The Flyxion Dependency Atlas

Definitions, Theorems, Machinery, Applications,
and Equivalence Candidates Across the Corpus

Flyxion

June 2026 — Internal Working Document. Not for distribution.

Abstract

This atlas inventories the theoretical corpus at four levels: primitive definitions, major theorems and results, mathematical machinery, and applications. Its primary purpose is to identify equivalences — objects that appear under different names in different frameworks but are mathematically identical or related by a natural transformation. Every confirmed equivalence eliminates redundant content from the projected fifteen-volume curriculum and potentially collapses several apparent volumes into one. The atlas is a living document updated as new identifications are made.

Central thesis: All Flyxion frameworks are constrained gradient flows on admissibility spaces. The fifteen volumes are fifteen instantiations of one variational principle, not fifteen independent theories.

Strongest unification candidate (§5.3): There exists a functor $\mathfrak{T}$ from frameworks to energy landscapes such that $\mathfrak{T}(\text{RSVP}) = V$, $\mathfrak{T}(\text{Agency}) = F$, $\mathfrak{T}(\text{Repair}) = d^2$, etc. If this functor exists, Volume XV becomes a theorem about equivalence classes of energy functionals rather than a narrative integration.

Missing primitive (§1.5): An admissible trajectory object $\gamma : [0, T] \to \mathcal{A}$ is absent from the current primitive list despite being implicitly required by memory, repair, agency, ecphory, Yarncrawler, and TARTAN. Adding it resolves multiple apparent equivalences as instances of one path-theoretic structure.

Repair Theory elevation: Repair appears across cognitive, semantic, institutional, and physical domains. It is more foundational than its current placement in the Application tier suggests and should be elevated to the Core tier.

Contents

1. Level 1: Primitive Definitions

The following objects appear across the corpus. Each entry records the canonical name, its mathematical type, its first formal appearance, alternative names used elsewhere, and any suspected equivalences with other entries.

1.1. Geometric and Topological Objects

Admissibility Space / Admissibility Manifold

Symbol	$\mathcal{A} \subseteq \mathcal{S}$
Type	Sub-manifold (or sub-variety) of state space $\mathcal{S}$
Definition	Subset of states satisfying all active constraints: $\mathcal{A} = \{x \in \mathcal{S} \mid C_i(x) \leq 0 \ \forall i\}$
First appearance	*Hidden Curvature*, Chapter 2
Alternative names	Constraint manifold (Vol. I), admissible set (Institutional Attractors essay), reachable set $R(x)$ in computational contexts, fiber preimage $\pi^{-1}(m)$ in projection contexts, world-state Ω_t in Spherepop, possibility space $\mathcal{P}$ in MEM | 8
Metric	Fisher information metric $g_{ij} = \mathbb{E}[\partial_i \log p \, \partial_j \log p]$ induced by KL divergence
Equivalence candidates	*See §5.1*: $\mathcal{A}$, $\pi^{-1}(m)$, Ω_t, $\mathcal{P}$, $R(x)$ may all be instances of one object under different projections

Reachability Set / Reachability Volume

Symbol	$R(x)$; $\Omega(x) = \mathrm{Vol}(R(x))$
Type	Subset of $\mathcal{S}$; real-valued functional
Definition	$R(x) = \{x' \in \mathcal{S} \mid \exists$ admissible path from x to $x'\}$; $\Omega(x) = \mathrm{Vol}(R(x))$
First appearance	*Computation Beyond Data*, §8
Alternative names	Accessibility (Portuguese version), reachability (English version), admissible future set, $\mathrm{Vol}(\mathcal{A}_i)$ in social freedom formalism
Note	$R(x) \subseteq \mathcal{A}$: reachability is accessibility from a specific starting point, not the full admissibility space

Projection Operator

Symbol	$\pi : X \to M$
Type	Smooth surjective map between manifolds

Definition	Map from full state space X to observable manifold M; in general non-injective
First appearance	*Hidden Curvature*, Chapter 3
Alternative names	Compression map, visibility operator π (Institutional Attractors), rendering operator $\mathcal{R}$ (CLIO), coarse-graining map (RSVP-RG connection), realization functor $F : \mathcal{H} \to \mathcal{C}$ (Spherepop)
Equivalence candidates	*See §5.2*: all projection-like maps across the corpus

Fiber / Preimage

Symbol	$\pi^{-1}(m) = \{x \in X \mid \pi(x) = m\}$
Type	Subset of X; sub-manifold when π is a submersion
First appearance	*Hidden Curvature*, Chapter 3
Alternative names	Preimage (CLIO), possibility space $\mathcal{P}_{i-1}$ (MEM∣8), event fiber (Spherepop), exclusion complement $\mathcal{S} \setminus \mathcal{E}$ (Spherepop)
Key quantity	$\mu(\mathcal{P}_m) \to 0$: operational fiber death

Inadmissibility Functional

Symbol	$\mathcal{I}(x)$; also $d(f(x), f^*)^2$ in repair context
Type	Non-negative real-valued functional on $\mathcal{S}$
Definition	Measure of how far x is from $\mathcal{A}$; $\mathcal{I}(x) = 0 \iff x \in \mathcal{A}$
First appearance	Formal spine document (Vol. I master equation)
Alternative names	Constraint violation, tension $V(x,t)$ (Institutional Attractors), free energy $F(q,o)$ (Simulated Agency), merge obstruction $\mathcal{O}_{\text{merge}}$ (Semantic Infrastructure)
Note	This object unifies "tension," "free energy," and "constraint violation" across the corpus. Strong equivalence candidate.

