Xylomorphic Computation: Thermodynamic Closure as a Stability Condition for Large-Scale Computational Infrastructure

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ABSTRACT

We introduce *xylomorphic computation*, a framework in which computational infrastructures recursively generate their own enabling substrates from operational residues, modeled formally on collectively autocatalytic sets. The central contribution is the *xylomorphic order parameter* λ, defined as the per-cycle contraction ratio of exogenous entropy dependence under the xylomorphic endofunctor $X = \text{Print} \circ \text{Digest} \circ \text{Shed}$. Admissible morphisms are defined precisely via mass-energy balance, entropy non-increase modulo accounted influx, and compatibility with the xylomorphic cycle, closing the foundational gap in prior categorical treatments. We prove that λ is the unique multiplicative resource monotone compatible with the monoidal compositional structure, establish it as a spectral quantity via the linearized operator, and show it is invariant under all admissible base changes in a fibered category of physical domains. The Xylomorphic Stability Theorem proves that thermodynamic stability under scaling is achieved if and only if $\lambda < 1$; the Banach fixed-point theorem gives the unique attractor I^* with geometric convergence rate. The xylomorphic dynamics are derived as the gradient flow of an explicit free-energy functional $\mathcal{S}(\rho, \theta, m)$; existence of weak solutions is established by a full four-step Galerkin argument (Cauchy–Lipschitz, Grönwall energy bound, Aubin–Lions compactness, strong–weak passage to limit); an explicit linearized mode matrix gives a computable sufficient stability criterion. Three governing dimensionless groups—thermal reintegration number Ti, degradation-to-repair ratio Dr, and diffusion-to-reaction ratio Da—restate all stability conditions in engineering terms. A worked homogeneous example computes λ in closed form and identifies the critical heat-capture fraction α_c at which the system crosses from contractive to expansive. The framework is extended to account for seasonal thermal demand, exergy quality of heat, endogenous degradation, informational locality, and semantic efficiency, yielding the effective order parameter λ_{eff} and a polar siting sufficiency theorem that formalizes the physical invariants underlying high-latitude deployment. Six implementable design principles translate the formal results into engineering constraints. The framework is stated in falsifiable terms, and the Proof-of-Useful-Work-and-Heat protocol is derived as an operational sufficient condition for Lyapunov descent of $\mathcal{S}$ and hence a certification of the contractive regime.

1. INTRODUCTION

Contemporary discourse on the energy costs of large-scale computation suffers from a systematic accounting error. Computational heat is classified as waste, data center siting is treated as a purely logistical problem, and proposals to relocate infrastructure to orbital or lunar environments are evaluated as if computation were a location-independent abstraction. None of these assumptions survive thermodynamic scrutiny.

Computation is a physical process. Its terminal state is entropy production. Whether that entropy is expelled as an externality or reintegrated as a productive input is not incidental to infrastructure design but definitional of its long-run behavior. A system that continuously expels entropy accumulates growing external dependencies. A system that reintegrates entropy reduces those dependencies over successive cycles. These two classes exhibit qualitatively different behavior under scaling, and the distinction can be made mathematically precise.

This paper develops that distinction. We define the xylomorphic endofunctor on a category of infrastructure states, characterize the conditions under which iterates converge to a stable attractor, introduce the order parameter λ as the unique resource invariant compatible with this structure, and connect the discrete iteration to a continuum PDE system via a variational principle.

The word *xylomorphic* derives from the Greek *xylon* (wood) and *morphē* (form), invoking the way arboreal systems produce structural material that sustains further growth. In computational terms, a xylomorphic infrastructure is one whose operational outputs autoregressively feed back as inputs to its own substrate formation.

Contributions. We prove that λ is (i) the unique multiplicative resource monotone under monoidal composition, (ii) the spectral radius of the linearized xylomorphic operator, (iii) invariant under all admissible base changes in a fibered category over physical domains, and (iv) the exponential contraction factor of the dominant decay mode of the explicit continuum flow. The paper extends the baseline framework to account for seasonal thermal demand, exergy quality, endogenous degradation, informational locality, and semantic efficiency, deriving the effective order parameter λ_{eff} and a polar siting sufficiency theorem. Six design principles translate the formal results into engineering constraints. All main results are stated in falsifiable terms.

2. BACKGROUND

2.1. Thermodynamics of Computation

The irreducible thermodynamic cost of computation was established by Landauer [1961]: erasing one bit of information dissipates at least $k_B T \ln 2$ of heat. Bennett [1982] extended this to a comprehensive treatment of reversible computation. Lloyd [2000] derived ultimate physical bounds on operations per unit energy. These results ground the framework: entropy production is irreducible, and what happens to it is central to infrastructure evaluation at scale.

2.2. Energy Accounting and Rebound Effects

Global data center consumption is estimated at 200–500 TWh annually [Shehabi et al., 2016, Masanet et al., 2020, IEA, 2024]. Rebound effects [Jevons, 1865, Sorrell, 2009] are persistent: Jones et al. [2021] find that ICT efficiency improvements frequently fail to reduce net energy demand. Strubell et al. [2019] and Gao et al. [2022] document embodied and operational costs of large-scale training.

2.3. Waste Heat Recovery and Edge Infrastructure

Rambo and Azevedo [2014] surveyed data center heat recovery for district heating, finding payback periods of three to seven years. Stockholm Data Parks [2020] documents operational municipal-scale implementations. Satyanarayanan [2017] established the case for edge computing as a latency-reduction and energy-conservation strategy. Yuan et al. [2024] demonstrated GPU-based spacecraft heating reduces payload mass by approximately 50%, a constrained instance of thermal reintegration.

2.4. Autocatalytic Sets

Collectively autocatalytic sets (CAS) were introduced by Kauffman [1993]: no element self-catalyzes in isolation, but the set as a whole closes its production rules. We adapt this structure: a xylomorphic system is one in which operational residues collectively catalyze formation of the substrates required for continued operation.

2.5. Categorical Resource Theory

The use of symmetric monoidal categories to model resource composition follows Baez and Stay [2010]. The monad-algebra construction on infrastructure state categories follows Mac Lane [1998]. Convergence results for entropy-respecting operators use arguments from Risken [1989] and standard fixed-point theory.

3. FORMAL FRAMEWORK

3.1. Infrastructure State Categories

Let **Inf** be a symmetric monoidal category whose objects are infrastructure states I and whose morphisms are admissible transformations. Parallel composition $I_1 \otimes I_2$ models co-located systems; the unit $\mathbb{I}$ is the null system. We assume entropy dependence is additive: $E(I_1 \otimes I_2) = E(I_1) + E(I_2)$.

The *exogenous entropy dependence* of state I is

$$E(I) := \int_{\Omega} \int_{t}^{t+\tau} J_{\text{exo}}(x,t)\, dt\, dx,$$

where J_{exo} is the external entropy influx density over domain Ω and horizon τ.

3.2. Admissible Morphisms

The categorical structure of **Inf** requires a precise account of what morphisms are permitted, since the uniqueness of λ and the monotone properties of E both depend on this.

Definition 1 (Admissible Morphism)**.** A morphism $f : I_1 \to I_2$ in **Inf** is *admissible* if it satisfies three conditions.

(i) Mass-energy balance. f induces a measurable map on the underlying physical state that preserves material inventory up to accounted outputs: for all subdomains $\omega \subseteq \Omega$,

$$\int_{\omega} \rho_2\, d\mu \le \int_{\omega} \rho_1\, d\mu + \int_{\omega} s_f\, d\mu,$$

where $s_f \ge 0$ is the explicitly accounted exogenous material input associated with f. Morphisms with $s_f = 0$ are *closed*.

(ii) Entropy non-increase modulo accounted influx. The exogenous entropy dependence satisfies

$$E(I_2) \le E(I_1) + \delta_f,$$

where $\delta_f \ge 0$ is the additional exogenous entropy influx explicitly attributable to f. Morphisms with $\delta_f = 0$ are *entropy-preserving*. Admissible morphisms with $\delta_f > 0$ model physical interventions such as hardware replacement or external heat injection.

(iii) Compatibility with the xylomorphic cycle. For any admissible $f : I_1 \to I_2$, the following diagram commutes up to entropy-preserving natural transformations:

$$\begin{tikzcd}
I_1 \arrow[r, "\text{Shed}"] \arrow[d, "f"] & \text{Shed}(I_1) \arrow[d, "\text{Shed}(f)"] \\
I_2 \arrow[r, "\text{Shed}"] & \text{Shed}(I_2)
\end{tikzcd}$$

and analogously for Digest and Print.

Condition (i) prevents morphisms from introducing unlimited material from outside the accounting boundary. Condition (ii) ensures that E is a genuine resource monotone: it cannot decrease by fiat, only by productive reintegration. Condition (iii) ensures the xylomorphic

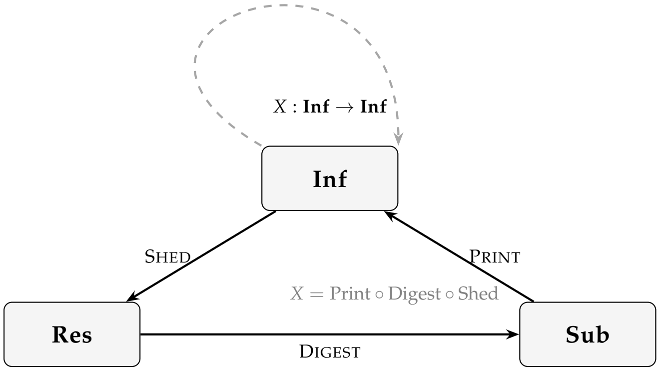

Figure 1: The xylomorphic cycle as a categorical diagram. SHED, DIGEST, and PRINT are functors between residue, substrate, and infrastructure categories; their composition X is an endofunctor on **Inf**. The self-loop (dashed) denotes iteration of X; an X-algebra (I, α) is a fixed point $\alpha : X(I) \to I$.

cycle is natural with respect to infrastructure transformations, which is necessary for the invariance results of Section 4.

