic gravity theory are two distinct points in the admissibility geometry of quantum theories that project to compatible observable physics at low energies.

7. Quadratic Gravity as Effective Admissibility-Flow Geometry

7.1. Geometry as the Shadow of Obstruction

In the admissibility framework, spacetime curvature is not a fundamental field. It arises from the obstruction to globally consistent projection: the shadow cast by the failure of Π to be globally flat.

Consider parallel transport of an admissibility constraint around a closed loop in M_{adm}. If the constraint closes upon return, the loop contributes no curvature to the projection geometry. If it fails to close, the discrepancy defines a curvature tensor measuring the local non-triviality of Π. The Riemann tensor is therefore a first-order correction to admissibility flow: a measure of how much Π departs from a globally flat projection. The Einstein field equations are the conditions for admissibility-flow consistency at leading order:

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu}. \tag{21}$$

These equations equate the curvature generated by matter (right side) with the admissibility-flow correction required by the projection geometry (left side). General relativity is the theory of the admissibility-flow constraint at leading order.

7.2. Quadratic Gravity as the Second-Order Term

If the Riemann tensor measures first-order departures of Π from flatness, quadratic curvature terms measure second-order departures. The full quadratic gravity action is the first two terms of a systematic admissibility-flow expansion:

$$S = \int d^4x \sqrt{-g}\, \mathcal{L}_{\mathrm{adm}}\big(\Pi^{(0)}, \Pi^{(1)}, \Pi^{(2)}, \ldots\big)\,. \tag{22}$$

The Einstein–Hilbert term captures the zeroth and first-order corrections; the quadratic terms capture the second-order correction. The theory is not an ad hoc extension of general relativity but the natural next term in a principled expansion.

Under this interpretation, the massive spin-2 ghost is a pole in the propagator on M_{adm} arising from the second-order correction. Whether it induces an observable pathology depends on whether Π maps its contribution to M_{obs} or to the fiber of Π. The Turok–Bateman proposal is, in essence, a claim that—at least in the scalar-mode limit—the ghost pole lies in the fiber of Π.

7.3. The Effective Interface Hierarchy

Different approximations to the admissibility-flow dynamics generate different effective theories:

At zeroth order, the admissibility flow is trivial: spacetime is flat and matter propagates on it without back-reaction. This is quantum field theory in a fixed background.

At first order, the back-reaction of matter on geometry is included: this is general relativity, or its quantization in the semiclassical approximation.

At second order, the higher-derivative corrections are included: this is quadratic gravity, with its ghost pole and its potential for UV completeness.

Each level is an effective interface theory—a description of M_{obs} obtained by truncating the admissibility-flow expansion at a given order. The ghost is not a pathology of nature but a feature of the second-order interface that is absent at lower-order interfaces.

Principle 7.1 (Effective Interface). Einstein gravity, quadratic gravity, and their gauge-theoretic analogues are effective descriptions of Π at successively higher orders in the admissibility-flow expansion: interface approximations rather than competing fundamental theories.

7.4. Gauge Symmetry as Fiber Structure

Two elements $h_1, h_2 \in M_{\mathrm{adm}}$ satisfying $\Pi(h_1) = \Pi(h_2)$ represent the same observable history by different internal paths. Gauge transformations generate exactly such equivalences: a gauge orbit in M_{adm} is a set of histories projecting to a single element of M_{obs}. Gauge symmetry is therefore not a feature of M_{obs} but of the fiber of Π—the representational redundancy of the internal description.

Diffeomorphism invariance, U(1) invariance in electrodynamics, and SU(3) invariance in QCD are three instances of the same phenomenon: the observable sector is carved out of a larger admissibility manifold by a projection whose fibers are gauge orbits. The ghost-parity symmetry of Turok and Bateman is another instance: a discrete symmetry of M_{adm} needed to make the projection to M_{obs} well-defined.

8. Observational–Interventional Separation and Latent Fundamentalism

8.1. The Underdetermination Theorem

Definition 8.1 (Observational Equivalence of Admissibility Manifolds)**.** *Two admissibility manifolds $M^{(1)}_{\mathrm{adm}}$ and $M^{(2)}_{\mathrm{adm}}$ equipped with projections Π_1 and Π_2 are* observationally equivalent *if*

$$\Pi_1\big(M^{(1)}_{\mathrm{adm}}\big) = \Pi_2\big(M^{(2)}_{\mathrm{adm}}\big) \quad \textit{as measurable spaces with probability structure.} \tag{23}$$

We denote the observational equivalence class of M_{adm} by $[M_{\mathrm{adm}}]_\Pi$.

Theorem 8.2 (Observational–Interventional Separation)**.** *The map $[M_{\mathrm{adm}}]_\Pi \mapsto M_{\mathrm{obs}}$ is not injective: the observational equivalence class does not determine the internal geometry of M_{adm}. In particular, two manifolds that differ in inner product signature, symplectic structure, or gauge group may belong to the same observational equivalence class.*

The proof is immediate: Π is a surjection from a larger space to a smaller one, so its fibers are non-trivial. Its consequence is substantive. The sign of the inner product on M_{adm} is underdetermined by any finite collection of observable transition probabilities. Requiring positive-definiteness of the inner product on M_{adm} is imposing a constraint on the fiber of Π—on the unobservable interior of the theory—without empirical justification.

8.2. Interface Geometry via Jensen–Shannon Divergence

The admissibility distance defined above still carries a residual commitment to the ontology it is trying to dissolve. The inf-sup formula

$$d_A\big(M^{(1)}, M^{(2)}\big) = \inf_{\phi} \sup_{h} \big\|\Pi_1(h) - \Pi_2(\phi(h))\big\|$$

compares images of the two projections, but the infimum runs over maps ϕ between the underlying admissibility manifolds. It still reaches into M_{adm} to do its work. A fully interface-native distance would live entirely within M_{obs}, requiring no reference to the internal structure of the theories being compared.

The Jensen–Shannon divergence provides exactly this. Given two probability distributions P and Q on M_{obs}—the observable interfaces of two admissibility manifolds, $P = \Pi_1(\mu_1)$ and $Q = \Pi_2(\mu_2)$ for some measures μ_i on $M^{(i)}_{\text{adm}}$—define

$$\text{JSD}(P\|Q) = H\left(\frac{P+Q}{2}\right) - \frac{1}{2}\big(H(P) + H(Q)\big), \tag{24}$$

where $H(\cdot)$ denotes the Shannon entropy. The quantity $\text{JSD}^{1/2}(P\|Q)$ is a proper metric on the space of probability distributions [23], and the entire expression is defined without reference to either $M^{(1)}_{\text{adm}}$ or $M^{(2)}_{\text{adm}}$: it depends only on the observable distributions P and Q.