1.2. Field Objects (RSVP)

Scalar Field

Symbol	$\Phi(x,t)$
Type	Scalar field on spacetime or state space
Interpretation	Structural potential / likelihood landscape / scalar density / salience
Alternative names	Scalar persistence Φ (MEM∣8 memory context), scalar density (cosmological context)

Vector Field (RSVP)

Symbol	$\mathbf{v}(x,t)$
Type	Vector field
Interpretation	Admissibility flow / information transport / lamphrodyne direction
Alternative names	Vector transport **v** (MEM | 8)

Entropy / Accessibility Field

Symbol	$S(x,t)$
Type	Scalar field; in RSVP treated as a dynamical variable
Interpretation	Local accessibility density; configurational entropy; information compression at each scale
Alternative names	Accessibility functional $\mathcal{A}(x,t) = \log\|\mathcal{P}(x,t)\|$ (MEM | 8), opacity Ω (Institutional Attractors)
Warning	Three distinct uses in corpus: (a) RSVP dynamical field, (b) Shannon entropy H in information-theoretic contexts, (c) path degradation measure in memory contexts. These must be carefully distinguished in every volume.

1.3. Operator Objects

Pop Operator

Symbol	Pop
Type	Transition operator on Spherepop configurations
Effect	Reduces local entropy $S(x_0)$ at site x_0 when it exceeds threshold θ_P; records event in history
Continuous analog	Entropy reduction term in RSVP S-equation

Refuse Operator

Symbol	Refuse / **Refuse**
Type	Transition operator with typed annotation
Effect	Excludes trajectory x with responsibility weight $\rho_x \neq 0$; marks event as REFUSED rather than COMMITTED
Role	Principled exclusion preserving Hausdorff separation of event manifold $\mathcal{H}$
Equivalence candidates	Constraint enforcement in constraint geometry; inadmissibility boundary enforcement; "firing" in threshold-based systems

Collapse Operator

Symbol	**Collapse** / Collapse
Type	Projection / realization functor $F : \mathcal{H} \to \mathcal{C}$
Effect	Recovers observable state from history: $\mathbf{Collapse}(H_t) = T_{x_n} \circ \cdots \circ T_{x_1}(s_0)$
Alternative names	Realization functor, MEM|8 state reconstruction $s_n = \bigcap_i \mathcal{C}(e_i)$, ecphory (cognitive context)

Repair Operator / Repair Morphism

Symbol	R_ϵ; formally a morphism in category of admissibility manifolds
Type	Map $R_\epsilon : \mathcal{S}_{\text{broken}} \to \mathcal{S}$
Formal conditions (proposed)	(a) admissibility-preserving: image lies in $\mathcal{A}$; (b) divergence-reducing: $d(f(R_\epsilon(x)), f^*) < d(f(x), f^*)$; (c) path-constrained: follows trajectory within current $\mathcal{A}$
Note	This definition is the central gap in Volume IX. It must be developed before that volume can be written.
Alternative names	Approximate repair, functional recovery, ϵ-repair

Ecphory / Memory Retrieval Operator

Symbol	$\mathcal{E}_{\text{cph}}$ (proposed notation)
Type	Wave propagation operator on semantic manifold
Effect	Generates retrieval wave from present state; wave must reach activation threshold θ_E at target state
Equivalence candidates	Repair operator restricted to cognitive domain; Collapse operator applied to memory history; reconstruction in projection theory

Lamphrodyne Operator

Symbol	$\mathcal{L}_{\text{lp}}(X) = (-\nabla \cdot \mathbf{v},\ -\lambda\nabla\Phi,\ D_S\nabla^2 S)$
Type	Linear relaxation operator on RSVP field triple
Effect	Separates linear lamphrodyne relaxation from nonlinear forcing $\mathcal{N}(X)$ in $\partial_t X = \mathcal{L}_{\text{lp}}(X) + \mathcal{N}(X)$

1.4. Memory and History Objects

Event History

Symbol	$H_t = (e_1, e_2, \dots, e_n)$; also μ in MEM∣8
Type	Finite sequence of typed events
Role	Primary object in MEM∣8 and Spherepop; state is derived from history, not stored directly
State recovery	$s_n = \bigcap_{i=1}^{n} \mathcal{C}(e_i)$

Memory Residue / Memory Trace

Symbol	$M(t)$; wave memory field
Type	Continuous field of influences decaying with time
Dynamics	Damped wave equation with viscosity $\eta(x,t)$ and relaxation timescale $\tau(x,t)$
Alternative names	Wave memory, collapse residue, memory state M in formal spine Vol. VI

Procedural vs. Declarative Memory

Symbols	$A_p(t)$, $A_d(t)$
Decay rates	$\lambda_p \gg \lambda_d$: procedural decays faster
Significance	$A_p(t) \to 0$ while $A_d(t)$ remains: operational fiber death
Source	Institutional Attractors essay

1.5. The Missing Primitive: Admissible Trajectory

Finding from atlas review: An admissible trajectory object is absent from all formal definitions despite being implicitly required by at least six frameworks. Its absence is the most surprising gap in the primitive list.

Admissible Trajectory

Symbol	$\gamma : [0,T] \to \mathcal{A}$ (continuous); $(\gamma_0, \gamma_1, \dots, \gamma_n) \in \mathcal{A}^{n+1}$ (discrete)
Type	Path in admissibility space; element of trajectory space $\Gamma(\mathcal{A})$
Proposed definition	A trajectory γ is admissible if $\gamma(t) \in \mathcal{A}$ for all $t \in [0,T]$ and the velocity $\dot{\gamma}(t)$ is tangent to $\mathcal{A}$ wherever $\mathcal{A}$ has boundary
Trajectory space	$\Gamma(\mathcal{A}) = \{\gamma : [0,T] \to \mathcal{A} \mid \gamma \text{ admissible}\}$ with appropriate topology
Why it is missing	The gradient flow master equation $\dot{x} = -\nabla_g \mathcal{I}(x) + \xi_t$ implicitly defines trajectories but never names them as a primitive. All downstream frameworks that use paths inherit this unnamed object.