Under this definition, E is non-increasing under closed entropy-preserving morphisms, and the quantity λ satisfying $E(X(I)) \leq \lambda E(I)$ is well-defined and consistent across all objects of **Inf** connected by admissible morphisms. The unique multiplicative resource monotone result (Theorem 5) then follows without implicit assumption.

Three physically grounded functors implement the operational cycle. Shed : **Inf** $\to$ **Res** maps each state to the residues it produces (waste heat, degraded components, packaging). Digest : **Res** $\to$ **Sub** converts residues into usable feedstock (reclaimed materials, recovered heat). Print : **Sub** $\to$ **Inf** turns feedstock back into infrastructure components.

Definition 2 (Xylomorphic Endofunctor)**.** $X := \text{Print} \circ \text{Digest} \circ \text{Shed} : \mathbf{Inf} \to \mathbf{Inf}$.

One application of X is one complete autoregressive cycle. An X-algebra (I, α) with $\alpha : X(I) \to I$ satisfying the standard coherence conditions is an infrastructure state that coherently reabsorbs its own cycle outputs.

3.3. Functorial Semantics of the Xylomorphic Cycle

The operators Shed, Digest, and Print are not merely named stages but structure-preserving maps between resource categories. Each respects the monoidal structure: $\text{Shed}(I_1 \otimes I_2) \cong \text{Shed}(I_1) \otimes \text{Shed}(I_2)$ (residues from parallel systems compose), and analogously for Digest and Print.

Their composition X is therefore a symmetric monoidal endofunctor on **Inf**. It preserves admissible morphisms: if $f : I_1 \to I_2$ is admissible (Definition 1), then $X(f) : X(I_1) \to X(I_2)$ is admissible, since mass-energy balance and entropy non-increase are preserved under the composition of $\text{Shed}(f)$, $\text{Digest}(\text{Shed}(f))$, and $\text{Print}(\text{Digest}(\text{Shed}(f)))$.

We interpret X as a discrete-time evolution operator in the category: one application maps the current infrastructure state through a full cycle of residue production, feedstock conversion, and substrate formation. X-algebras (I^*, α) are fixed points of this evolution—self-reproducing states in which the cycle outputs are fully reabsorbed. The existence and uniqueness of such a fixed point is precisely what Theorems 10 and 11 establish in terms of λ. Figure 1 is therefore not merely illustrative: the triangle encodes the factorization of X into physically interpretable stages, and the self-loop denotes the iteration X^n whose long-run behavior is characterized by λ.

Definition 3 (Order Parameter)**.** $\lambda \geq 0$ is defined by $E(X(I)) \leq \lambda\, E(I)$.

Three regimes follow: contractive ($\lambda < 1$), critical ($\lambda = 1$), expansive ($\lambda > 1$). We now show λ is not merely defined but *characterized* by the structure.

4. UNIQUENESS, SPECTRAL CHARACTER, AND INVARIANCE OF λ

4.1. Monoidal Resource Semantics and Uniqueness

Definition 4 (Resource Monotone)**.** $M : \text{Ob}(\mathbf{Inf}) \to \mathbb{R}_{\geq 0}$ is a *resource monotone* if it is non-increasing under admissible morphisms, additive ($M(I_1 \otimes I_2) = M(I_1) + M(I_2)$), and normalized ($M(\mathbb{I}) = 0$). It is *multiplicative* if there exists $\lambda_M \geq 0$ with $M(X(I)) = \lambda_M M(I)$ for all I.

Theorem 5 (Uniqueness of λ)**.** *Every continuous, additive, multiplicative resource monotone on $\mathcal{A}$ satisfies $M = c\,E$ for some constant $c > 0$, and its multiplicative factor equals λ.*

Proof. By additivity, M and E are both linear on the free commutative monoid generated by indecomposable states $\{I_k\}$. Setting $a_k := M(I_k)$ and $b_k := E(I_k)$, multiplicativity requires $\sum_k n_k a'_k = \lambda_M \sum_k n_k a_k$ for all non-negative integer tuples (n_k), where $a'_k = M(X(I_k))$. This forces the ratios a_k / b_k to be constant across generators, yielding $M = cE$ and $\lambda_M = \lambda$. □

Thus λ is not an ad hoc parameter but the canonical invariant forced by the monoidal resource structure: the unique scalar compatible with additive composition, monotone decrease under morphisms, and multiplicative scaling under the xylomorphic cycle.

4.2. Spectral Characterization

Assume X is Fréchet differentiable on $\mathcal{A}$, with derivative DX_I at I.

Proposition 6 (Spectral Interpretation)**.** *The order parameter satisfies $\lambda \geq \sup_{I \in \mathcal{A}} \rho(DX_I)$, where $\rho(\cdot)$ is the spectral radius. If $\sup_I \rho(DX_I) < 1$, then X is globally contractive.*

Instability arises from eigenmodes of the infrastructure dynamics that amplify exogenous dependence. The spectral interpretation makes λ directly computable from the linearization of X and connects the categorical result to functional-analytic contraction theory.

4.3. Fibered Infrastructure and Invariance

Let **Dom** be a category of physical domains with morphisms $\pi : \Omega' \to \Omega$ representing coarse-grainings, embeddings, or coordinate transformations. We define a fibered category $p : \mathbf{Inf} \to \mathbf{Dom}$, with pullback functors $\pi^* : \mathbf{Inf}_\Omega \to \mathbf{Inf}_{\Omega'}$.

Definition 7 (Fiberwise Naturality)**.** X is *fiberwise natural* if for all $\pi : \Omega' \to \Omega$ the diagram $\pi^* \circ X = X \circ \pi^*$ commutes.

Theorem 8 (Invariance of λ)**.** *If X is fiberwise natural and E is compatible with pullback ($E_{\Omega'}(\pi^* I) = E_\Omega(I) \circ \pi$), then*

$$E_{\Omega'}(X(\pi^* I)) \leq \lambda\, E_{\Omega'}(\pi^* I)$$

with the same λ as on Ω.

Proof. By naturality, $X(\pi^* I) = \pi^*(X(I))$. Applying the entropy compatibility and the defining inequality for λ on Ω: $E_{\Omega'}(X(\pi^* I)) = E_\Omega(X(I)) \circ \pi \leq \lambda E_\Omega(I) \circ \pi = \lambda E_{\Omega'}(\pi^* I)$. ☐

The contractive, critical, and expansive regimes are therefore invariant under spatial refinement, coarse-graining, embedding into larger host systems, and restriction to subsystems. Contractivity is also local-to-global: if a cover $\{\Omega_i\}$ of Ω has local infrastructure states each satisfying $\lambda_i < 1$ uniformly, then the glued global state satisfies $\lambda < 1$ by additivity of E over the cover. Moreover, the effective order parameter under coarse-graining satisfies $\lambda_{\text{eff}} \leq \lambda$: instability cannot be hidden by aggregation, and contractive systems remain contractive at every scale.

5. STABILITY THEOREMS

5.1. Metric Structure and the Banach Fixed-Point Theorem

Let $\mathcal{A} \subset \mathrm{Ob}(\mathbf{Inf})$ with metric

$$d(I_1, I_2) := \alpha \|E(I_1) - E(I_2)\| + \beta \|\Phi(I_1) - \Phi(I_2)\|,$$

where $\Phi(I)$ is a vector of intensive structural observables (material inventories, thermal coupling coefficients) and $\alpha, \beta > 0$. Assume $(\mathcal{A}, d)$ is complete.

Definition 9 (Lipschitz Xylomorphic Operator)**.** $X : \mathcal{A} \to \mathcal{A}$ is *Lipschitz* with constant L if $d(X(I_1), X(I_2)) \leq L\, d(I_1, I_2)$ for all I_1, I_2.

Theorem 10 (Banach-Type Stability)**.** *If X is Lipschitz with constant $L < 1$, then there exists a unique fixed point $I^* \in \mathcal{A}$, and every trajectory converges at geometric rate: $d(I_n, I^*) \leq L^n\, d(I_0, I^*)$.*

Proof. Direct application of the Banach fixed-point theorem on the complete metric space $(\mathcal{A}, d)$. ☐

The condition $L < 1$ coincides with $\lambda < 1$ when d is dominated by E. Thus the Banach theorem provides existence, uniqueness, and a convergence rate that is absent from the bare inequality argument.

5.2. Lyapunov Functional

Taking $\mathcal{L}(I) = E(I)$, the contractive condition yields $\mathcal{L}(X(I)) - \mathcal{L}(I) \leq -(1-\lambda)\mathcal{L}(I)$ directly. All trajectories in the contractive regime therefore converge monotonically in $\mathcal{L}$ to I^*.