Definition 8.3 (Interface Distance)**.** *Let $P = \Pi_1(\mu_1)$ and $Q = \Pi_2(\mu_2)$ be the observable interfaces of two admissibility manifolds equipped with reference measures μ_1, μ_2. The* interface distance *between the two theories is*

$$d_{\text{int}}(P, Q) = \text{JSD}^{1/2}(P\|Q) = \left[H\left(\frac{P+Q}{2}\right) - \frac{1}{2}\big(H(P) + H(Q)\big)\right]^{1/2}. \tag{25}$$

The shift from d_A to d_{int} is not merely notational. It represents a change in what the theory of theories is *about*. The inf-sup formulation of d_A still asks how close two internal structures can be made to look; it frames indistinguishability as an obstruction to reconstruction. The JSD formulation asks only how distinguishable two interfaces are from within the observable domain. No hidden state. No latent manifold. No appeal to a true underlying description. The geometry of comparison is intrinsic to the interface itself.

The entropy form (24) admits a decomposition that connects directly to the admissibility program:

$$\mathcal{D}(P, Q) \;=\; H\left(\frac{P+Q}{2}\right) - \frac{1}{2}\big(H(P) + H(Q)\big) \;=\; \text{JSD}(P\|Q). \tag{26}$$

The first term measures the entropy of the *merged* interface—the distribution seen by an observer who cannot distinguish which theory generated a given outcome. The second term measures the average entropy of the *separate* interfaces. Their difference is exactly the information that disappears when the distinction between the two interfaces is forgotten. We may call $\mathcal{D}(P, Q)$ the *distinction functional*: it measures how much observational content is carried by the difference between P and Q. The Jensen–Shannon divergence is its metric realisation.

Gauge Orbits as JSD-Zero Equivalence Classes

The JSD formulation provides a purely interface-native characterisation of the structures previously described in terms of internal geometry. Two histories $h, h' \in M_{\mathrm{adm}}$ are *observationally indistinguishable* when

$$\mathrm{JSD}\big(\Pi(h),\, \Pi(h')\big) = 0. \tag{27}$$

This is the correct definition of a gauge orbit from the interface perspective: a gauge transformation is a deformation of the internal history that induces zero change in the distinction functional. The internal structure of the gauge orbit—whether it is generated by a Lie group, a BRST operator, a ghost-parity discrete symmetry, or the harmonic gauge symmetry of the dimension-zero scalars—is irrelevant to its observable characterisation. All that matters is that the deformation annihilates $\mathcal{D}$.

Likewise, a *hidden admissibility direction* in the sense of Definition 3.1 can now be given a metric characterisation: it is a deformation of the internal theory along which the interface distance remains arbitrarily small,

$$\lim_{\epsilon \to 0} d_{\mathrm{int}}\big(\Pi(h),\, \Pi(h + \epsilon\, \delta h)\big) = 0 \quad \text{while} \quad \delta h \neq 0. \tag{28}$$

The ghost sector of a Krein-space theory is a hidden admissibility direction in exactly this sense: the deformation of the internal state space that adds a $\mathcal{K}_-$ component does not change the interface distance between observable transition probabilities, because ghost-parity symmetry ensures the additional contribution cancels in the traced probability functional.

Fisher Information Geometry of the Interface Space

For a parametric family of projections Π_λ, the interface distance induces a Riemannian structure on the parameter space. For infinitesimally separated interfaces Π_λ and $\Pi_{\lambda+\delta\lambda}$, the squared interface distance expands as

$$d^2_{\mathrm{int}}\big(\Pi_\lambda,\, \Pi_{\lambda+\delta\lambda}\big) = \frac{1}{8}\, g_{ij}(\lambda)\, \delta\lambda^i\, \delta\lambda^j + O(\delta\lambda^3), \tag{29}$$

where the metric tensor is

$$g_{ij}(\lambda) = \frac{\partial^2}{\partial\lambda^i\,\partial\lambda^j}\,\mathrm{JSD}^2\big(\Pi_\lambda,\,\Pi_{\lambda+\delta\lambda}\big)\Big|_{\delta\lambda=0}. \tag{30}$$

For small separations this reduces to the Fisher information metric on the manifold of probability distributions [24], establishing that $(\mathcal{M}, d_{\mathrm{int}})$ is locally a statistical manifold in the sense of information geometry.

This is not the curvature of spacetime. It is not the curvature of a quantum state manifold. It is the curvature of *distinguishability*: the rate at which the observable interfaces of nearby theories diverge as a function of the theory parameters. The admissibility geometry of quantum theories—the space $(\mathcal{M}, d_{\mathrm{int}})$—is therefore a well-defined Riemannian manifold whose geodesics are paths through theory space that minimise the total accumulated distinguishability between successive observable interfaces.

Loss Becomes Relational

The philosophical consequence of adopting d_{int} as the primitive metric on theory space is a reorientation that runs deeper than any particular result about ghosts or quadratic gravity.

Under the inf-sup formulation of d_A, the question one asks about two theories is: *what hidden content is inaccessible?* The comparison reaches into the fibers of Π and asks how different the internal structures are from each other. Loss is ontological: something exists in M_{adm} that does not appear in M_{obs}.

Under the JSD formulation of d_{int}, the question becomes: *how distinguishable are the interfaces?* The comparison lives entirely within M_{obs} and asks how much observational content separates the two probability distributions. Loss is relational: the distinction functional $\mathcal{D}(P, Q)$ measures how much information disappears when one stops distinguishing between the two interfaces. There is no commitment to a hidden object that is being lost—only to a geometry of distinction that measures how much the forgetting costs.

This reorientation places the framework in a precise relationship to the admissibility program as a whole. The program began with the conviction that reachability, admissibility, and constraint closure are the correct ontological primitives. The JSD formulation now suggests a further reduction: the primitive is not the admissibility manifold M_{adm} but the distinguishability structure on M_{obs}. The admissibility manifold is a representational tool for generating and organising interface distributions; the geometry of those distributions, measured by d_{int}, is what physics is ultimately about. In this sense the framework converges on an information-geometric version of the distinction program:

$$\text{Reality} \longrightarrow \text{Distinctions} \longrightarrow \text{Distances between distinctions,} \tag{31}$$

where the final object is the metric space $(\mathcal{M}, d_{\text{int}})$ and the Jensen–Shannon divergence is the first mathematically natural metric that lives entirely at that level without requiring access to anything underneath.

Remark 8.1. Theories in the same observational equivalence class (Definition 8.1) have $d_{\text{int}} = 0$ by construction. Theories at $d_{\text{int}} > 0$ make empirically distinguishable predictions. The Derived Interface Conjecture of Section 9 asserts that quadratic gravity and Hilbert-space quantum gravity satisfy $d_{\text{int}} = 0$ below the Planck scale, and $d_{\text{int}} > 0$ above it.

8.3. Against Latent Fundamentalism

Latent fundamentalism is the tendency to grant ontological status to internal coordinates merely because they appear in a successful model. It manifests systematically in the physics of higher-derivative theories.

In ghost arguments: the internal coordinates of the extended phase space are granted direct physical interpretation, and their unbounded range is treated as a certificate of physical instability rather than as a coordinate property of the representation.

In the Hilbert-space requirement: the positive-definiteness of the inner product on the state space is treated as a necessary condition for physical admissibility when it is at most a sufficient one.

In multiverse arguments: the proliferation of branches in the quantum state of the universe is treated as physically real on the ground that the wave function is the fundamental object of quantum mechanics. But the wave function is an element of M_{adm}, not of M_{obs}.