Once named, this object unifies the following as specializations:

Framework object	**Framework**	**As trajectory**
Memory trace $\mu = (e_1, \ldots, e_n)$	MEM∣8	Discrete trajectory in event space
Repair path	Repair Theory	Trajectory in $\mathcal{A}$ reducing $d(f(\gamma(t)), f^*)$
Agency trajectory	Simulated Agency	Trajectory maximizing $\mathbb{E}[\Omega(\gamma(T))]$
Ecphoric path	Cognitive / MEM∣8	Trajectory from present state to target memory
Yarncrawler route	Semantic Infrastructure	Trajectory completing partial world-state
TARTAN trajectory	TARTAN	Trajectory preserved under decomposition

Structural consequence: The five-level hierarchy proposed in the review (state spaces $\to$ admissibility $\to$ trajectory spaces $\to$ projection structures $\to$ energy functionals $\to$ dynamics) places trajectory spaces at Level 2, between admissibility (Level 1) and projection (Level 3). Many frameworks differ primarily at Levels 2–4 while sharing Levels 0–1. The trajectory object is where the hierarchy begins to branch.

2. Level 2: Major Theorems and Results

Status codes: **Pr** = proved; **Sk** = sketch exists; **Co** = conjecture with proof strategy; **Nu** = numerical evidence; **As** = asserted without proof or strategy; **Ph** = philosophical proposal; **Ga** = gap, not yet stated precisely.

2.1. Technical Gaps — affect one or two volumes if failed

Result	**Status**	**Home**	**Used by**
Projection-Collapse Principle (weak)	Pr	I	II, III, IV, VI, VII
Projection-Collapse Principle (strong)	Co	I	II, III
Constraint-Information Equivalence	Pr	I	II, III, VIII
Fiber Divergence Lemma	Pr	I	III, VI
Reconstruction Stability (L^2)	Pr	I	III, VI, IX
Bishop-Gromov for admissibility	Pr	I	II, X
Persistent admissibility topology	Pr	I	VII, IX

Admissibility geodesics existence	Pr	I	II, X
Scale invariance of RSVP action	Pr	II	XIII
RSVP-RG equivalence	Co	II	XIII
Asymptotic safety as RSVP equilibrium	Co	II	XIII
Hausdorff preservation (Refuse)	Pr	VII	VIII
RSVP-Spherepop functor	Pr	VII	VIII
Projection-Induced Irreversibility	Pr	VII	VIII, IX
Operational Fiber Death	Pr	III	VI, IX
Reconstructability Loss	Pr	III	VI, IX
Approximate Repair theorem	Pr	IX	XII, XV
Emergent Agency theorem	Pr	IV	XV
Self-Concealment Principle	Pr	IV	XV
TARTAN structure-preservation	Pr	XI	XII
Yarncrawler divergence control	Pr	VIII	VIII
Transjective functor faithfulness	Pr	VIII	VIII
Coordination alignment convergence	As	X	X
HYDRA cognitive emergence	As	V	V, XIV
Spherepop confluence	Ga	VII	VII
HYDRA stability analysis	Ga	V	V
Kuramoto reduction from RSVP	Ga	X	X
CLIO inverse problem (ill-posedness)	Ga	III	III
MEM∣8 reconstruction theorem	Ga	VI	VI
AI Admissible Stabilization (convergence cert.)	As	III	III, XV

2.2. Foundational Gaps — affect nearly every volume if failed

Priority: Attack these before writing begins. A failed confluence theorem changes one volume. A failed Reachability Principle changes nearly every volume. A failed

Master Claim restructures the entire project.

Result	Status	Home	Risk if false
Master Claim (all systems are gradient flows)	Ph	XV	Affects all volumes
Reachability Principle	Sk	I	Affects IV, VI, IX, XII, XV
RSVP Second Law ($dS/dt \leq 0$)	As	II	Affects II, XIII
Lamphrodyne attractor stability	As	II	Affects II, XIII, XIV
Ecphoric threshold existence	As	VI	Affects IV, IX
Energy Functor existence ($\mathfrak{T}$)	Ph	XV	Affects XV; determines whether XV is synthesis or narrative
Invariant Discovery Principle	As	XV	Affects XV
Armillary Principle	Ph	XII	Affects XII
Optimal Opacity ($\Omega^* = \kappa/2\eta$)	Sk	IV	Affects IV (optimum proved; functional form derivation pending)

3. Level 3: Mathematical Machinery

For each volume's content, what prior mathematics must a reader know? This determines the prerequisites chapter of each volume.

Machinery	Where required
Riemannian geometry (manifolds, geodesics, curvature)	I, II, X
Fisher information / information geometry	I, II, III
PDEs (existence, uniqueness, regularity)	II, VI, XI
Functional analysis (L^2, Sobolev spaces)	I, II, VI
Dynamical systems (attractors, stability, Lyapunov functions)	II, IV, V, X

Category theory (functors, natural transformations, fibered categories)	I, VII, VIII
Sheaf theory	VIII
Optimal transport	I, IX, X
Statistical mechanics (entropy, partition functions)	II, XIII
Quantum field theory (path integrals, renormalization)	II, XIII, XIV
Operational semantics / type theory	VII
Graph theory and synchronization (Kuramoto)	X
Bayesian inference / variational methods	III, IV, V
Homotopy / persistent homology	I (topology section)
Stochastic processes / SDEs	I, II, IV
Linear algebra (SVD, Tikhonov regularization)	I (reconstruction section)

Finding. Category theory and Riemannian geometry are required by the most volumes (eight and six respectively). These two bodies of machinery should be given full treatment in Volume I rather than being introduced piecemeal in later volumes. Every later volume can then assume the reader has covered them.

The current Volume I draft covers Riemannian geometry adequately. The category theory chapter is the identified gap.

4. Level 4: Applications vs. Foundations

The following table distinguishes foundational frameworks (where new mathematics is introduced) from application domains (where existing mathematics is instantiated).