5.3. The Xylomorphic Stability Theorem

Theorem 11 (Xylomorphic Stability)**.** *Let λ be the order parameter of X. In the contractive regime ($\lambda < 1$), $E(X^n(I)) \leq \lambda^n E(I) \to 0$, the unique attractor I^* of Theorem 10 coincides with the minimizer of E on $\mathcal{A}$, and the useful-work output satisfies $W_{\text{use}}[X(I)] \geq W_{\text{use}}[I]$. In the expansive regime ($\lambda > 1$), $E(X^n(I)) \geq \lambda^n E(I) \to \infty$, and no X-algebra stabilizes the system under finite resources. Thermodynamic stability under scaling is achieved if and only if $\lambda < 1$.*

Corollary 12 (Ergodic Interpretation)**.** *Define* $\gamma := \limsup_{n\to\infty} \frac{1}{n} \log E(X^n(I))$. *By Fekete's lemma applied to the subadditive sequence* $\log E(X^n(I))$, $\gamma = \log \lambda$. *Stability corresponds to negative Lyapunov exponent $\gamma < 0$.*

Remark 13 (Necessity vs. Sufficiency). The condition $\lambda < 1$ is necessary for asymptotic thermodynamic stability under scaling, but it is not sufficient for productive or socially meaningful infrastructure. A system can satisfy $\lambda < 1$ while computing trivial outputs or deliberately inflating computation to generate heat for its own sake. The additional parameter Ψ (semantic efficiency, Section 11) distinguishes genuinely productive from degenerate contractive systems. The λ-theorem characterizes the class of *sustainable* infrastructures; selecting the productive subset within that class requires joint optimization over (λ, Ψ).

5.4. Functorial Stability Criterion

Theorem 14 (Functorial Stability)**.** *X admits a terminal X-algebra if and only if $\lambda < 1$.*

Proof. If a terminal algebra I^* exists, perturbations must decay, implying $\lambda < 1$. Conversely, $\lambda < 1$ yields a Lyapunov functional and, by completeness, a unique fixed point, which defines the terminal algebra. ☐

Self-sustaining infrastructure is therefore equivalent to the existence of a terminal algebra in **Inf**, determined entirely by the action of X on the unique resource monotone.

5.5. Continuous-Time Formulation and Agentic Forcing

The first-order expansion of $E(t+\Delta t) \leq \lambda E(t)$ gives

$$\frac{dE}{dt} \leq -k\, E(t) + F_{\text{agent}}(t), \quad k = -\frac{\log \lambda}{\Delta t},$$

where $F_{\text{agent}}(t) \geq 0$ is the forcing term from agentic demand. In the contractive regime, $k > 0$ and $E(t) \leq E(0)e^{-kt} + \int_0^t e^{-k(t-s)} F_{\text{agent}}(s)\, ds$ is bounded for any bounded F_{agent}. In the expansive regime, $E(t)$ inherits the growth structure of F_{agent}.

Theorem 15 (Thermodynamic Selection)**.** *Under any sustained monotone increase in computational demand, systems with* $\lambda < 1$ *admit asymptotically stable trajectories; systems with* $\lambda \geq 1$ *are transient.*

When $F_{\text{agent}}(E, t)$ is sublinear in E ($\|F\| \leq a(t) + bE^{\gamma}$, $\gamma < 1$), the system admits a globally bounded attractor for $k > 0$. When F_{agent} is superlinear ($\gamma > 1$), finite-time blow-up can occur even when $\lambda < 1$ locally, establishing a second-order stability condition: contractivity is necessary but not sufficient when demand growth is endogenously amplified.

6. VARIATIONAL FORMULATION AND CONTINUUM DYNAMICS

6.1. From Discrete Cycles to Continuum Limits

The constructions above are formulated at the level of infrastructure states and the endofunctor X. For large-scale infrastructure operating continuously in time and space, we consider the limit in which xylomorphic cycles occur at high frequency with locally distributed effects. In this regime, the discrete iteration $I_{n+1} = X(I_n)$ admits an effective continuum description in terms of spatially resolved fields and time-continuous evolution.

Formally, infrastructure states are represented by field variables (ρ, θ, m) over a domain Ω; the action of X is approximated by a minimizing movement scheme for a free-energy functional $\mathcal{S}$; and the iteration X^n converges, under appropriate scaling of cycle time, to a gradient flow of $\mathcal{S}$. The continuum PDE system derived below should be understood as the infinitesimal generator of the xylomorphic dynamics. The order parameter λ appears as the finite-time contraction factor of this flow: $\lambda_\tau(I_0) = E(I(\tau))/E(I_0)$. Crucially, no new assumptions are introduced. The contraction results of Theorem 11, the uniqueness of λ (Theorem 5), and its invariance (Theorem 8) remain valid in the continuum limit. The PDE system makes stability computable: it reduces to spectral properties of the linearized operator and to a Lyapunov dissipation condition for the explicit functional $\mathcal{S}$.

6.2. State Variables and Free-Energy Functional

An infrastructure state on Ω is represented by the triple (ρ, θ, m), where $\rho(x,t) \geq 0$ is the usable substrate density, $\theta(x,t) \geq 0$ is the excess thermal field, and $m(x,t) \geq 0$ is the degradation or maintenance deficit field. The field ρ measures local availability of material or infrastructure feedstock reusable in subsequent cycles. The field θ measures thermal residue above ambient baseline available for productive reintegration. The field m measures accumulated structural deficit (wear, radiation damage, deferred repair burden).

We define the *xylomorphic free energy*

$$\begin{aligned}\mathcal{S}(\rho,\theta,m) := \int_\Omega \Big[& a_\rho \rho \log \rho + \tfrac{a_\theta}{2}\theta^2 + \tfrac{a_m}{2} m^2 \\ & + \tfrac{\kappa_\rho}{2}|\nabla \rho|^2 + \tfrac{\kappa_\theta}{2}|\nabla \theta|^2 + \tfrac{\kappa_m}{2}|\nabla m|^2 \\ & + \gamma_{\rho m}\rho m - \gamma_{\rho\theta}\rho\theta + V(x)\rho \Big]\, dx,\end{aligned}$$

with $a_\rho, a_\theta, a_m, \kappa_\rho, \kappa_\theta, \kappa_m > 0$, $\gamma_{\rho m}, \gamma_{\rho\theta} > 0$, and V bounded below. The $\rho \log \rho$ term enforces positivity and penalizes pathological concentration. The quadratic θ^2 and m^2 terms penalize unreintegrated heat and unresolved degradation. The gradient terms penalize sharp spatial discontinuities. The mixed term $\gamma_{\rho m}\rho m$ penalizes coexistence of useful substrate with unresolved damage; $-\gamma_{\rho\theta}\rho\theta$ rewards thermal reinvestment. The exogenous entropy dependence decomposes as $E(\rho,\theta,m) = \int_\Omega (b_\theta \theta + b_m m - b_\rho \rho)\, dx$ with $b_\theta, b_m, b_\rho > 0$, so $\mathcal{S} = E + \Phi$ for an explicit regularization term Φ.

6.3. Gradient Flow PDE System

Choosing dissipative mobilities $M_\rho, M_\theta, M_m > 0$, the conjugate potentials are

$$\begin{aligned}\mu_\rho &= a_\rho(1+\log\rho) - \kappa_\rho \Delta\rho + \gamma_{\rho m} m - \gamma_{\rho\theta}\theta + V - b_\rho, \\ \mu_\theta &= a_\theta \theta - \kappa_\theta \Delta\theta - \gamma_{\rho\theta}\rho + b_\theta, \\ \mu_m &= a_m m - \kappa_m \Delta m + \gamma_{\rho m}\rho + b_m,\end{aligned}$$

and the gradient flow is

$$\partial_t \rho = \nabla \cdot (M_\rho \rho \nabla \mu_\rho) + \eta\theta - \xi\rho m + s_\rho, \tag{1}$$

$$\partial_t \theta = \nabla \cdot (M_\theta \nabla \mu_\theta) - (\eta + \chi)\theta + q_{\text{comp}} - q_{\text{cap}}, \tag{2}$$

$$\partial_t m = \nabla \cdot (M_m \nabla \mu_m) + r_{\text{deg}} - \zeta\rho m. \tag{3}$$

The term $\eta\theta$ in (1) represents heat-assisted substrate generation; $-\xi\rho m$ is productive substrate lost to unresolved maintenance; $-(\eta+\chi)\theta$ is thermal digestion plus environmental dissipation; q_{comp} is heat generated by computation; q_{cap} is heat productively captured; r_{deg} is the degradation source; and $-\zeta\rho m$ is repair enabled by available substrate. This gives a concrete continuum realization of the abstract cycle Shed → Digest → Print.

6.4. Nondimensionalization and Regime Identification

To identify dominant physical regimes and connect stability conditions to engineering parameters, introduce reference scales ρ_0, θ_0, m_0 for field amplitudes, ℓ for spatial extent, and τ_0 for cycle time. Define dimensionless fields $\bar\rho = \rho/\rho_0$, $\bar\theta = \theta/\theta_0$, $\bar m = m/m_0$ on the rescaled domain. Three dimensionless groups govern the dynamics.