In each case the admissibility perspective offers the same corrective: internal coordinates are instruments of representation. Their pathologies are coordinate pathologies until proven otherwise. The physical question is always: what does Π map them to?

Turok makes this argument directly, and extends it to the assumptions behind string theory: the requirement that theories live in Hilbert spaces was "probably the strongest assumption" behind the conclusion that quantum gravity requires extra dimensions. Now that an example exists of a UV-complete quantum theory of gravity that does not live in a Hilbert space, the necessity argument for strings loses its basis. The admissibility framework generalizes this critique: the question of which internal structure corresponds to the observable world is empirical, not a priori.

8.4. A Systematic Table of Coordinate and Observable Pathologies

The distinction between coordinate and observable pathologies applies across a range of well-understood physical contexts. The following table illustrates the pattern:

Context	Coordinate Pathology	Status in M_{obs}
Lorenz-gauge QED	Negative-norm photons	Absent from physical states
BRST gauge theories	Off-shell Faddeev–Popov ghosts	Absent from BRST cohomology
Pais–Uhlenbeck model	Unbounded Hamiltonian	Bounded periodic trajectories
Ostrogradsky (gravity)	Runaway phase-space direction	Cosmological expansion
Krein-space QFT	Indefinite inner product	Positive traced probabilities
Spin-2 ghost (QG)	Negative propagator pole	Status under investigation
Gauge orbits	Representational redundancy	Single observable history
Wave function branches	Proliferating M_{adm} directions	Observable record underdetermined

The table illustrates that coordinate pathologies are the rule rather than the exception in well-understood physical theories. The history of physics is substantially a history of learning to identify which features of a formalism are representational and which are physical.

8.5. The Philosophical Position of Quadratic Gravity

The gravitational entropy program of Turok connects to the admissibility perspective in a striking way. Rather than deriving the large-scale homogeneity and isotropy of the universe from special initial conditions or a smoothing mechanism like inflation, Turok asks whether the observed universe is simply a *typical* element of M_{obs} for the relevant M_{adm}.

This is exactly the strategy of the admissibility program applied to cosmology: rather than seeking a dynamical mechanism that drives the universe toward observed states, one asks how large the admissible trajectory space is for states consistent with observation. In the admissibility framework, the gravitational entropy associated with a cosmological history is the logarithm of the volume of the corresponding admissibility set:

$$S_{\text{grav}} \sim \log \text{Vol}(\mathcal{A}_H). \tag{32}$$

The observed universe is typical if $\mathcal{A}_H$ is large—if many distinct elements of M_{adm} project to the observed large-scale structure. The problem of cosmological initial conditions is thereby reframed not as a question about dynamics but as a question about the geometry of Π.

9. The Derived Interface Conjecture

9.1. Statement

The arguments of the preceding sections converge on a single conjecture. We state it in general and then in the specific form relevant to quantum gravity.

Conjecture 9.1 (Derived Interface, General Form). Observable physical theories are effective interfaces generated by projection from a larger admissibility manifold M_{adm}. The structural features of such theories—Hilbert space, positive inner product, gauge symmetry, metric positivity—are properties of the interface, not of M_{adm} itself.

Conjecture 9.2 (Derived Interface, Quantum Gravity Form). Observable spacetime theories and their quantizations are effective interfaces of the following kind:

(i) Einstein gravity is the leading-order interface description of admissibility-flow consistency on M_{adm}.

(ii) Quadratic gravity is the second-order description, retaining first admissibility-curvature corrections. The massive ghost pole corresponds to a negative-metric direction in the fiber of Π that does not reach M_{obs} at sub-Planckian energies.

(iii) Krein-space sectors correspond to hidden admissibility directions in M_{adm} that survive internally but cancel under Π at accessible energies, provided ghost-parity symmetry is preserved.

(iv) Gauge symmetry encodes the fiber structure of Π: redundant directions in M_{adm} mapping to a single element of M_{obs}.

(v) The Hilbert Assumption is the zeroth-order interface approximation, valid when the fiber of Π is negligible and all internal degrees of freedom are observable.

9.2. Falsifiability and Empirical Content

The Derived Interface Conjecture is falsifiable. A physical effect—a scattering amplitude, a cosmological observable, a precision measurement—that could not be reproduced by any admissibility-consistent projection from any M_{adm} would refute the general form.

For the quantum gravity form, the most direct test is the one Turok identifies: whether the ghost-parity symmetry extends to the spin-2 sector of quadratic gravity. If it does, the spin-2 ghost is a hidden admissibility direction and the specific Conjecture 9.2(ii) gains strong support. If it does not, the spin-2 ghost generates observable negative probabilities at energies where it goes on-shell, and the theory as stated must be modified.

The constructive reading of the conjecture defines a research programme: given M_{obs} corresponding to current observations, what is the space of admissibility manifolds M_{adm} that project onto it? This *admissibility geometry of quantum theories* has Hilbert-space quantum mechanics as one distinguished point. Krein-space theories are neighboring points differing in the inner-product signature. Higher-derivative theories are points at greater distance, differing in kinetic structure. Exploring the geometry of this space—which regions project onto compatible M_{obs}, which do not, what distinguishes neighboring points empirically—is the task that the admissibility framework makes precise.

9.3. On the Necessity of Positive Norm

The compact summary of the paper's principal conclusion is:

$$\text{positive norm on } M_{\mathrm{adm}} \implies \text{observable admissibility,} \tag{33}$$

but the converse fails in general. Positive norm is a sufficient condition, not a necessary one. Hilbert-space quantum mechanics occupies a privileged and well-understood region of the admissibility geometry, but it is a region, not the entire geometry. Theories with indefinite inner product on M_{adm} may be perfectly admissible in M_{obs}, and the burden of proof lies with those who claim otherwise.

The implication for the programme of quantizing gravity is substantial. The conclusion that gravity requires extra dimensions or strings was predicated on the assumption that admissibility is imposed on M_{adm} rather than on M_{obs}. Relaxing this assumption does not guarantee that quadratic gravity is the correct theory of quantum gravity; it may not be. But it removes the a priori impossibility of four-dimensional quantum gravity without exotic structure, and restores the question to the domain of empirical investigation where it belongs.

10. Conclusion

We have argued that the standard dismissal of higher-derivative quantum gravity rests on the Hilbert Assumption: the identification of the full internal state space of a theory with its observable sector. Relaxing this assumption opens a coherent framework—the admissibility manifold hierarchy—in which negative-norm sectors, Krein-space structures, and quadratic gravity appear not as pathologies but as coordinate features of a larger internal space projecting onto the observable world.

The central structural results are:

The *Projection Sufficiency Principle*: physical constraints are conditions on M_{obs}, not on M_{adm}.

The *Projection Pathology Separation Theorem*: coordinate pathologies in M_{adm}: negative inner products, unbounded Hamiltonians, gauge redundancies—need not propagate to observable pathologies in M_{obs}.

The *Observable Admissibility Theorem*: a Krein-space quantum theory with ghost-parity symmetry produces non-negative observable probabilities summing to unity, regardless of the sign of individual ghost-state inner products. This is the content of the Turok–Bateman proposal, derived from the general framework.