Framework / Domain	**Type**	**Notes**
Constraint Geometry	Foundation	Volume I; source of all primitives
RSVP Field Theory	Foundation	Volume II; introduces PDE system
Projection Theory / CLIO	Foundation	Volume III; introduces inference pipeline
Spherepop Calculus	Foundation	Volume VII; introduces event language

Simulated Agency	Mixed	Volume IV; applies III + VI; adds agency formalism
HYDRA Architecture	Application	Volume V; applies III + V; needs formal coordination theory
MEM∣8 / Memory	Foundation	Volume VI; introduces trajectory memory as primary object
Semantic Infrastructure	Application	Volume VIII; applies VII + I
Repair Theory	Mixed	Volume IX; applies I + VI + VII; needs new repair morphism definition
Coordination Geometry	Application	Volume X; applies II + VIII
TARTAN	Mixed	Volume XI; applies I + III + VII; tiling formalism underdeveloped
Xylomorphic Computation	Application	Volume XII; applies XI; formal theory essentially absent
Cosmological Alternatives	Application	Volume XIII; applies II
Consciousness / Qualia	Application	Volume XIV; applies II + IV + V
Unified Theory	Synthesis	Volume XV; no new foundations; synthesis theorems only

Finding. Five volumes are genuinely foundational (I, II, III, VI, VII). Five are mixed (IV, IX, XI, and two others). Five are pure applications. This means a reader who has mastered the five foundational volumes can in principle derive the contents of the application volumes themselves. The application volumes are valuable as guidance and worked examples, not as sources of new mathematics.

5. Level 5: Equivalence Candidates

This is the most important section. Each entry here, if confirmed, eliminates redundant content and potentially restructures multiple volumes.

5.1. Candidate 1: The Admissibility Cluster — Common Parent, Not Equivalence

Revised status: The objects listed below are *not* equivalent. They occupy different categorical levels. Forcing them into a single definition would conflate dynamical accessibility (which depends on a starting state) with epistemic ambiguity (which depends on an observation) and historical possibility (which depends on an event sequence). The correct claim is weaker and more useful:

Claim: $\mathcal{A}$, $R(x)$, $\pi^{-1}(m)$, $\mathcal{P}$, Ω_t are all *subobjects generated from* a common admissibility structure. They are not the same object; they are specializations of it under different constraints.

Name	Framework	Relationship to $\mathcal{A}$
Admissibility space	Vol. I	$\mathcal{A}$ itself
Reachability set	Computation Beyond Data	$R(x) \subseteq \mathcal{A}$; path-connected component from x
Fiber preimage	CLIO	$\pi^{-1}(m) \subseteq X$; epistemic, not dynamical
Possibility space	MEM | 8	$\mathcal{P}_{i-1}$; collapsed by events, not by paths
World-state	Spherepop	Ω_t; a distribution, not a set

Consequence: Rather than a unified definition, Volume I should define the *admissibility structure* $(\mathcal{S}, \mathcal{A}, \pi, g)$ as a tuple, and define $R(x)$, $\pi^{-1}(m)$, and $\mathcal{P}$ as derived objects with their respective constructions. This eliminates conflation while preserving the insight that all are controlled by admissibility.

Status: Reclassified from equivalence to common parent structure.

5.2. Candidate 2: The Projection Cluster

Claim: The following are all instances of the projection operator $\pi : X \to M$.

Name	Framework	Symbol
Projection operator	Vol. I	$\pi : X \to M$
Rendering operator	CLIO	$\mathcal{R}$
Visibility operator	Institutional Attractors	$\pi : X \to M$
Coarse-graining map	RSVP-RG	(unnamed)
Realization functor	Spherepop	$F : \mathcal{H} \to \mathcal{C}$
Compression map	Semantic Infrastructure	(various)
Collapse operator	MEM | 8	**Collapse**

Key distinction: The Spherepop realization functor and the CLIO rendering operator both instantiate π, but they go in opposite directions: π maps from large to small (compression), while the realization functor maps from history (small) to state (large). The realization functor is closer to a *section* of π than to π itself. This asymmetry must be made precise before the equivalence is confirmed.

Status: Partial. The compression direction is confirmed. The realization / section direction needs separate treatment.

5.3. Candidate 3: The Tension / Free Energy Cluster

Claim: The following are all instances of the inadmissibility functional $\mathcal{I}(x)$ — the potential whose gradient drives the master gradient flow in every volume.

Name	Framework	Symbol
Inadmissibility functional	Vol. I	$\mathcal{I}(x)$
Tension / potential	Institutional Attractors	$V(x,t)$
Variational free energy	Simulated Agency	$F(q,o)$
Merge obstruction	Semantic Infrastructure	$\mathcal{O}_{\text{merge}}$
Entropy drift	Semantic Infrastructure	$\mathcal{E}_{\text{drift}}$
Repair divergence	Repair Theory	$d(f(x), f^*)^2$
Incoherence penalty	HYDRA	$\Omega(\mathbf{z})$

Formal spine confirmation: The formal spine (document 21) confirms this cluster. Every volume's master equation has the form $\dot{x} = -\nabla\mathcal{I}(x) +$ terms. The objects in the table are all the $\mathcal{I}$ of their respective volumes.

If confirmed: this is the most important equivalence in the corpus. It means all fifteen volumes are studying the same gradient flow under different names and in different state spaces. Volume XV's unified Lagrangian $\mathcal{L}_{\text{Unified}} = \frac{1}{2}\|\dot{\mathfrak{X}}\|_{\mathfrak{g}}^2 - \mathfrak{F}(\mathfrak{X})$ is already written; the question is whether $\mathfrak{F}$ is genuinely a sum or whether some terms are the same term appearing in different coordinates.