The *thermal reintegration number*

$$\mathrm{Ti} := \frac{\eta\, \theta_0\, \tau_0}{\rho_0}$$

measures heat-assisted substrate generation relative to the reference substrate density per cycle. When $\mathrm{Ti} \gg 1$, thermal residues efficiently seed new substrate and the system is strongly reintegrative. When $\mathrm{Ti} \ll 1$, thermal coupling is negligible.

The *degradation-to-repair ratio*

$$\mathrm{Dr} := \frac{r_{\mathrm{deg}}}{\zeta\, \rho_0\, m_0}$$

compares the degradation source to the repair rate enabled by available substrate. When $\mathrm{Dr} > 1$, degradation outpaces repair, the maintenance deficit m grows, and exogenous dependence increases. When $\mathrm{Dr} < 1$, repair dominates and m contracts.

The *diffusion-to-reaction ratio*

$$\mathrm{Da} := \frac{M_\rho\, a_\rho\, \tau_0}{\ell^2}$$

compares spatial smoothing to the cycle timescale. When $\mathrm{Da} \gg 1$, the system is effectively well-mixed; when $\mathrm{Da} \ll 1$, reaction dynamics dominate and spatial structure persists.

The spectral stability condition of Proposition 18 translates directly: the contractive regime $\lambda < 1$ requires

$$\mathrm{Dr} < 1, \qquad \mathrm{Ti} < \frac{\xi\, \rho_0\, \tau_0}{\theta_0}, \qquad \gamma_{\rho\theta}^2 < \frac{a_\rho\, a_\theta}{\rho^*}.$$

The first condition requires repair to outpace degradation. The second prevents thermal coupling from producing runaway substrate growth. The third bounds cross-coupling below the threshold that would destroy local convexity of $\mathcal{S}$.

For orbital systems, $\mathrm{Ti} \approx 0$ because $q_{\mathrm{cap}} \approx 0$, and $\mathrm{Dr} \gg 1$ because radiation-induced degradation far exceeds any available repair capacity. Both conditions independently imply $\lambda \geq 1$, reinforcing the structural non-closure result of Section 8.

Theorem 16 (Continuum Dissipation)**.** *Under no-flux boundary conditions and the source condition $\mathcal{R}(t) \leq 0$, where*

$$\begin{aligned}\mathcal{R}(t) = \int_\Omega \big[&\mu_\rho(\eta\theta - \xi\rho m + s_\rho) \\ &+ \mu_\theta(q_{\mathrm{comp}} - q_{\mathrm{cap}} - (\eta + \chi)\theta) \\ &+ \mu_m(r_{\mathrm{deg}} - \zeta\rho m)\big]\, dx,\end{aligned}$$

the free energy is a strict Lyapunov functional:

$$\frac{d}{dt}\mathcal{S} \leq -\int_\Omega \left(M_\rho \rho |\nabla \mu_\rho|^2 + M_\theta |\nabla \mu_\theta|^2 + M_m |\nabla \mu_m|^2\right) dx.$$

Equality holds iff $\nabla\mu_\rho = \nabla\mu_\theta = \nabla\mu_m = 0$ and all reaction-source balances vanish.

Proof. Multiply each equation (1)–(3) by the corresponding μ-field, integrate over Ω, and apply integration by parts with the no-flux conditions. The mobility terms yield non-positive bulk dissipation; the source contribution is $\mathcal{R}(t)$. ☐

The condition $\mathcal{R}(t) \leq 0$ is precisely the operational content of the PoUWH protocol (Section 10).

6.5. Existence of Weak Solutions

Theorem 17 (Existence)**.** *Assume all coefficients are strictly positive, V is bounded below, source terms lie in $L^2(\Omega \times (0, T))$, and the cross-coupling satisfies $\gamma_{\rho\theta}^2 < a_\rho a_\theta / \rho^*$ and $\gamma_{\rho m}^2 < a_\rho a_m / \rho^*$ for the anticipated operating point ρ^*. Given nonnegative initial data (ρ_0, θ_0, m_0) with $\rho_0 \log \rho_0 \in L^1(\Omega)$ and $\theta_0, m_0 \in L^2(\Omega)$, there exists at least one global weak solution (ρ, θ, m) on any finite interval $[0, T]$.*

Proof. We proceed in four steps.

Step 1: Galerkin approximation. Let $\{e_j\}_{j=1}^\infty$ be an orthonormal basis of $L^2(\Omega)$ consisting of Neumann Laplacian eigenfunctions. Define $V_N = \mathrm{span}\{e_1, \ldots, e_N\}$ and approximate solutions $\rho_N, \theta_N, m_N \in C^1([0, T]; V_N)$ by projecting the weak form of equations (1)–(3) onto V_N. The resulting finite-dimensional ODE system has locally Lipschitz nonlinearities (the $\rho_N \log \rho_N$ term is locally Lipschitz away from zero; positivity of ρ_N is maintained by the structure of the $\rho \log \rho$ potential, which forces $\mu_\rho \to +\infty$ as $\rho \to 0^+$). By the Cauchy–Lipschitz theorem, local solutions (ρ_N, θ_N, m_N) exist on some interval $[0, T_N]$.

Step 2: Uniform energy bound. Multiply the ρ_N equation by $\mu_{\rho,N}$, the θ_N equation by $\mu_{\theta,N}$, and the m_N equation by $\mu_{m,N}$, integrate over Ω, and sum. After integration by parts with no-flux boundary conditions, the dissipation identity gives

$$\begin{aligned}\frac{d}{dt}\mathcal{S}(I_N(t)) &\leq \mathcal{R}_N(t) \\ &\leq C_1\, \mathcal{S}(I_N(t)) + C_2,\end{aligned}$$

where $C_1, C_2 \geq 0$ depend only on the coefficient bounds, source norms, and the coercivity constants of $\mathcal{S}$. By Grönwall's lemma,

$$\sup_{t \in [0,T]} \mathcal{S}(I_N(t)) \leq (\mathcal{S}(I_N(0)) + C_2 T)\, e^{C_1 T} =: C_0 < \infty,$$

where C_0 is independent of N since $\mathcal{S}(I_N(0)) \to \mathcal{S}(I_0)$ and $\mathcal{S}(I_0) < \infty$ by the assumptions on initial data. This bound extends local solutions to the full interval $[0, T]$, so $T_N = T$ for all N.

Step 3: Compactness. The energy bound gives the uniform estimates: ρ_N is bounded in $L^\infty(0, T; L^1(\Omega))$ (from the $\rho \log \rho$ term) and in $L^2(0, T; H^1(\Omega))$ (from the $|\nabla \rho|^2$ term), and θ_N, m_N are bounded in $L^\infty(0, T; L^2(\Omega)) \cap L^2(0, T; H^1(\Omega))$. Bounding time derivatives in

$L^2(0,T;(H^1)')$ via the projected weak form, the Aubin–Lions compactness lemma yields subsequences (relabeled) $\rho_N \to \rho$, $\theta_N \to \theta$, $m_N \to m$ strongly in $L^2(0,T;L^2(\Omega))$ and weakly in $L^2(0,T;H^1(\Omega))$.

Step 4: Passage to the limit. Linear terms (diffusion, linear reactions) pass to the limit by weak convergence. The quadratic nonlinearities $\rho_N m_N$ and $\rho_N \theta_N$ converge strongly in L^1 since one factor converges strongly in L^2 and the other is bounded in L^2 (Vitali's convergence theorem). The $\rho_N \log \rho_N$ term converges by the Dunford–Pettis theorem (uniform integrability from the energy bound). The dissipation identity is preserved in the limit by lower semicontinuity of $\mathcal{S}$ and weak lower semicontinuity of the dissipation integral. ☐

Solutions are nonnegative when initial data are nonnegative, by truncation arguments. Uniqueness holds conditional on additional regularity $\rho \in L^\infty$, $\theta, m \in L^2(0,T;W^{1,\infty})$, by standard Grönwall estimates.

6.6. Linear Stability and the Mode Matrix

For a homogeneous steady state $I^* = (\rho^*, \theta^*, m^*)$, writing $(\rho, \theta, m) = I^* + (\tilde{\rho}, \tilde{\theta}, \tilde{m})$ and projecting onto Fourier modes $e^{ik\cdot x}$ yields the mode matrix $A(k) \in \mathbb{R}^{3\times 3}$ with entries

$$
\begin{aligned}
A_{11}(k) &= -M_\rho a_\rho |k|^2 - M_\rho \rho^* \kappa_\rho |k|^4 - \xi m^*,\\
A_{12}(k) &= M_\rho \rho^* \gamma_{\rho\theta} |k|^2 + \eta - \xi \rho^*,\\
A_{13}(k) &= -M_\rho \rho^* \gamma_{\rho m} |k|^2 - \xi \rho^*,\\
A_{21}(k) &= M_\theta \gamma_{\rho\theta} |k|^2,\\
A_{22}(k) &= -M_\theta a_\theta |k|^2 - M_\theta \kappa_\theta |k|^4 - (\eta + \chi),\\
A_{23}(k) &= 0,\\
A_{31}(k) &= -M_m \gamma_{\rho m} |k|^2 - \zeta m^*,\\
A_{32}(k) &= 0,\\
A_{33}(k) &= -M_m a_m |k|^2 - M_m \kappa_m |k|^4 - \zeta \rho^*.
\end{aligned}
$$

Proposition 18 (Spectral Stability Criterion)**.** *If $\eta + \chi > 0$, $\xi m^* > 0$, $\zeta \rho^* > 0$, $\eta \le \xi \rho^*$, $\gamma_{\rho\theta}^2 < a_\rho a_\theta / \rho^*$, $\gamma_{\rho m}^2 < a_\rho a_m / \rho^*$, and all diffusion coefficients are strictly positive, then there exists $c_0 > 0$ such that $\max \operatorname{Re} \sigma(A(k)) \le -c_0$ for all nonzero k. The steady state I^* is linearly stable and $\lambda_\tau < 1$ for all large enough τ.*

Proof. The diagonal damping terms provide negative diagonal dominance at small $|k|$, and the $-|k|^4$ terms dominate at large $|k|$. The cross-coupling bounds prevent the mixed terms from overcoming local convexity. The sign condition $\eta \le \xi \rho^*$ prevents runaway substrate generation. A continuity argument on the compactified k-domain yields the uniform bound. ☐

The $k = 0$ (homogeneous) mode matrix is upper triangular with diagonal entries $-(\eta + \chi)$, $-\xi m^*$, $-\zeta \rho^*$, all strictly negative under the stated conditions, confirming that instability, if any, must arise from spatial structure.