The *Observable Stability Principle*: physical stability is determined by the admissibility of projected trajectories, not by the sign of the Hamiltonian. The Ostrogradsky theorem is a statement about M_{adm}; its implications for M_{obs} must be determined via Π.

The *Observational–Interventional Separation Theorem*: the observable record underdetermines the structure of M_{adm}, establishing a principle of ontological restraint on the interpretation of internal coordinates.

These results converge on the *Derived Interface Conjecture*: observable spacetime theories are effective interfaces generated by projection from a larger admissibility manifold. Einstein gravity and quadratic gravity are the first two terms of a systematic admissibility-flow expansion. Ghost sectors are hidden admissibility directions in the fiber of Π. Gauge symmetry is fiber structure. The Hilbert Assumption is the zeroth-order interface approximation.

Hilbert-space quantum mechanics remains the best-confirmed physical framework we possess. The argument here is not that it is wrong but that it may occupy a privileged region inside a larger admissibility geometry rather than constituting that geometry in its entirety. Krein-space theories, higher-derivative theories, and quadratic quantum gravity represent neighboring admissible sectors of that geometry—sectors that the observable interface has not, at accessible energies, distinguished from the interior.

The ghost need not be exorcised. It need only be projected.

Appendix A: The Dimension-Zero Scalar Programme

The arguments of this paper connect to a research programme, developed by Boyle and Turok, that addresses several foundational puzzles simultaneously through the introduction of *dimension-zero scalar fields.* We outline this programme here because it exemplifies, at a concrete level, how the admissibility framework leads to new physics rather than mere reinterpretation of existing results.

A.1 Ordinary versus Dimension-Zero Scalars

An ordinary conformally coupled scalar φ of mass dimension one is described by the action

$$S_2[\varphi] = \tfrac{1}{2} \int d^4x \sqrt{g}\, \varphi\, \Delta_2\, \varphi, \tag{34}$$

where $\Delta_2 = -\Box + \frac{1}{6}R$ is the unique conformally invariant second-order operator. The action is invariant under $g_{\mu\nu} \to \Omega^2 g_{\mu\nu}$, $\varphi \to \Omega^{-1}\varphi$.

A dimension-zero conformally coupled scalar is instead described by

$$S_4[\varphi] = \pm\tfrac{1}{2} \int d^4x \sqrt{\pm g}\, \varphi\, \Delta_4\, \varphi, \tag{35}$$

where the sign convention follows from demanding positivity of the Euclidean action and then Wick-rotating, and Δ_4 is the unique conformally invariant fourth-order operator,

$$\Delta_4 = \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \tfrac{2}{3}R\,\Box + \tfrac{1}{3}(\nabla^\mu R)\nabla_\mu. \tag{36}$$

This action is invariant under $g_{\mu\nu} \to \Omega^2 g_{\mu\nu}$, $\varphi \to \Omega^0\varphi$: the field does not transform under Weyl rescalings, which accounts for the dimension-zero designation.

In flat spacetime the action reduces to

$$S_4 = \pm\tfrac{1}{2} \int d^4x\, \varphi\, \Box^2\varphi = \pm\tfrac{1}{2} \int d^4x\, (\Box\varphi)^2. \tag{37}$$

This is a fourth-order theory, so one should expect ghost states in the propagator. However, as Bogoliubov et al. established and Boyle and Turok exploited, the action (A.3) possesses an infinite-dimensional gauge symmetry

$$\varphi \;\longrightarrow\; \varphi + \chi, \quad \Box\chi = 0. \tag{38}$$

This symmetry shifts φ by any harmonic function, and once it is properly taken into account, the theory has *no local degrees of freedom at all.* Every local gauge-invariant operator commutes, and the theory possesses a unique physical state: the vacuum. This is the decisive fact about dimension-zero scalars. Their ghost problem is not resolved by projecting onto a positive Hilbert subspace but by the gauge symmetry making all local degrees of freedom pure gauge.

A.2 The Vacuum Energy and the Weyl Anomaly

The dimension-zero scalar contributes to the vacuum energy per Fourier mode with a coefficient of opposite sign to ordinary scalars. Combining the contributions from dimension-one scalars (n_0), Weyl fermions ($n_{1/2}$), gauge bosons (n_1), and dimension-zero scalars (n_0'), one finds

$$E_k^{(0)} = \frac{\hbar k}{2}\big[n_0 - 2n_{1/2} + 2n_1 + 2n_0'\big]. \tag{39}$$

Similarly, the trace of the stress-energy tensor in curved spacetime—the Weyl anomaly—takes the form

$$\langle T^\mu_\mu \rangle = c\,C^2 - a\,E + \xi R, \tag{40}$$

where $C^2 = C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$ is the square of the Weyl tensor, $E = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$ is the Euler density, and the coefficients at one loop are

$$a = \frac{1}{360(4\pi)^2}\big[n_0 + \tfrac{11}{2}n_{1/2} + 62n_1 - 28n_0'\big], \tag{41}$$

$$c = \frac{1}{120(4\pi)^2}\big[n_0 + 3n_{1/2} + 12n_1 - 8n_0'\big]. \tag{42}$$

The key observation of Boyle and Turok is that dimension-zero scalars contribute *negatively* to both a and c, providing an alternative to supersymmetry as a mechanism for Weyl anomaly cancellation.

A.3 The Three-Generation Prediction

Consider the standard model gauge group SU(3) × SU(2) × U(1) with $n_1 = 12$ gauge bosons, and suppose the Higgs is composite or emergent so that $n_0 = 0$. The conditions for simultaneous cancellation of the vacuum energy and both terms in the Weyl anomaly—i.e. setting $E_k^{(0)} = 0$, $a = 0$, $c = 0$—constitute three equations in the four unknowns $\{n_0, n_{1/2}, n_1, n_0'\}$. The unique solution is

$$n_{1/2} = 4n_1, \qquad n_0' = 3n_1, \qquad n_0 = 0. \tag{43}$$

For $n_1 = 12$, this gives $n_{1/2} = 48 = 3 \times 16$: exactly three generations of standard model fermions, each including a right-handed neutrino. The number of dimension-zero scalars required is $n_0' = 36$.

This is a remarkable result: the number of fermion generations is not a free parameter of the model but a consequence of requiring simultaneous cancellation of three independent gravitational anomalies. The result is non-trivial in that $E_k^{(0)}$,

a, and c are completely independent linear combinations of $\{n_0, n_{1/2}, n_1, n_0'\}$; their simultaneous vanishing could easily have been inconsistent.

A.4 Admissibility Interpretation

In the language of this paper, the dimension-zero scalar programme is exactly an instance of the Projection Sufficiency Principle. The fourth-order action in M_{adm} carries coordinate pathologies—ghost poles in the propagator, an indefinite inner product. But once the gauge symmetry $\varphi \to \varphi + \chi$ is accounted for, the projection Π to M_{obs} eliminates all local degrees of freedom. The observable sector is simply the vacuum, with its scale-invariant fluctuation spectrum. The ghost never reaches M_{obs}.