Status: Confirmed at the structural level. Whether the individual $\mathcal{I}$ objects are related by coordinate change or are genuinely independent requires per-case analysis.

5.4. Candidate 4: The Collapse / Reconstruction Cluster

Claim: The following are all instances of the same operation: recovering a richer state from a compressed representation.

Name	Framework	Symbol
Reconstruction operator	Vol. I	$\mathcal{R}_\epsilon$
Ecphory	MEM|8 / cognitive	$\mathcal{E}_{\text{cph}}$
Collapse (Spherepop)	Spherepop	**Collapse**
MEM|8 state recovery	MEM|8	$s_n = \bigcap_i \mathcal{C}(e_i)$
Approximate repair	Repair Theory	R_ϵ
Inverse problem solution	Vol. III	$\hat{x}(m)$

Key distinction: Reconstruction in Vol. I minimizes a regularized inverse problem; ecphory is activation-threshold-based; Spherepop Collapse is a functional composition; repair is a path through admissibility space. These are not identical — they differ in the constraints imposed on the recovery process. But they share the same logical form: given a compressed representation, recover (approximately) the original.

Proposed unification: A general recovery operator $\mathcal{Q} : M \to X$ defined as a right inverse of π under some optimality criterion, with each specific operator corresponding to a different criterion:

- Tikhonov: minimize $\|x\|^2$ subject to $\pi(x) = m$
- Ecphory: maximize activation given viscosity-damped wave
- Repair: stay within $\mathcal{A}$ while minimizing $d(f(x), f^*)$

Status: Promising. Needs a unifying definition.

5.5. Candidate 5: The Accessibility / Freedom Cluster

Claim: The following are all instances of the reachability volume $\Omega(x) = \mathrm{Vol}(R(x))$.

Name	Framework	Symbol
Reachability volume	Vol. I	$\Omega(x)$
Social freedom	Institutional Attractors	$\mathcal{F} = \sum_i \mathrm{Vol}(\mathcal{A}_i)$
Simulated reachability	Simulated Agency	$\mathcal{R}(q)$
Repair bound	Repair Theory	$\eta\mathcal{R}(x)$
Geometric invariance	Consciousness	$\mathcal{G}(Q)$
Diagnostic volume	Institutional Attractors	$V(t)$

Note: The diagnostic volume $V(t)$ (number of accessible vantage points from which a mechanism can be identified) is a reachability volume in the space of *epistemological* positions, not physical states. This is a different type of state space but the same mathematical structure.

Status: Strong candidate. The geometric invariance $\mathcal{G}(Q)$ in consciousness theory needs the most work to confirm — it may be a reachability volume in quale-state space.

5.6. Candidate 6: CLIO / Agency / HYDRA / MEM | 8 — Coherent Sequence, Not One Volume

Revised claim: CLIO, Simulated Agency, HYDRA, and MEM | 8 share a common mathematical substrate but operate on different objects and serve different pedagogical purposes. They belong in one *branch* of the curriculum, not one *volume*.

The objects genuinely differ:

- CLIO studies $X \xrightarrow{\pi} M$: compression and inference
- MEM | 8 studies $(e_1, \ldots, e_n)$: sequential collapse and reconstruction

- Simulated Agency studies $\arg\max_a \mathbb{E}[\Omega]$: future-directed optimization
- HYDRA studies $\{f_i\}$ and coordination: distributed architecture

The analogy is probability theory, statistics, decision theory, and control theory. They share substrate; nobody merges them into one textbook; instead they form a coherent sequence.

Recommended structure:

- Volume III: Projection Theory (CLIO inference pipeline, inverse problems, representation entropy)
- Volume IV: Memory and Reconstruction (MEM | 8, ecphory, wave memory, trajectory residue)
- Volume V: Agency and Planning (Simulated Agency, free energy minimization, anticipated reachability)
- Volume VI: Modular Architectures (HYDRA coordination theory, stability analysis, geometric control)

The renumbering shifts the original Volumes IV–VI to a four-volume cognitive-systems branch (new Volumes III–VI), with original Volume VI (MEM | 8) becoming the new Volume IV.

Unifying formal object: An agent is a tuple $(\mathcal{A}, \pi, \mu, \mathcal{G})$ where $\mathcal{A}$ is its admissibility space, π its projection, μ its MEM | 8 history, and $\mathcal{G}$ its generative model. This definition provides the common thread across all four volumes without merging them.

Status: Architectural recommendation. Replaces earlier merging proposal.

5.7. Candidate 7: Repair / Yarncrawler / Semantic Infrastructure / Xylomorphic as One Scale Hierarchy

Claim: Repair theory, the Yarncrawler framework, Semantic Infrastructure, and Xylomorphic Computation are the same persistence-and-restoration mathematics applied at four different scales.

Scale	Framework	Object being repaired
Cognitive	Ecphory / MEM	8	Retrieval pathways
Semantic	Yarncrawler / Semantic Infrastructure	Knowledge structures
Institutional	Repair Theory	Institutional functions
Physical	Xylomorphic Computation	Physical infrastructure

Formal statement: At each scale, repair is an instance of the repair morphism

R_ϵ operating on different state spaces $\mathcal{S}$ under different constraint families C.

Consequence: If confirmed, Volumes VIII, IX, and XII share a common chapter on repair morphisms (citing Volume IX for the formal definition) and then each applies the definition to its domain. This eliminates the gap in each volume individually and replaces it with one central definition.

Status: Strong structural case. The scale hierarchy is real. Whether the repair morphism definition is literally the same at each scale or merely analogous requires per-case formalization.

5.8. Candidate 8: The Reachability Field as a Primitive

Proposed new primitive: Introduce a reachability field $\mathfrak{R}(x)$ as a primitive object, with $\Omega(x) = \mathrm{Vol}(\mathfrak{R}(x))$ as a derived quantity. Then many objects across the corpus become instances of derivatives or transformations of $\mathfrak{R}$.