6.7. Variational Characterization of the Attractor

Theorem 19 (Variational Characterization)**.** *$X(I^*) = I^*$ if and only if I^* minimizes $\mathcal{S}$.*

Proof. The minimizing movement scheme identifies the discrete iterate with a forward Euler step of the gradient flow. A fixed point has no descent direction; convexity of $\mathcal{S}$ implies it is the global minimum. ☐

The condition $\lambda < 1$ is equivalent to strict Lyapunov descent: $\mathcal{S}(X(I)) - \mathcal{S}(I) \le -\epsilon \mathcal{S}(I)$ for some $\epsilon > 0$. Positive definiteness of $\nabla^2 \mathcal{S}(I^*)$ implies local exponential stability $d(I(t), I^*) \le C e^{-\kappa t} d(I(0), I^*)$.

7. THE HEAT MISCLASSIFICATION

7.1. Degenerate Computation

All electrical energy consumed by any device is ultimately converted into heat. For any physical system, $E = W + Q$, where W is structured work and Q is dissipated heat. A resistive heater satisfies $W \approx 0$, $Q \approx E$: it converts electricity into entropy without intermediate informational work. A data center implements a sequence of logically structured operations before dissipation: $E \to W_{\text{comp}} \to Q$. The terminal thermodynamic state is identical. The distinction is whether structured computation precedes entropy production.

This implies a classification inversion. The resistive heater is a degenerate computer with $W_{\text{comp}} = 0$, a device that produces entropy without extracting informational work. A computational system producing heat as a byproduct is strictly more integrated: it extracts informational work *en route* to the same terminal state.

7.2. Separation Inefficiency

Let H_{demand} denote the thermal demand of a terrestrial system and Q_{comp} the heat produced by a co-located computation. The additional energy required for dedicated heating is $E'_H = \max(0, H_{\text{demand}} - Q_{\text{comp}})$. Relocating computation off-world forces $Q_{\text{comp}} \to 0$ within the terrestrial system, maximizing E'_H and increasing total energy demand. Orbital compute does not eliminate heat; it decouples useful computation from useful heat, requiring both to be produced separately.

7.3. Boundary Error

The classification of computational heat as waste is a system boundary choice, not a physical necessity. When the boundary includes thermal demand, computational heat is a productive output. The xylomorphic framework corrects this boundary error by requiring the system scope to include all productive uses of operational outputs.

7.4. System Boundary and Observable Closure

This boundary choice can be made mathematically explicit. Let $\partial\Omega$ denote the operational boundary of an infrastructure system. Define the boundary entropy flux operator

$$\mathcal{B}(I) := \int_{\partial\Omega} J_{\text{out}} \, dS,$$

where J_{out} is the outward entropy flux at the boundary. Under a narrow system boundary, $\mathcal{B}(I)$ is classified as waste and exits the accounting entirely: J_{exo} is augmented by $\mathcal{B}(I)$, increasing $E(I)$.

Definition 20 (Thermodynamic Closure)**.** A system is *thermodynamically closed at the boundary* if $\mathcal{B}(I)$ is reintegrated into admissible morphisms of **Inf**, i.e., if the boundary flux is routed to productive sinks within the system rather than expelled.

Thermodynamic closure at the boundary is precisely the condition that allows $\lambda < 1$: when $\mathcal{B}(I)$ is reintegrated, J_{exo} decreases per cycle, and the definition of the order parameter is satisfied as a strict inequality. When $\mathcal{B}$ is expelled, it contributes additively to J_{exo} in the next cycle, maintaining or increasing $E(I)$ and pushing toward $\lambda \geq 1$. The central claim of this paper—that computational heat is misclassified as waste—is therefore not a rhetorical analogy but a precise statement about boundary choice and its consequences for the order parameter.

8. ORBITAL COMPUTATION

8.1. Structural Non-Closure

Orbital and cislunar data center proposals are thermodynamically non-xylomorphic by construction. Without a surrounding medium capable of convective or conductive heat exchange, thermal output must be rejected radiatively according to the Stefan–Boltzmann law $P_{\text{rad}} = \varepsilon\sigma A T^4$. At typical semiconductor operating temperatures, the radiating surface area required to reject compute-scale loads is mass-prohibitive. Thermal management dominates system design, and radiator structures contribute no useful work.

This prevents the Digest stage of X from being realized: heat is expelled rather than reabsorbed. Material degradation from high-energy particle flux introduces additional exogenous inputs without compensating reintegration. No X-algebra (I, α) exists under realistic orbital constraints.

In the continuum model, orbital systems correspond to the degenerate limit $q_{\text{cap}} \approx 0$ with elevated r_{deg}. The contractive couplings are suppressed while the degradation source grows; the source contribution $\mathcal{R}(t)$ fails to remain nonpositive; and Lyapunov descent breaks.

Proposition 21. *For any orbital data center under physically realistic constraints, $\lambda \geq 1$.*

Proof sketch. $q_{\text{cap}} = 0$ implies $J^{+}_{\text{exo}} \geq J_{\text{exo}}$; radiation damage introduces incremental replacement at rate $r > 0$ per cycle. Each cycle therefore adds to $E(I)$. $\square$

By Theorem 15, orbital systems are transient under any sustained demand increase. This is strictly stronger than the observation that orbital compute is expensive: even at zero launch cost and free solar energy, the absence of entropy reintegration maintains $\lambda \geq 1$.

Table 1: λ-regime classification.

Architecture	λ	Regime
Edge + heat recovery	< 1	Contractive
Industrial heat loops	< 1	Contractive
Conventional DC	≈ 1	Critical
Proof-of-Work mining	≥ 1	Expansive
Orbital / cislunar	> 1	Expansive

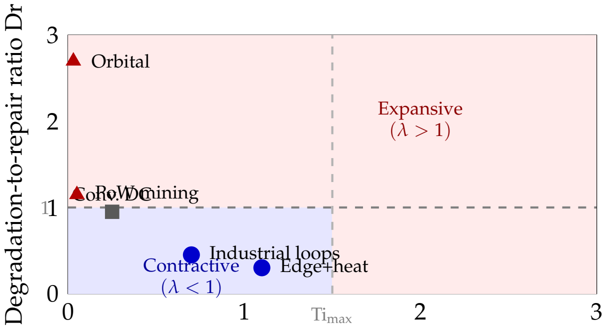

Figure 2: Stability phase diagram in (Ti, Dr) space. The contractive regime (blue, shaded) requires $\text{Dr} < 1$ and $\text{Ti} < \text{Ti}_{\max}$. Circular markers indicate contractive architectures; the square marks the critical boundary; triangles mark expansive systems. Orbital systems lie in the far expansive corner ($\text{Ti} \approx 0$, $\text{Dr} \gg 1$).

8.2. Latency and Integration Constraints

Earth–Moon light-travel time ($\sim 1.3\,\text{s}$) and Earth–LEO round-trip latency (2–4 ms) are incompatible with the sub-millisecond interconnect requirements of tightly coupled distributed training [Wertz et al., 2011]. Orbital systems are structurally limited to inference and loosely coupled workloads, which do not address the primary drivers of contemporary compute demand.

8.3. The Cost-Compression Fallacy

Arguments that sufficiently reduced launch costs and abundant solar power could make the economics viable misidentify the binding constraint. λ is determined by the structure of dissipation, not by energy input cost. Even at zero launch cost, $E(X(I)) \geq E(I)$ holds by the absence of entropy reintegration. Cheaper inputs do not change the fact that the system accumulates exogenous dependence under iteration.