The debate between Boyle–Turok and their critics (Cline and Hell, 2025) concerns exactly whether the gauge symmetry is sufficient to remove the ghost from M_{obs}, or whether the ghost survives under certain background conditions (curved spacetime, time-dependent background field). In the admissibility framework, this is the question of whether the projection Π is sufficiently robust across all physical backgrounds, not merely in flat spacetime. It is a concrete instance of the general question posed in Conjecture 9.2: does the relevant symmetry—whether ghost-parity or the harmonic-function gauge symmetry—extend from the restricted sector to the full theory?

Appendix B: The CPT-Symmetric Universe and Gravitational Entropy

B.1 The CPT Universe

Boyle, Finn, and Turok proposed in 2018 that the state of the universe does not spontaneously violate CPT: the universe after the Big Bang is the CPT image of the universe before it, both classically and quantum mechanically. The pre- and post-bang epochs comprise a universe/anti-universe pair emerging from a conformal singularity at $\tau = 0$.

The key classical observation is that the radiation-dominated flat FRW metric takes the form $ds^2 = a^2(\tau)(-d\tau^2 + d\mathbf{x}^2)$ with $a(\tau) \propto \tau$ near the bang. Analytically continuing across $\tau = 0$ reveals a new isometry: time reversal $\tau \to -\tau$. The CPT symmetry condition on the tetrad is

$$e^a_\mu(\tau, \mathbf{x}) = -e^a_\mu(-\tau, \mathbf{x}), \tag{44}$$

which immediately implies that $a(\tau)$ is an odd function with a simple zero at $\tau = 0$.

The quantum consequences are substantial. In a general curved spacetime, the choice of vacuum is ambiguous: different observers at different epochs define inequivalent vacua related by Bogoliubov transformations. The CPT symmetry, however, selects a unique preferred vacuum $|0_0\rangle$ invariant under $CPT|0_0\rangle = |0_0\rangle$. This CPT-symmetric vacuum is the geometric mean of the vacua preferred by observers in the far past and far future.

The CPT condition on the scalar perturbations selects the regular, growing mode and eliminates the singular decaying mode near $\tau = 0$—precisely the boundary condition responsible for the acoustic oscillations in the CMB power spectrum that are usually attributed to inflation. For tensor perturbations, the same condition eliminates primordial gravitational waves: the CPT-symmetric universe predicts $r = 0$ to this order, distinguishing it from inflationary models.

B.2 The Gravitational Entropy Calculation

Boyle and Turok developed a thermodynamic explanation for the observed flatness, homogeneity, and isotropy of the universe based on a new calculation of gravitational entropy. The central observation is that cosmological solutions with radiation and a cosmological constant can be analytically continued to Euclidean instantons via a combined conformal and Wick rotation (CW rotation), in which both $n \to -iN$ (standard Wick rotation) and $a(t) \to ib(t)$ (conformal rotation). The latter is required because $a(\tau)$ has a zero at the bang and becomes imaginary under standard Wick rotation.

The gravitational entropy of a cosmological solution is the semiclassical exponent of the Euclidean path integral:

$$S_g = iS_{\text{Lorentzian}} = -S_E. \tag{45}$$

For a universe with cosmological constant ρ_Λ, radiation entropy S_r, and spatial curvature κ, the calculation yields

$$S_g \sim S_\Lambda^{1/4} S_r, \tag{46}$$

where $S_\Lambda = 24\pi^2 M_{\text{Pl}}^4/\rho_\Lambda$ is the de Sitter entropy.

The physical interpretation is as follows. The partition function of the cosmological ensemble is $Z = e^{S_g}$. The total entropy is maximized when the universe is flat, homogeneous, and isotropic with a small positive cosmological constant. The observed large-scale geometry of the cosmos is therefore typical rather than fine-tuned: it is the most probable macrostate of the gravitational ensemble. Inflation is no longer needed as a dynamical smoothing mechanism.

In the language of the admissibility framework, the gravitational entropy is the logarithm of the volume of the admissibility set of cosmological histories consistent with the observed large-scale parameters:

$$S_g \sim \log \mathrm{Vol}(\mathcal{A}_H^{\mathrm{cosm}}). \tag{47}$$

The observed universe is typical because $\mathcal{A}_H^{\mathrm{cosm}}$ is large: many distinct elements of M_{adm} project to the observed flat, homogeneous observable sector. This is a reachability-volume argument applied to cosmological histories, and it connects the gravitational entropy programme directly to the admissibility geometry of Section 9.

B.3 Dark Matter as a CPT Consequence

In the CPT-symmetric universe, the right-handed neutrinos (which must exist by the three-generation prediction of Appendix A) are produced from the CPT-symmetric vacuum by the same mechanism that produces Hawking radiation from black holes. The ν_R field equation is regular at the bang; the CPT-symmetric vacuum state then determines a unique initial condition for ν_R production, from which the density of right-handed neutrinos in the observable universe can be calculated without free parameters.

If one of the three right-handed neutrinos is stable—which follows from a $\mathbb{Z}_2$ symmetry that simultaneously makes the lightest neutrino massless—its dark matter density matches the observed Ω_{DM} when its mass is $M \approx 5 \times 10^8\,\mathrm{GeV}$. This is a prediction, not a fit: the dark matter density is determined by the same CPT boundary conditions that fix the CMB power spectrum.

The theory also predicts: (i) the three light neutrinos are Majorana and permit neutrinoless double beta decay; (ii) the lightest neutrino is massless; (iii) no primordial long-wavelength gravitational waves ($r = 0$ at this order).

Appendix C: The Criticism and the Admissibility Response

The criticism of Cline and Hell (2025) targets the dimension-zero scalar programme directly. Their argument is that the ghost in the $S_4[\varphi]$ theory is not eliminated by the harmonic gauge symmetry and persists as an observable instability, particularly in curved spacetime backgrounds.

Their central technical point is the following. In flat spacetime, the Hubbard–Stratonovich decomposition of S_4 introduces an auxiliary field σ and yields the two-derivative Lagrangian

$$S \simeq \int d^4x\big(-\partial_\mu\sigma\,\partial^\mu\varphi - \tfrac{1}{2}\sigma^2\big)\,, \tag{48}$$

whose kinetic matrix has eigenvalues ± 1, revealing a ghost. The corresponding Hamiltonian density

$$\mathcal{H} = \dot{\varphi}\,\dot{\sigma} + \nabla\varphi \cdot \nabla\sigma + \tfrac{1}{2}\sigma^2 \tag{49}$$

is unbounded below, and explicit wave-like solutions demonstrate locally unbounded energy density. In FRW backgrounds, Cline and Hell show that the coefficient c_1 of the kinetic term for scalar perturbations becomes negative at large wave numbers, indicating superluminal ghost modes. They also argue that coupling to standard model fields induces a confining fifth force that rules out the dimension-zero scalars as a source of primordial perturbations.

From the admissibility perspective, the disagreement is a disagreement about whether the projection Π maps the ghost to the fiber of Π or to M_{obs}. Boyle and Turok's claim is that the gauge symmetry $\varphi \to \varphi + \chi$ removes the ghost from any physical prediction because all local gauge-invariant operators commute. Cline and Hell's claim is that this argument applies only in flat spacetime and that the gauge symmetry does not extend to FRW backgrounds in a way that continues to eliminate the ghost from the physical spectrum.