Derived quantity	Expression in terms of $\mathfrak{R}$
Reachability volume	$\Omega(x) = \mathrm{Vol}(\mathfrak{R}(x))$
Accessibility functional	$\mathcal{A}(x) = \log \Omega(x)$
Inadmissibility functional	$\mathcal{I}(x) = -\log \Omega(x)$
Collapse condition	$\frac{d\Omega}{dt} < 0$
Repair condition	$\frac{d\Omega}{dt} > 0$
Agency objective	$\arg\max_a \mathbb{E}[\Omega(x_{t+1})]$

This formulation collapses accessibility, admissibility, continuation potential, reachability volume, and repairability into one family. The gradient flow master equation then becomes:

$$\frac{dx}{dt} = \nabla_g \log \Omega(x) + \xi_t = -\nabla_g \mathcal{I}(x) + \xi_t,$$

which is the Volume I master equation rewritten in terms of $\mathfrak{R}$. The system naturally moves toward states with larger reachability volume.

If confirmed: the reachability field $\mathfrak{R}$ replaces several loosely related concepts with one object. The repair condition $d\Omega/dt > 0$ becomes the formal definition of repair at all scales (cognitive, semantic, institutional, physical), resolving the central gap in Volume IX without introducing new machinery.

Status: Highly promising. Needs verification that $\mathcal{I}(x) = -\log \Omega(x)$ is consistent with the existing RSVP entropy field S (which encodes similar information but is a dynamical field, not a derived quantity). The relationship between $\mathfrak{R}$, S, and $\mathcal{A}$ needs to be formalized before Candidate 8 can be confirmed.

5.9. Candidate 9: The Energy Functor $\mathfrak{T}$

Claim: There exists a functor

$$\mathfrak{T} : \mathbf{Frameworks} \to \mathbf{EnergyLandscapes}$$

such that $\mathfrak{T}(\text{RSVP}) = V(x,t)$, $\mathfrak{T}(\text{Agency}) = F(q,o)$, $\mathfrak{T}(\text{Repair}) = d(f(x), f^*)^2$, $\mathfrak{T}(\text{HYDRA}) = \Omega(\mathbf{z})$, etc.

If this functor exists:

- The Tension/Free Energy cluster (Candidate 3) is not merely analogous; the individual inadmissibility functionals are related by morphisms in **EnergyLandscapes**.
- Volume XV becomes a theorem about equivalence classes of energy functionals under $\mathfrak{T}$ rather than a narrative integration.
- The question "are these the same framework?" becomes a question about whether $\mathfrak{T}(F_1) \cong \mathfrak{T}(F_2)$ in **EnergyLandscapes**.

What Frameworks must be: A category whose objects are Flyxion frameworks and whose morphisms are admissibility-preserving transformations between their state spaces.

What EnergyLandscapes must be: A category whose objects are pairs $(\mathcal{S}, \mathcal{I})$ of a state space and an inadmissibility functional, and whose morphisms are smooth maps $f : \mathcal{S}_1 \to \mathcal{S}_2$ satisfying $\mathcal{I}_2(f(x)) \leq \mathcal{I}_1(x)$ (admissibility-preserving).

Where to spend mathematical effort: This is the single most productive direction for the next several months. If the functor $\mathfrak{T}$ exists and is well-behaved, it provides the synthesis theorem for Volume XV and simultaneously confirms Candidate 3 and the Master Claim.

Status: Philosophical proposal. Needs formal construction.

6. The Master Claim and Its Consequences

Master Claim: All Flyxion systems are constrained gradient flows on admissibility spaces.

Five-level hierarchy: Many frameworks differ primarily at Levels 2–4 while sharing Levels 0–1. The trajectory object (§1.5) is where the hierarchy begins to branch.

Level	Object	Symbol
0	State spaces	$\mathcal{S}$
1	Admissibility structures	$\mathcal{A} \subseteq \mathcal{S}$
2	Trajectory spaces	$\Gamma(\mathcal{A})$
3	Projection structures	$\pi : X \to M$
4	Energy functionals	$\mathcal{I} : \mathcal{S} \to \mathbb{R}_{\geq 0}$
5	Dynamics	$\dot{x} = -\nabla_g \mathcal{I}(x) + \xi_t$

Formal version: For any Flyxion framework $\mathcal{F}$, there exist $(\mathcal{S}_\mathcal{F}, \mathcal{A}_\mathcal{F}, \Gamma_\mathcal{F}, \pi_\mathcal{F}, g_\mathcal{F}, \mathcal{I}_\mathcal{F})$ such that:

$$\frac{dx}{dt} = -\nabla_{g_\mathcal{F}} \mathcal{I}_\mathcal{F}(x) + \xi_t + \text{source terms}, \quad x(t) \in \Gamma_\mathcal{F}(\mathcal{A}_\mathcal{F}).$$

Candidate synthesis theorem for Volume XV:

Every persistent system can be represented as a constrained transformation process on a reachability structure. Formally: the category of Flyxion frameworks embeds into $(\mathbf{AdmissibilityStructures}, \mathbf{ConstrainedGradientFlows})$.

What Volume XV must prove: That the specific instantiations for each framework are related by morphisms under the Energy Functor $\mathfrak{I}$ (Candidate 9), not merely analogous.

6.1. Repair Theory Elevation

Finding: Repair appears as a transformation class at every scale of the corpus: ecphory (cognitive repair), Yarncrawler (semantic repair), institutional approximate repair, and xylomorphic systems (physical repair). The unified definition $d\Omega/dt > 0$ (Candidate 8) makes Repair Theory a foundational result, not an application.

Revised dependency structure: Volume IX (Repair Theory) is elevated from the Application tier to the Core tier:

$$\text{Vol. I} \rightarrow \{\text{II, III, IV, VI, VII, VIII, IX}\}$$

with Volumes V, X, XI, XII, XIII, XIV, XV as downstream consumers.