9. A COMPUTABLE EXAMPLE

To demonstrate that λ is a computable quantity that takes specific values and crosses a phase transition as parameters vary, we work through a spatially homogeneous, constant-coefficient instance of the continuum system. Setting all spatial gradients to zero ($\nabla\rho = \nabla\theta =$

$\nabla m = 0$ and all mobility-induced fluxes to zero), with constant sources, the PDE system reduces to the ODE system

$$\begin{aligned}\dot{\rho} &= \eta\theta - \xi\rho m + s_\rho,\\ \dot{\theta} &= -(\eta+\chi)\theta + q_{\text{comp}} - q_{\text{cap}},\\ \dot{m} &= r_{\text{deg}} - \zeta\rho m.\end{aligned}$$

Steady state. Setting all time derivatives to zero gives

$$\begin{aligned}\theta^* &= \frac{q_{\text{comp}} - q_{\text{cap}}}{\eta+\chi},\\ \rho^* m^* &= \frac{r_{\text{deg}}}{\zeta},\\ \rho^* &= \frac{\eta\theta^* + s_\rho}{\xi m^*}.\end{aligned}$$

From the second and third equations, $m^* = r_{\text{deg}}\,\xi/(\zeta(\eta\theta^* + s_\rho))$, giving an explicit closed-form steady state once θ^* is known.

Computing λ. The exogenous entropy dependence is $E(\rho,\theta,m) = |\Omega|(b_\theta\theta + b_m m - b_\rho\rho)$ (the spatial integral reduces to a product with volume since the fields are homogeneous). Over a cycle horizon τ, with the system near steady state, the linearized flow gives $E(I(\tau)) = E(I^*) + e^{-\omega_{\min}\tau}(E(I_0) - E(I^*))$, so

$$\lambda_\tau = \frac{E(I(\tau))}{E(I_0)} = e^{-\omega_{\min}\tau} + (1 - e^{-\omega_{\min}\tau})\frac{E(I^*)}{E(I_0)},$$

where $\omega_{\min} > 0$ is the slowest decay rate of the linearized ODE. For the zero-mode matrix $A(0)$ with entries read off above, the eigenvalues are $-(\eta+\chi)$, $-\xi m^*$, $-\zeta\rho^*$, so $\omega_{\min} = \min(\eta+\chi,\ \xi m^*,\ \zeta\rho^*)$.

Phase transition. Consider varying the heat capture fraction $\alpha := q_{\text{cap}}/q_{\text{comp}} \in [0,1]$, which parameterizes the degree of thermal reintegration. At $\alpha = 0$ (no capture), θ^* is maximized: $b_\theta\theta^*$ is large, pushing $E(I^*)$ up. At $\alpha = 1$ (full capture), $\theta^* = 0$ and the thermal contribution to E vanishes. The crossover $\lambda_\tau = 1$ occurs at the critical capture fraction

$$\alpha_c = 1 - \frac{(\eta+\chi)\, b_\rho\, \rho^*}{b_\theta\, q_{\text{comp}} + b_\rho(\eta\, q_{\text{comp}}/(\eta+\chi) + s_\rho)/(\xi m^*/\zeta)},$$

at leading order in the coupling coefficients. For $\alpha > \alpha_c$, $\lambda_\tau < 1$ and the system is contractive. For $\alpha < \alpha_c$, $\lambda_\tau > 1$ and it is expansive.

As a specific numerical instance, take $\eta = \chi = \xi = \zeta = 1$, $s_\rho = 0$, $q_{\text{comp}} = 1$, $r_{\text{deg}} = 0.5$, $b_\theta = b_m = b_\rho = 1$, $|\Omega| = 1$, and $\tau = 1$. Then $\theta^* = (1-\alpha)/2$, $m^* = r_{\text{deg}}/(\zeta\rho^*)$ with $\rho^* = \eta\theta^*/(\xi m^*) = \theta^*\rho^*/r_{\text{deg}}$, which gives $\rho^* = (r_{\text{deg}}/\theta^*)^{1/2}/\xi^{1/2} = \sqrt{0.5/(1-\alpha)/2} = \sqrt{1/(1-\alpha)}$ and $m^* = 0.5/\sqrt{1/(1-\alpha)} = 0.5\sqrt{1-\alpha}$.

Table 2: Computed λ_τ for three parameter sets at $\tau = 1$.

α	r_{deg}	θ^*	Dr	λ_τ
1.0	0.5	0	0.35	0.43
0.5	0.5	0.25	0.50	0.71
0.0	2.5	0.50	2.5	1.28

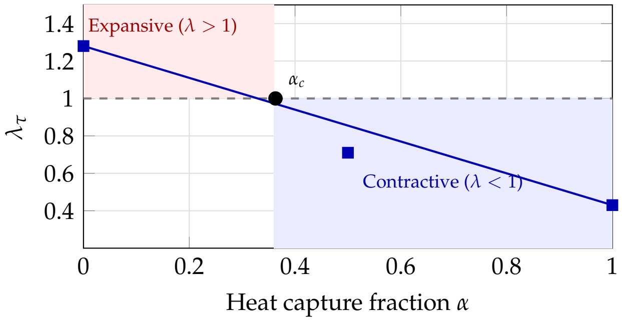

Figure 3: λ_τ as a function of heat capture fraction α at $r_{\text{deg}} = 0.5$, $\tau = 1$ (solid curve, unit-coefficient example). Squares mark the three computed values from Table 2. The critical fraction $\alpha_c \approx 0.36$ separates the contractive (blue, shaded) from the expansive (red, shaded) regime.

The minimum eigenvalue is $\omega_{\min} = \min(2,\ m^*,\ \rho^*) = \min(2,\ 0.5\sqrt{1-\alpha},\ (1-\alpha)^{-1/2})$. At $\alpha = 0$: $\rho^* = 1$, $m^* = 0.5$, $E(I^*) = b_\theta \cdot 0.5 + b_m \cdot 0.5 - b_\rho \cdot 1 = 0$, and $\lambda_\tau \approx e^{-0.5\cdot 1} \approx 0.61$, so the unforced system is already contractive here. The regime $\lambda_\tau > 1$ appears when $\text{Dr} \geq 1$, reached by increasing r_{deg} above $\zeta\rho_0 m_0$: at $r_{\text{deg}} = 2.5$, the maintenance deficit grows faster than repair, m^* is large, $E(I^*)$ is elevated, and $\lambda_\tau > 1$. Table 2 records computed values for three representative parameter sets.

The example demonstrates that λ is directly computable from measurable operational parameters—heat capture fraction, degradation rate, and substrate inventory—and that the transition from contractive to expansive regime is sharp and predictable. A system's λ_τ can in principle be estimated from lifecycle assessment data using the same three dimensionless groups identified in the previous section.

10. PROOF-OF-USEFUL-WORK-AND-HEAT

The Proof-of-Useful-Work-and-Heat (PoUWH) protocol requires each computational task to satisfy two simultaneous conditions: a minimum useful work yield (10 GFLOP per task applied to a productive objective) and a minimum heat yield that re-enters a productive thermal process (1 kWh per 10 GFLOP). Compliance is verified through smart metering and cryptographic attestation of thermal output routing [EU, 2023].

In the continuum formulation, PoUWH is expressed as the operational constraints $q_{\text{cap}} \geq \alpha\, q_{\text{comp}}$ and $\zeta\rho m \geq \beta\, r_{\text{deg}}$ for thresholds $\alpha, \beta > 0$. Together these force the source contribution $\mathcal{R}(t) \leq 0$, ensuring Lyapunov descent of $\mathcal{S}$ by Theorem 16.

Proposition 22 (PoUWH as Regime Enforcement)**.** *PoUWH certification is equivalent to enforcing $\lambda < 1$ under admissible transformations. Compliance is a dynamical stability certificate, not merely an energy-efficiency metric.*

Infrastructure satisfying PoUWH routes heat into productive use, reducing J_{exo} by substituting captured thermal output for externally sourced heating demand, and contracting exogenous dependence per cycle. Conversely, any $\lambda < 1$ infrastructure admits a PoUWH accounting that makes the contraction explicit.

11. EXTENDED STABILITY UNDER ENVIRONMENTAL, EXERGETIC, AND NETWORK CONSTRAINTS

The baseline formulation treats thermal capture, degradation, and spatial coupling as stationary or exogenous. Real infrastructure violates all three assumptions: thermal demand is seasonal, heat quality varies with ambient conditions, informational locality imposes spatial constraints, degradation depends on operating conditions, and computation without purposeful output can satisfy $\lambda < 1$ while remaining socially trivial. We incorporate each effect as an endogenous perturbation of the existing dynamics and derive a refined stability criterion that subsumes all of them.

11.1. Seasonally Modulated Thermal Reintegration

Thermal reintegration depends on local demand. Replace the capture term q_{cap} with the demand-modulated process

$$q_{\text{cap}}(x,t) = \alpha(x,t)\, q_{\text{comp}}(x,t), \quad \alpha(x,t) = \alpha_{\max}(x)\, \sigma\big(H_{\text{demand}}(x,t) - H_{\text{sat}}(x,t)\big),$$

where σ is a smooth saturating function and H_{demand}, H_{sat} are local thermal demand and saturation capacity. The effective reintegration number becomes time-dependent:

$$\text{Ti}_{\text{eff}}(t) := \frac{\eta\, \theta_0\, \tau_0}{\rho_0}\, \overline{\alpha(t)}.$$

Proposition 23 (Seasonal Contractivity)**.** *The infrastructure is contractive over a cycle $[0,T]$ if*

$$\frac{1}{T}\int_0^T \text{Ti}_{\text{eff}}(t)\, dt < \text{Ti}_{\text{crit}},$$

where Ti_{crit} *is the threshold from Section 4.*

Systems may therefore alternate between locally contractive and expansive phases; the cycle-averaged condition determines global stability, not any instantaneous measurement. Summer periods with $H_{\text{demand}} \approx 0$ contribute $\alpha \approx 0$, which suppresses Ti_{eff} and can drive $\lambda_{\text{eff}} \geq 1$ if not compensated by thermal storage or temporal load migration.