This is precisely the kind of case-by-case question that the Projection Pathology Separation Theorem identifies as needing separate analysis. The theorem establishes that coordinate pathologies *can* fail to propagate to M_{obs}; it does not establish that they always do so. Whether the harmonic gauge symmetry is robust against curved-background perturbations is a technical question that the general framework leaves open, to be resolved by detailed calculation.

The exchange between Boyle–Turok and Cline–Hell is therefore a concrete instantiation of the admissibility programme in action: the ghost in M_{adm} is identified; the question is whether a specific symmetry maps it to the fiber of Π under all physically relevant projection conditions. The current status is that this question is not fully resolved, and the outcome will determine whether the dimension-zero scalar programme is viable.

This is characteristic of the general situation identified in Section 9: the admissibility framework does not make theories easy; it makes the right question precise. Whether a ghost is an observable pathology or a hidden admissibility direction is always a question about Π, and answering it requires both the general framework and the specific technical analysis of the theory in question.

Appendix D: Scale-Invariant Primordial Perturbations Without Inflation

One of the most striking predictions of the combined programme is that the dimension-zero scalars can generate the observed scale-invariant spectrum of primordial density perturbations *without inflation*. We sketch the mechanism here because it connects the UV structure of the theory (the dimension-zero scalar action) to the IR observations (the CMB power spectrum), illustrating how the admissibility framework connects across scales.

The two-point function of the dimension-zero scalar in flat spacetime is

$$\langle \varphi(t, \mathbf{x})\, \varphi(t, \mathbf{x}') \rangle = \int \frac{d^3k}{(2\pi)^3}\, e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}\, \frac{1}{4k^3}. \tag{50}$$

The factor $1/4k^3$ is scale invariant: the power spectrum $P(k) \propto k^{-3}$ means that each logarithmic interval of k contributes equally to $\langle \varphi^2 \rangle$. This is *Harrison–Zel'dovich* scale invariance, the same spectrum observed in the CMB, arising here from the conformal invariance of S_4 rather than from inflationary dynamics.

In a flat FRW spacetime, conformal invariance ensures that the scale factor $a(\tau)$ does not appear in the action (A.3), so the same calculation applies cosmologically: the vacuum fluctuations of φ are cosmologically scale invariant.

The coupling between the dimension-zero scalars and standard model running couplings generates a slight red tilt. The running coupling constant $g(\mu)$ varies with energy scale, and this variation introduces a small breaking of scale invariance. For the standard model gauge couplings extrapolated to the Planck scale, the predicted spectral index is approximately

$$n_s \approx 0.958, \tag{51}$$

in good agreement with the Planck satellite measurement $n_s = 0.959 \pm 0.006$. The tensor-to-scalar ratio is predicted to be zero (no primordial gravitational waves), consistent with the current upper bound $r < 0.036$ from BICEP/Keck.

The connection to the admissibility framework is via the unistochastic picture of Section 6. The primordial scalar perturbations are transition probabilities from the initial vacuum configuration to the late-time CMB configuration, computed by integrating over field trajectories in M_{adm} weighted by $|\Phi|^2$. The scale invariance of the perturbation spectrum reflects the conformal invariance of the S_4 dynamics on M_{adm}; the slight red tilt reflects the running of couplings that breaks conformal invariance at finite energy. The observable CMB spectrum is Π applied to this ensemble of field trajectories.

Appendix E: Consolidated Proof Scaffold

The following collects tightened proofs of the principal results, filling gaps identified in the main text and adding several results that complete the synthesis. Notational conventions follow the main text.

E.1 Projection Sufficiency as a Consequence of Definition

Theorem 10.1 (Projection Sufficiency, formal version)**.** *Let $A_O : M_{\text{obs}} \to \{0,1\}$ be the observable admissibility predicate, and let $A_H(h) = A_O(\Pi(h))$ be its pullback to M_{adm}. If physical predictions depend only on $A_O(\Pi(h))$, then no condition on $h \in M_{\text{adm}}$ is physically necessary unless it changes $\Pi(h)$.*

Proof. Let $h, h' \in M_{\text{adm}}$ satisfy $\Pi(h) = \Pi(h')$. Then $A_H(h) = A_O(\Pi(h)) = A_O(\Pi(h')) = A_H(h')$. Any property distinguishing h from h' while leaving $\Pi(h)$ fixed has no observable effect. Physical constraints may therefore be imposed entirely on M_{obs} without imposing them on every coordinate direction of M_{adm}. □

Corollary 10.2 (Hilbert Assumption as injective special case)**.** *Positive-definite Hilbert quantum mechanics is the special case in which Π is injective. When Π is injective, $\Pi(h) = \Pi(h')$ implies $h = h'$, so every fiber is a singleton and there are no hidden admissibility directions. Every condition on internal coordinates is immediately a condition on observable predictions.*

Proof. Injectivity of Π means $\Pi^{-1}(o) = \{h\}$ for each $o \in M_{\text{obs}}$. There is therefore no $h \neq h'$ with $\Pi(h) = \Pi(h')$; the fiber is trivial, the internal coordinate is fully observable, and the Hilbert Assumption holds. □

E.2 Ghost States: Negative Norm Is Not Negative Probability

The following makes precise the claim that negative norm is a coordinate pathology rather than an observable pathology, using the Krein space structure of Section 3.

Proposition 10.3. *Let $g \in \mathcal{K}_-$ so that $[g,g]_{\mathcal{K}} < 0$. Unless the observable probability functional is literally equal to the Krein inner product, $[g,g]_{\mathcal{K}} < 0$ does not imply $P_{\text{obs}} < 0$.*

Proof. Observable probability is a functional on M_{obs}, not on M_{adm}. It has the form $P_{\text{obs}}(F \mid I) = f(\Pi(g))$ for some function f on M_{obs}. The statement $[g,g]_{\mathcal{K}} < 0$ is a statement about the Krein inner product on M_{adm}. Unless $f(\Pi(\cdot)) = [\cdot,\cdot]_{\mathcal{K}}$ identically — which is precisely the Hilbert Assumption in Krein form — the sign of $[g,g]_{\mathcal{K}}$ is independent of the sign of P_{obs}. □

E.3 Ostrogradsky: State Pathology versus History Pathology

Proposition 10.4. *Ostrogradsky unboundedness is a pathology of the extended phase-space representation, not of projected observable histories.*

Proof. Ostrogradsky's theorem establishes

$$\inf_{(Q,P)\in T^*M_{\mathrm{adm}}^{\mathrm{ext}}} H(Q,P) = -\infty.$$

Observable stability is the assertion that projected histories remain admissible:

$$A_O(\Pi(\gamma(t))) = 1 \quad \text{for all } t \in [0,T].$$

The first statement quantifies over all points in the extended phase space. The second quantifies over actual dynamical trajectories and their projections. A trajectory $\gamma(t)$ may avoid the unbounded regions of $T^*M_{\mathrm{adm}}^{\mathrm{ext}}$ entirely — the Pais–Uhlenbeck oscillator does exactly this, as its solutions are bounded periodic orbits despite the Hamiltonian being unbounded below. Therefore H unbounded below does not imply $\Pi(\gamma(t))$ unstable. □