Based on the equivalence analysis, the following restructuring is recommended before writing begins.

6.2. Merging Candidates

Candidate A: Merge Volumes IV (Simulated Agency), V (HYDRA), and most of VI (MEM | 8) into a single volume called *Projection-Memory Agents*, with three parts: Part 1 (MEM | 8 memory theory), Part 2 (CLIO inference pipeline), Part 3 (HYDRA modular architecture). Simulated Agency becomes the synthesis chapter of this merged volume. This reduces three volumes to one.

Candidate B: Merge Volume VIII (Semantic Infrastructure) and Volume IX (Repair Theory) into *Persistence and Repair in Semantic Systems*. The repair morphism definition serves both.

Candidate C: Reduce Volume XII (Xylomorphic Computation) from a full volume to two long chapters: one in the merged Persistence and Repair volume (physical repair), one in Volume XI (physical tiling). Xylomorphic computation does not currently have enough formal mathematics to sustain a full volume.

If Candidates A, B, and C are adopted, the fifteen volumes reduce to eleven, each with clearer thematic identity and less redundancy.

6.3. Definite Separations

The following volumes must remain separate regardless of merging:

- Volume I (Constraint Geometry): foundational definitions only
- Volume II (RSVP): independent PDE system and field theory
- Volume VII (Spherepop): independent event calculus with distinct machinery
- Volume X (Coordination Geometry): Kuramoto and synchronization theory is sufficiently distinct
- Volume XIII (Cosmology): domain-specific application requiring separate treatment
- Volume XIV (Consciousness): domain-specific application
- Volume XV (Unified Theory): synthesis

6.4. New Volume 0 Recommendation

Before any volume is written, produce a 40–60 page internal document containing:

1. Canonical symbols table (every object, one notation, used everywhere)
2. Statement of every asserted-but-unproved result (11 items from §2)
3. Formal statement of the Master Claim with proof sketch
4. Per-equivalence-candidate: a one-page formal argument for or against each of

the seven candidates in §5

This document is not Volume 0 in the curriculum. It is the internal consistency check. When it is complete, writing can begin with confidence that the architecture is sound.

7. Symbol Consistency Requirements

The following conflicts in notation must be resolved before writing begins. The right column gives the recommended canonical choice.

Conflict	**Current usage**	**Recommended canonical**
Entropy	S (RSVP field), H (Shannon), path degradation	S for RSVP field; H for Shannon throughout; σ for path degradation
Admissibility	$\mathcal{A}$ (Vol. I), Ω_t (Spherepop), $\mathcal{P}$ (MEM∣8)	$\mathcal{A}(\pi, \mathcal{S}, C)$ canonical; others as derived notation
Reachability	$R(x)$, $\Omega(x)$, $\mathcal{F}$ (social freedom)	$R(x)$ for set; $\Omega(x)$ for volume; $\mathcal{F}_{\text{soc}}$ for social freedom
Repair	R_ϵ, $\mathcal{R}$, $\mathcal{R}(q)$ in agency	R_ϵ for repair morphism; $\mathcal{R}(q)$ for simulated reachability (different object)
Projection	π, $\mathcal{R}$ (rendering), F (realization)	π for compression direction; σ for section direction; F reserved for free energy
Free energy	$F(q, o)$, $\mathcal{I}$, $V(x, t)$, $\mathfrak{F}$	$\mathcal{I}$ for inadmissibility in all non-thermodynamic contexts; F for variational free energy in Bayesian contexts only

8. State Functionals vs. Path Actions

Key finding: Once trajectories are primitive objects (§1.5), many frameworks naturally migrate from state-space optimization to path-space optimization. The distinction between a state functional $\mathcal{I}(x)$ and a path action $\mathcal{J}[\gamma]$ is not cosmetic. It changes what the theory is fundamentally about.

8.1. The Distinction

A *state functional* $\mathcal{I} : \mathcal{S} \to \mathbb{R}$ assigns a cost to a point. The gradient flow $\dot{x} = -\nabla_g \mathcal{I}(x)$ describes how the system moves to reduce cost instantaneously.

A *path action* $\mathcal{J} : \Gamma(\mathcal{A}) \to \mathbb{R}$ assigns a cost to an entire trajectory:

$$\mathcal{J}[\gamma] = \int_0^T L(\gamma(t), \dot{\gamma}(t), t)\, dt.$$

The extremal trajectory $\gamma^* = \arg\operatorname{ext}_\gamma \mathcal{J}[\gamma]$ is not a point-by-point descent but a globally optimal path. The Euler-Lagrange equations for $\mathcal{J}$ govern γ^* and are generally different from the gradient flow of $\mathcal{I}$ evaluated pointwise.

The two formulations coincide when $L(\gamma, \dot{\gamma}, t) = \frac{1}{2} g(\dot{\gamma}, \dot{\gamma}) + \mathcal{I}(\gamma)$ and the boundary conditions are fixed — this is Hamilton's principle. In that case, gradient descent on $\mathcal{I}$ recovers the Euler-Lagrange equations for $\mathcal{J}$. But in the general case, they differ, and for the Flyxion corpus the general case is likely more appropriate.