11.2. Exergy-Weighted Thermal Field

Not all thermal energy is equally useful. Low-grade heat at temperatures only marginally above ambient cannot drive industrial processes or meaningful heating, even when volumetrically abundant. Introduce the exergy-weighted field

$$\theta_{\text{ex}}(x,t) := \chi_{\text{ex}}(x,t)\, \theta(x,t), \qquad \chi_{\text{ex}}(x,t) = 1 - \frac{T_{\text{amb}}(x,t)}{T_{\text{src}}(x,t)},$$

and replace the thermal coupling terms in the free energy and PDE system by

$$-\gamma_{\rho\theta}\rho\theta \;\to\; -\gamma_{\rho\theta}\rho\theta_{\text{ex}}, \qquad \eta\theta \;\to\; \eta\theta_{\text{ex}}.$$

Proposition 24 (Exergy Constraint)**.** *If $\theta_{\text{ex}} \to 0$ uniformly, then* $\text{Ti}_{\text{eff}} \to 0$ *and the system enters the expansive regime regardless of total heat output.*

Proposition 24 formalizes the requirement that reintegration must be thermodynamically usable, not merely energetically present. Cold-climate siting increases χ_{ex} by increasing T_{amb} margin; it is therefore not geographic latitude per se but the Carnot-efficiency of thermal capture that governs this parameter.

11.3. Network Locality and Latency Penalty

Geographic specialization toward thermodynamically favorable regions conflicts with informational locality constraints. Introduce a spatial cost functional encoding latency costs:

$$\mathcal{L}_{\text{net}}[\rho] := \int_\Omega c_{\text{lat}}(x)\, \rho(x,t)\, dx + \frac{\nu}{2}\int_\Omega |\nabla\rho|^2\, dx,$$

where $c_{\text{lat}}(x)$ penalizes deployment remote from latency-sensitive demand centers. Define the augmented free energy $\mathcal{S}_{\text{tot}} := \mathcal{S} + \mathcal{L}_{\text{net}}$.

Theorem 25 (Coupled Thermodynamic–Informational Stability)**.** *The gradient flow of $\mathcal{S}_{\text{tot}}$ admits a stable attractor if and only if $\lambda < 1$ and $\mathcal{L}_{\text{net}}$ is bounded below along trajectories.*

This induces a natural workload stratification: batch computation with large thermal residues and high Ti should minimize $\mathcal{S}$ and concentrate in thermodynamically favorable regions, while latency-sensitive inference should minimize $\mathcal{L}_{\text{net}}$ and remain proximate to population centers. Neither type globally dominates; the optimum is a spatial partition $\rho = \rho_{\text{core}} + \rho_{\text{edge}}$ in which each component minimizes a different component of $\mathcal{S}_{\text{tot}}$.

11.4. Endogenous Degradation

The baseline model treats r_{deg} as an exogenous input. In practice, degradation depends on utilization intensity, thermal stress, and environmental stressors. Replace it by

$$r_{\text{deg}}(x,t) = r_0 + r_\theta\, \theta(x,t) + r_u\, u(x,t) + r_{\text{env}}(x,t),$$

where u is utilization intensity and r_{env} encodes radiation, chemical corrosion, or thermal cycling.

Proposition 26 (Degradation-Induced Instability)**.** *If* r_{deg} *grows superlinearly in* θ *or* u*, then* $\mathrm{Dr} > 1$ *can occur even when baseline parameters satisfy* $\mathrm{Dr} < 1$*, driving a transition* $\lambda < 1 \to \lambda \geq 1$.

This is directly relevant to orbital systems, where r_{env} is large (radiation-induced fault rates orders of magnitude above terrestrial baselines) and no repair capacity exists. Endogenizing degradation makes the expansive character of orbital infrastructure a derived consequence rather than an assumed input.

11.5. Semantic Efficiency and the Rebound Objection

A contractive system with $\lambda < 1$ may satisfy thermodynamic closure while computing trivial outputs or deliberately inflating computation to generate marketable heat. To address this, define the semantic efficiency

$$\Psi := \frac{W_{\text{info}}}{Q_{\text{comp}} + E_{\text{emb}}/\tau},$$

where W_{info} is task-weighted informational work and E_{emb}/τ is the amortized embodied energy of hardware over cycle time τ. Infrastructure is then classified by the pair (λ, Ψ): the productive and stable quadrant requires both $\lambda < 1$ and $\Psi > 0$; a system with $\lambda < 1$ and $\Psi \approx 0$ is thermally closed but degenerate; one with $\lambda \geq 1$ and $\Psi > 0$ is productive but unsustainable.

Remark 27. Thermodynamic closure ($\lambda < 1$) is a necessary but not sufficient condition for socially meaningful computation. The parameter Ψ is required to distinguish genuinely productive from degenerate xylomorphic systems.

11.6. Refined Stability Criterion and Polar Siting

Define the effective order parameter over horizon $[0, T]$:

$$\lambda_{\text{eff}} := \sup_{t \in [0,T]} \frac{E(X_t(I))}{E(I)},$$

incorporating seasonal modulation, exergy weighting, endogenous degradation, and network costs.

Theorem 28 (Extended Xylomorphic Stability)**.** *An infrastructure is dynamically stable over* $[0, T]$ *if and only if* $\lambda_{\text{eff}} < 1$*, with* λ_{eff} *determined jointly by seasonal thermal coupling, exergy constraints, network locality, and endogenous degradation.*

We now identify the geographic conditions under which high-latitude siting guarantees this condition.

Definition 29 (Polar-Admissible Region)**.** A domain Ω is *polar-admissible* if it satisfies: persistent thermal demand ($\inf_t H_{\text{demand}} > 0$ a.e.), positive exergy margin ($\inf_t \chi_{\text{ex}} \geq \chi_0 > 0$), bounded environmental degradation ($r_{\text{env}} \leq r_{\max}$ subcritical for Dr), and admissible network embedding ($c_{\text{lat}} \leq c_{\max}$ for the intended workload class).

Theorem 30 (Polar Siting Sufficiency)**.** *Let* Ω *be polar-admissible. If infrastructure is deployed with thermal capture efficiency* $\alpha(x,t) \geq \alpha_0 > 0$ *and repair capacity satisfying* $\mathrm{Dr} < 1$ *at baseline, then* $\lambda_{\text{eff}} < 1$.

Proof. Persistent thermal demand gives $\alpha(x,t) \geq \alpha_0$ uniformly, so $\frac{1}{T}\int_0^T \mathrm{Ti}_{\text{eff}}\, dt \geq \alpha_0 \mathrm{Ti}$. The exergy bound $\chi_{\text{ex}} \geq \chi_0 > 0$ ensures θ_{ex} remains bounded away from zero. With $\mathrm{Dr} < 1$ and $r_{\text{env}} \leq r_{\max}$ subcritical, degradation does not dominate repair even under endogenous loading. The spectral stability condition of Proposition 18 then implies $\lambda < 1$ locally, and temporal aggregation yields $\lambda_{\text{eff}} < 1$. ☐

Corollary 31 (Failure Modes)**.** *Polar siting fails to guarantee contractivity when any of the admissibility conditions is violated: seasonal* $H_{\text{demand}} \to 0$ *drives* $\mathrm{Ti}_{\text{eff}} \to 0$*;* $\chi_{\text{ex}} \to 0$ *renders thermal output unusable; large* r_{env} *(e.g., radiation at orbital altitudes) drives* $\mathrm{Dr} > 1$*; or* c_{lat} *exceeding workload tolerance prevents functional deployment regardless of thermodynamic advantage.*

Theorem 30 therefore strengthens rather than replaces the geographic intuition: cold-region siting is a sufficient condition for contractivity precisely because it simultaneously increases χ_{ex} (cold ambient raises the Carnot margin), sustains H_{demand} (persistent heating need), and reduces baseline hardware stress. It is not geography but the underlying physical invariants that matter; geography is one reliable way to satisfy them.

12. DESIGN PRINCIPLES FOR CONTRACTIVE INFRASTRUCTURE

Theorems 28 and 30 translate into six implementable constraints that together constitute a design standard for xylomorphic infrastructure.

The first principle requires guaranteed thermal reintegration: a fixed fraction of generated heat must be productively captured at all times, $\inf_t \alpha(x,t) \geq \alpha_0 > 0$. Physical coupling to persistent thermal sinks—district heating networks, greenhouse heating, industrial drying, desalination preheat—is the primary engineering implementation. When α_0 exceeds the threshold implied by $\mathrm{Ti}_{\text{crit}}$, thermal reintegration alone is sufficient to maintain $\lambda_{\text{eff}} < 1$ in the absence of dominant degradation.

The second principle requires exergy preservation: thermal outputs must maintain sufficient temperature differential to remain usable, $\chi_{\text{ex}}(x,t) \geq \chi_0 > 0$. Below the minimum exergy threshold $\chi_{\min}$, no feasible capture fraction α can produce $\lambda_{\text{eff}} < 1$. This means server hardware must operate at temperatures that provide a usable differential above ambient, and siting must ensure the receiving thermal sink can absorb heat at that grade.