E.4 Born Rule as a Projection from Rotation-Scaling Transport

Proposition 10.5. *The Born rule is a projection from the amplitude space (rotation-scaling geometry) to the observable intensity space.*

Proof. Write a quantum transition amplitude $a_{FI} = r_{FI}e^{i\theta_{FI}} \in \mathbb{C}$. The Born rule assigns probability $P(F \mid I) = |a_{FI}|^2 = r_{FI}^2$. The map $a_{FI} \mapsto |a_{FI}|^2$ is many-to-one: any two amplitudes $re^{i\theta_1}$ and $re^{i\theta_2}$ with $\theta_1 \neq \theta_2$ produce the same probability r^2. The phase θ_{FI} lies in the fiber of this map; it is a hidden direction in amplitude space that does not appear in M_{obs}. The Born rule is therefore a surjective map from the complex amplitude geometry of M_{adm} to the non-negative real intensity geometry of M_{obs}: a projection in the precise sense of Definition 2.1. □

E.5 Irreversibility from Non-Invertibility of Projection

Theorem 10.6 (Irreversibility from Non-Invertibility). *If $\Pi : M_{\mathrm{adm}} \to M_{\mathrm{obs}}$ is non-injective, then no reconstruction map $R : M_{\mathrm{obs}} \to M_{\mathrm{adm}}$ satisfies $R \circ \Pi = \mathrm{id}_{M_{\mathrm{adm}}}$ on all of M_{adm}.*

Proof. Since Π is non-injective, there exist $h \neq h'$ in M_{adm} with $\Pi(h) = \Pi(h')$. If $R \circ \Pi = \mathrm{id}$, then $h = R(\Pi(h)) = R(\Pi(h')) = h'$, contradicting $h \neq h'$. Hence no perfect reconstruction exists. □

Definition 10.7 (Reconstruction Defect and Irreversibility). *Given a reconstruction map $R : M_{\text{obs}} \to M_{\text{adm}}$ and a metric d on M_{adm}, the* reconstruction defect *of h is*

$$\Delta_R(h) = d\big(h,\ R(\Pi(h))\big). \tag{52}$$

A system is irreversible at h *if $\Delta_R(h) > 0$ for every reconstruction map R.*

Remark 10.1. Under this definition, irreversibility is not a property of a dynamics but of a projection: it is the persistent failure of any map from observations back to the generating history. Thermodynamic irreversibility, memory loss, and the arrow of time all become special cases of the same structure — non-invertible projection — distinguished only by the space M_{adm} and metric d in question. This connects the admissibility framework to a trajectory-based account of the second law.

E.6 Jensen–Shannon Distance: Interface-Nativity and Fisher Expansion

The following consolidates the properties of d_{int} established in Section 8.2 into a single self-contained block.

Proposition 10.8 (Interface-Nativity). *The Jensen–Shannon divergence* $\text{JSD}(P\|Q)$ *compares observable interfaces without reference to any underlying admissibility manifold.*

Proof. The definition $\text{JSD}(P\|Q) = H(\frac{P+Q}{2}) - \frac{1}{2}(H(P)+H(Q))$ uses only P, Q, their mixture $M = \frac{1}{2}(P+Q)$, and the Shannon entropy H. No element of M_{adm} appears. The computation is therefore intrinsic to the observable distribution space. □

Theorem 10.9 (Gauge Equivalence as Zero Interface Distance). *Two histories $h, h' \in M_{\text{adm}}$ are observationally equivalent if and only if* $\text{JSD}(\Pi(h), \Pi(h')) = 0$.

Proof. Since JSD is a metric on probability distributions, $\text{JSD}(P,Q) = 0 \iff P = Q$. Therefore $\text{JSD}(\Pi(h), \Pi(h')) = 0 \iff \Pi(h) = \Pi(h')$), which is precisely the condition that h and h' lie in the same fiber of Π (Definition 8.1). Gauge orbits are therefore the zero-distance fibers of the interface metric. □

Proposition 10.10 (Fisher Expansion of Interface Metric). *For a parametric family P_λ of observable distributions,*

$$\text{JSD}^2(P_\lambda, P_{\lambda+\epsilon}) = \frac{1}{8}\sum_x \frac{(\delta P(x))^2}{P_\lambda(x)} + O(|\epsilon|^3) = \frac{1}{8} g^F_{ij}(\lambda)\, \epsilon^i \epsilon^j + O(|\epsilon|^3), \tag{53}$$

where $g^F_{ij}(\lambda) = \sum_x P_\lambda(x)\, \partial_i \log P_\lambda(x)\, \partial_j \log P_\lambda(x)$ is the Fisher information metric.

Proof. Let $\delta P(x) = P_{\lambda+\epsilon}(x) - P_\lambda(x)$ and $M(x) = P_\lambda(x) + \frac{1}{2}\delta P(x)$. For small ϵ, the KL expansion $D_{\text{KL}}(P\|M) = \sum_x P(x)\log(P(x)/M(x))$ yields $D_{\text{KL}}(P\|M) =$

$\frac{1}{8}\sum_x(\delta P(x))^2/P_\lambda(x) + O(|\epsilon|^3)$. Since $\mathrm{JSD}(P\|Q) = \frac{1}{2}D_{\mathrm{KL}}(P\|M) + \frac{1}{2}D_{\mathrm{KL}}(Q\|M)$, and both KL terms contribute equally at leading order, the result follows with $\delta P(x) = \partial_i P_\lambda(x)\,\epsilon^i$. □

E.7 Distinction Loss Functional

Definition 10.11 (Distinction Functional)**.** *For observable distributions P, Q on M_{obs}, the* distinction functional *is*

$$\mathcal{D}(P,Q) = H\left(\frac{P+Q}{2}\right) - \frac{1}{2}\big(H(P) + H(Q)\big) = \mathrm{JSD}(P\|Q). \tag{54}$$

Proposition 10.12. *$\mathcal{D}(P,Q)$ measures the information destroyed when the distinction between P and Q is erased by replacing both with their mixture.*

Proof. The entropy $H(\frac{P+Q}{2})$ is the uncertainty of an observer who cannot distinguish which interface generated each observation. The average $\frac{1}{2}(H(P) + H(Q))$ is the uncertainty when the distinction is retained. Their difference $\mathcal{D}(P,Q) \geq 0$ is the additional uncertainty caused by forgetting the distinction. It vanishes if and only if $P = Q$, in which case the distinction carries no information. □

E.8 Summary Correspondence Table

The following table translates every major concept of the admissibility framework into the language of interface geometry and JSD.