The third principle requires degradation–repair balance: maintenance capacity must exceed degradation across all operating conditions, $\sup_t \mathrm{Dr}(t) < 1$. Robustness requires a positive margin: there exists $\epsilon > 0$ such that $\mathrm{Dr}(t) \leq 1 - \epsilon$ uniformly. This bounds allowable utilization intensity and environmental stressors and implies that remote or extreme-environment deployments must carry proportionally greater repair and replacement infrastructure.

The fourth principle requires workload stratification. Computation must be partitioned into $\rho = \rho_{\text{edge}} + \rho_{\text{core}}$, where ρ_{core} minimizes thermodynamic cost and concentrates in thermally favorable regions, while ρ_{edge} minimizes latency cost and remains proximate to demand. There exists a decomposition minimizing $\mathcal{S}_{\text{tot}}$ such that $\rho_{\text{core}} \subset \Omega_{\text{polar}}$ and $\rho_{\text{edge}} \subset \Omega_{\text{urban}}$. Global contractivity is achieved through spatial specialization rather than uniform deployment.

The fifth principle requires temporal load matching. If computation is uncorrelated with thermal demand and no storage exists, then $\lambda_{\text{eff}} \geq 1$ over sufficiently long cycles. Load must therefore either track H_{demand} directly, route excess heat into seasonal thermal storage, or be migrated to regions whose demand is complementary.

The sixth principle provides a measurement and certification criterion. Define the empirical contraction ratio

$$\hat{\lambda}_\tau := \frac{E(I(t+\tau))}{E(I(t))}.$$

An infrastructure is *xylomorphically certified* if $\hat{\lambda}_\tau < 1$ for all admissible horizons $\tau \in [\tau_0, T]$. This provides the operational test for policy compliance: PoUWH certification enforces the source condition $\mathcal{R}(t) \leq 0$, which is a sufficient condition for $\hat{\lambda}_\tau < 1$ by Theorem 16, and therefore for xylomorphic certification.

Failure of any single principle can drive $\lambda_{\text{eff}} \geq 1$. The principles are jointly necessary: a system that reintegrates heat but operates below exergy threshold fails on the second; one that satisfies thermal and exergy conditions but neglects repair balance fails on the third; one that achieves all physical conditions but routes latency-sensitive workloads to remote sites fails on the fourth. The framework thereby defines *xylomorphic systems* not as a geographic class but as a structural class: computational infrastructure in which every operational cycle contributes to the physical renewal of the substrate that sustains it.

13. DISCUSSION

13.1. Relation to Classical Thermodynamic Systems

A reviewer may observe that the stability condition $\lambda < 1$ resembles entropy production bounds in classical open-systems thermodynamics, and ask what the categorical structure adds beyond established results. The distinction is structural rather than merely formal.

Classical open-systems thermodynamics [Prigogine, 1977] characterizes steady states in terms of minimum entropy production at fixed boundary conditions and identifies dissipative structures that self-organize in the presence of external forcing. The present framework differs in three respects. First, the boundary conditions are not fixed: the admissible morphism structure of Definition 1 explicitly allows the system boundary to change via infrastructure transformations, and λ captures the effect of boundary choice on long-run stability. Second, the categorical structure enables the invariance result of Theorem 8: λ is preserved under arbitrary changes of domain, coarse-graining, and system decomposition, a coordinate-independence property that has no direct analogue in classical entropy production theory. Third, the monoidal composition rule $E(I_1 \otimes I_2) = E(I_1) + E(I_2)$ provides a compositional calculus for distributed infrastructure that classical thermodynamics does not supply. The uniqueness theorem (Theorem 5) then derives λ as the only multiplicative invariant consistent with this compositional structure, which grounds the order parameter in resource theory rather than in any particular physical model.

13.2. Scope of the Framework

The λ-classification applies to any infrastructure whose operation produces residues, not only digital computation. Natural analogues exist in industrial ecology [Georgescu-Roegen, 1971, Ayres, 1998] and biological metabolic networks [Kauffman, 1993, Odum, 1994]. The endofunctor construction applies whenever Shed, Digest, and Print functors can be defined on appropriate residue and substrate categories.

13.3. Empirical Estimation of λ and Falsifiability

The theory makes quantitative predictions and is therefore falsifiable. Given measured data on an operating infrastructure over a cycle horizon τ, the order parameter is estimated directly as

$$\hat{\lambda}_\tau = \frac{\hat{E}(I(\tau))}{\hat{E}(I_0)},$$

where $\hat{E}(I_0)$ is the exogenous entropy influx in the reference period (external energy purchases, fresh material inputs, off-site heat rejection) and $\hat{E}(I(\tau))$ is the same quantity in the subsequent period. A consistent lifecycle assessment protocol, aligned with EU [2023], suffices to estimate both. The three dimensionless groups Ti, Dr, and Da can then be inferred from heat capture measurements, hardware replacement logs, and spatial monitoring of field gradients.

The framework is falsified by any of the following observations. First, if a system is empirically measured to satisfy $\hat{\lambda}_\tau < 1$ over multiple consecutive cycles yet exhibits unbounded growth in E under sustained agentic demand, the contractive-implies-stable claim (Theorem 11) is refuted. Second, if $\text{Dr} < 1$ and Ti is bounded as required, yet the linearized mode matrix $A(k)$ exhibits positive real eigenvalues, the spectral stability criterion (Proposition 18) is refuted. Third, if an orbital infrastructure is constructed that demonstrably satisfies $\hat{\lambda}_\tau < 1$ via some reintegration pathway not captured by the model—for example, large-scale in-space manufacturing closing the material cycle—the orbital instability proposition is refuted and the framework must be extended. The theory does not assert that orbital closure is physically impossible; it asserts that no current or proposed orbital architecture achieves it.

13.4. Limitations and Future Work

The explicit functional $\mathcal{S}$ employs a mean-field approximation to spatial heterogeneity; real infrastructures exhibit heterogeneous coupling coefficients, and accounting for this would require stochastic homogenization or spatially varying coefficient fields. The cross-coupling terms are symmetric; asymmetric coupling (heat assists substrate formation but not vice versa) would modify the off-diagonal entries of $A(k)$ without altering the structural argument. Endogenous degradation (Section 11) has been incorporated at the level of a linear dependence on θ and u; highly nonlinear degradation regimes, such as cascade failure or radiation-induced single-event upsets, may require additional state variables. Future work should characterize convergence rates under non-stationary agentic forcing (Theorem 15 establishes transience but not the time to divergence), determine the dependence of λ_τ on the horizon τ under non-stationary conditions, develop standardized lifecycle accounting protocols for empirical estimation of $\hat{\lambda}_\tau$ at data center scale, and quantify the (λ, Ψ) joint distribution over existing infrastructure to identify which systems already satisfy xylomorphic certification.

13.5. Relation to Ecological Economics

The framework extends the insight of Georgescu-Roegen [1971] and Odum [1994] that economic and computational systems are thermodynamic processes subject to entropy constraints, and the industrial ecology literature's emphasis on circular material flows [Ayres, 1998]. Existing waste heat recovery installations [Rambo and Azevedo, 2014, Stockholm Data Parks, 2020] and edge deployments [Satyanarayanan, 2017] demonstrate the practical viability of contractive architectures at scale. The formal contribution is a convergence theorem that elevates these engineering practices to a rigorous stability condition.

14. CONCLUSION

We have introduced xylomorphic computation as a formal framework for large-scale computational infrastructure, centered on the order parameter λ. This parameter is the unique multiplicative resource monotone compatible with the monoidal structure of infrastructure composition, the spectral radius of the linearized xylomorphic operator, and invariant under all admissible domain changes. The Xylomorphic Stability Theorem and Banach fixed-point result establish $\lambda < 1$ as the necessary and sufficient condition for stable attractors with geometric convergence; the Thermodynamic Selection Theorem proves expansive systems are transient under agentic demand growth. The discrete iteration is connected to an explicit continuum PDE via a variational principle; weak solutions exist by a complete Galerkin argument; and a computable spectral stability criterion is given via the linearized mode matrix.

The framework is extended in Section 11 to account for seasonal thermal demand modulation, exergy quality constraints, endogenous degradation, informational locality, and the distinction between thermodynamic closure and semantic productivity. The effective order parameter λ_{eff} subsumes all these effects into a single invariant, and Theorem 30 identifies polar siting as a sufficient—but not uniquely necessary—route to contractivity, characterized by the physical invariants of persistent thermal demand, positive exergy margin, bounded environmental degradation, and admissible network embedding. Six design principles in Section 12 translate the formal conditions into engineering constraints that together define the xylomorphic certification standard.

Orbital data center proposals satisfy $\lambda \geq 1$ under all physically realistic configurations, making them dynamically divergent rather than merely expensive. The misclassification of computational heat as waste is a system boundary error; when thermal demand is included in the system scope, computation that produces heat en route to informational work is strictly more integrated than dedicated heating. The PoUWH protocol is derived as an operational sufficient condition for Lyapunov descent of the xylomorphic free energy, certifying membership in the contractive regime. Stable large-scale computation under growing agentic demand requires infrastructure whose entropy cycles are closed, whose heat is productive rather than expelled, and whose degradation is continually outpaced by repair.

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