Admissibility concept	Interface-geometric translation
Gauge orbit	Zero-JSD fiber of Π: $\mathrm{JSD}(\Pi(h), \Pi(h')) = 0$
Hidden admissibility direction	Deformation with vanishing first-order δ_{obs}
Ghost state	Element of $\mathcal{K}_-$ mapping to zero-JSD deformation
Hilbert Assumption	Π injective: no non-trivial zero-JSD fibers
Observable pathology	$\mathcal{D}(P, P') > 0$ induced by the deformation
Irreversibility	Positive reconstruction defect $\Delta_R > 0$
Born rule	Projection $re^{i\theta} \mapsto r^2$ from amplitude to intensity
Theory comparison	$d_{\mathrm{int}}(P, Q) = \mathrm{JSD}^{1/2}(P\|Q)$
Fisher geometry of theories	$g_{ij} = \frac{1}{8} I_{ij}(\lambda)$ at leading order
Derived Interface Conjecture	$d_{\mathrm{int}}(\mathrm{QG}, \text{quadratic gravity}) = 0$ below M_{Pl}

E.9 The Mature Framework in One Chain

The progression from internal machinery to distinguishability geometry can be compressed into a single diagram:

$$M_{\mathrm{adm}} \xrightarrow{\Pi} M_{\mathrm{obs}} \xrightarrow{\mathrm{JSD}} (\mathcal{M},\, d_{\mathrm{int}}). \tag{55}$$

The first arrow discards internal structure; what survives is the observable interface. The second arrow measures distinguishability between interfaces; what survives is the geometry of distinction. The question

$$\text{“What exists?”} \tag{56}$$

is replaced by

$$\text{“What distinctions survive projection?”} \tag{57}$$

Ghost states, gauge orbits, and Hilbert-space structure are all answers to the question of what does *not* survive: they are zero-JSD fibers, hidden directions, and special cases of injective projection, respectively. The geometry of what does survive — the metric space $(\mathcal{M}, d_{\mathrm{int}})$ — is the object that the theory of physical representation is ultimately about.

References

[1] K. S. Stelle, "Renormalization of higher-derivative quantum gravity," *Physical Review D* **16**, 953–969 (1977).

[2] K. S. Stelle, "Classical gravity with higher derivatives," *General Relativity and Gravitation* **9**(4), 353–371 (1978).

[3] I. G. Avramidi and A. O. Barvinsky, "Asymptotic freedom in higher-derivative quantum gravity," *Physics Letters B* **159**(4–6), 269–274 (1985).

[4] A. Pais and G. E. Uhlenbeck, "On field theories with non-localized action," *Physical Review* **79**, 145–165 (1950).

[5] C. M. Bender and P. D. Mannheim, "No-ghost theorem for the fourth-order derivative Pais–Uhlenbeck oscillator model," *Physical Review Letters* **100**, 110402 (2008).

[6] M. V. Ostrogradsky, "Mémoires sur les équations différentielles, relatives au problème des isopérimètres," *Mémoires de l'Académie impériale des sciences de Saint-Pétersbourg* **VI**(4), 385–517 (1850).

[7] R. P. Woodard, "Ostrogradsky's theorem on Hamiltonian instability," *Scholarpedia* **10**(8), 32243 (2015).

[8] P. D. Mannheim, "Making the case for conformal gravity," *Foundations of Physics* **42**, 388–420 (2012).

[9] A. Salvio and A. Strumia, "Agravity up to infinite energy," *European Physical Journal C* **78**, 124 (2018).

[10] D. Anselmi and M. Piva, "A new formulation of Lee–Wick quantum field theory," *Journal of High Energy Physics* **2017**(6), 066 (2017).

[11] M. G. Kreĭn and H. Langer, "On some mathematical principles in the linear theory of damped oscillations of continua," *Integral Equations and Operator Theory* **1**(3), 364–399 (1978).

[12] K. Bleuler, "Eine neue Methode zur Behandlung der longitudinalen und skalaren Photonen," *Helvetica Physica Acta* **23**, 567–586 (1950).

[13] L. D. Faddeev and V. N. Popov, "Feynman diagrams for the Yang–Mills field," *Physics Letters B* **25**(1), 29–30 (1967).

[14] C. Becchi, A. Rouet, and R. Stora, "Renormalization of gauge theories," *Annals of Physics* **98**(2), 287–321 (1976).

[15] N. Turok and L. Boyle, "A minimal explanation of the primordial cosmological perturbations," *Physical Review Letters* **132**, 061402 (2024).

[16] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space*, Cambridge University Press, Cambridge, 1982.

[17] J. F. Donoghue, "General relativity as an effective field theory: the leading quantum corrections," *Physical Review D* **50**, 3874–3888 (1994).

[18] S. Weinberg, "Ultraviolet divergences in quantum theories of gravitation," in *General Relativity: An Einstein Centenary Survey*, ed. S. W. Hawking and W. Israel, Cambridge University Press, Cambridge, 1979, pp. 790–831.

[19] S. Deser, "Ostrogradski, ghosts, and gauge as shadows of higher derivatives," talk given at the International School on Strings and Fundamental Physics, Munich, 2009.

[20] T. Needham, *Visual Complex Analysis*, Oxford University Press, Oxford, 1997.

[21] J. A. Barandes, "The unistochastic quantum theory," *arXiv:2302.10778* [quant-ph] (2023).

[22] J. A. Barandes, "Is quantum mechanics just thermodynamics in disguise?" *arXiv:2407.09663* [quant-ph] (2024).

[23] D. M. Endres and J. E. Schindelin, "A new metric for probability distributions," *IEEE Transactions on Information Theory* **49**(7), 1858–1860 (2003).

[24] S.-I. Amari and H. Nagaoka, *Methods of Information Geometry*, American Mathematical Society, Providence, 2000.

[25] L. Boyle and N. Turok, "Cancelling the vacuum energy and Weyl anomaly in the standard model with dimension-zero scalar fields," *arXiv:2110.06258* [hep-th] (2022).

[26] L. Boyle, K. Finn, and N. Turok, "CPT-symmetric universe," *Physical Review Letters* **121**, 251301 (2018).

[27] L. Boyle, K. Finn, and N. Turok, "The Big Bang, CPT, and neutrino dark matter," *Annals of Physics* **438**, 168767 (2022).

[28] N. Turok and L. Boyle, "Gravitational entropy and the flatness, homogeneity and isotropy puzzles," *Physics Letters B* **849**, 138442 (2024).

[29] J. M. Cline and A. Hell, "Pathologies of dimension-zero scalar fields," *arXiv:2603.05683* [hep-th] (2025).

[30] L. Boyle and N. Turok, "Thermodynamic solution of the homogeneity, isotropy and flatness puzzles (and a clue to the cosmological constant)," *Physics Letters B* **849**, 138443 (2024).

[31] C. M. Bender and P. D. Mannheim, "Exactly solvable $\mathcal{PT}$-symmetric models in two dimensions," *Journal of Physics A: Mathematical and Theoretical* **41**, 244005 (2008).

[32] N. N. Bogolubov, A. A. Logunov, A. I. Oksak, and I. T. Todorov, *General Principles of Quantum Field Theory*, Kluwer Academic Publishers, Dordrecht, 1990.

[33] S. W. Hawking and T. Hertog, "Living with ghosts," *Physical Review D* **65**, 103515 (2002).

[34] M. J. Duff, "Twenty years of the Weyl anomaly," *Classical and Quantum Gravity* **11**, 1387–1404 (1994).

[35] R. Penrose, "Conformal treatment of infinity," in *Relativity, Groups and Topology*, eds. C. DeWitt and B. S. DeWitt, Gordon and Breach, London, 1964.