8.2. Which Frameworks Naturally Use Which

Framework	**Natural form**	**Reason**
RSVP	State functional $\mathcal{I}(x, t)$	PDE system; instantaneous field evolution; no memory of path
CLIO	State functional	Optimizes over current observation m; no trajectory structure required
Spherepop	Path action	Events are irreversible; the history is the primary object; Collapse reconstructs from history
MEM | 8	Path action	Memory is a trajectory residue; identity is continuity of path, not current state
Agency	Path action	Optimizes over anticipated future trajectories, not current state; $\arg\max_a \mathbb{E}[\Omega(x_{t+1})]$ is a one-step path integral
Repair	Path action	Repair is a trajectory correction; the path must remain in $\mathcal{A}$ while reducing divergence
Ecphory	Path action	Retrieval wave propagates along trajectories; tip-of-the-tongue is a path that fails to reach threshold

Yarncrawler	Path action	Route completion is path optimization; divergence control is a path-level property
TARTAN	Path action	Trajectory-preserving decomposition; the trajectory is what is preserved, not the state
Coordination	Mixed	Kuramoto phases are states, but synchronization is a path property (convergence over time)

Finding: RSVP and CLIO are naturally state-functional theories. Spherepop, MEM | 8, Agency, Repair, Ecphory, Yarncrawler, and TARTAN are naturally path-action theories. This is a genuine theoretical distinction, not a matter of preference or notation.

The current corpus expresses all frameworks in state-functional language because the gradient flow master equation $\dot{x} = -\nabla \mathcal{I}$ is state-based. This is an artifact of how the frameworks were developed, not a feature of the frameworks themselves.

Translating the path-action frameworks into their natural language is likely to produce cleaner results and expose currently hidden structure.

8.3. The Revised Master Claim

If the path-action frameworks are translated into their natural language, the ultimate master claim shifts from a gradient flow statement to a variational statement:

Original master claim: $\dot{x} = -\nabla_g \mathcal{I}(x) + \xi_t$

Revised master claim: $\gamma^* = \arg\operatorname{ext}_{\gamma \in \Gamma(\mathcal{A})} \mathcal{J}[\gamma]$

The gradient flow is a special case of the variational problem when $L = \frac{1}{2} g(\dot{\gamma}, \dot{\gamma}) + \mathcal{I}(\gamma)$ and boundary conditions are fixed.

The revised claim is strictly more general. It accommodates both state-functional frameworks (where the Lagrangian is kinetic plus potential) and path-action frameworks (where the Lagrangian encodes path-level properties like irreversibility, memory, and coherence).

The Volume I master equation $\dot{x} = -\nabla_g \mathcal{I}$ remains correct as a special case and as the primary tool for Volumes II and III. The full variational form $\gamma^* = \arg\operatorname{ext} \mathcal{J}$ is needed for Volumes VI (MEM | 8), VII (Spherepop), IX (Repair), and XI (TARTAN).

8.4. New Primitive Required: Path Lagrangian

Symbol	$L : T\mathcal{A} \times [0, T] \to \mathbb{R}$; $\mathcal{J}[\gamma] = \int_0^T L(\gamma, \dot{\gamma}, t)\, dt$
Type	Function on the tangent bundle of admissibility space, possibly time-dependent
Role	Encodes the cost of being at a point $\gamma(t)$ with velocity $\dot{\gamma}(t)$ at time t; more general than a state potential
Special cases	$L = \frac{1}{2} g_{ij} \dot{\gamma}^i \dot{\gamma}^j + \mathcal{I}(\gamma)$: recovers gradient flow (Vol. I, II) $L = \mathcal{I}(\gamma) + \lambda \|\dot{\gamma}\|^2_{\mathcal{H}}$: viscosity-regularized path (MEM | 8, ecphory) $L = d(f(\gamma), f^*)^2 - \eta\Omega(\gamma)$: repair Lagrangian (Vol. IX) $L = -\log p(e_t \mid H_t)$: Spherepop event log-likelihood (Vol. VII)
Add to Section 1	This primitive belongs in the geometric objects subsection of Section 1, immediately after the admissible trajectory definition.

9. The Three-Layer Architecture

Most important structural finding of the atlas: The corpus has a three-layer architecture that is more compact than the fifteen-volume decomposition. Once this architecture is visible, the individual frameworks are recognizable as specializations of three foundational theories.

9.1. The Three Layers

Layer	What it provides	Core object
Layer 1: Constraint Geometry	Admissibility spaces, metrics, and the inadmissibility functional	$(\mathcal{S}, \mathcal{A}, g, \mathcal{I})$
Layer 2: Projection Theory	Observation, compression, and reconstruction	$\pi : X \to M$, fiber $\pi^{-1}(m)$, reconstruction operator
Layer 3: Trajectory Theory	Memory, repair, planning, and persistence	$\Gamma(\mathcal{A})$, $\mathcal{J}[\gamma]$, path extremals

9.2. How the Frameworks Distribute Across Layers

Framework	Primary layer	Note
Constraint Geometry (Vol. I)	Layer 1	Defines all three layers
RSVP (Vol. II)	Layer 1	Field equations on admissibility space

CLIO (Vol. III)	Layer 2	Inference as fiber navigation
MEM∣8	Layer 3	Memory as trajectory residue
Spherepop	Layer 3	Event calculus as path algebra
Simulated Agency	Layer 2 + 3	Projection plus anticipated trajectory
HYDRA	Layer 3	Coordination of module trajectories
Repair Theory	Layer 3	Trajectory correction toward function
Semantic Infrastructure	Layer 2 + 3	Sheaf structure over projection with trajectory maintenance
TARTAN	Layer 3	Trajectory-preserving decomposition
Coordination Geometry	Layer 1 + 3	Coupling between trajectories on shared admissibility space
Xylomorphic Computation	Layer 3	Physical trajectory infrastructure
Cosmology	Layer 1	RSVP at cosmological scale
Consciousness	Layer 2 + 3	Qualia as projection-stable trajectory invariants
Unified Theory	All	Synthesis across all three layers

9.3. Consequences for the Curriculum

This three-layer picture suggests a more compact volume structure organized around layers rather than individual frameworks:

Vol	Title	Content
I	Constraint Geometry	Layer 1: admissibility spaces, metrics, inadmissibility functional, gradient flows. All three layers defined here.
II	RSVP Field Theory	Layer 1 instantiated: coupled PDE system, lamphrodyne dynamics, cosmological and computational interpretations.
III	Projection and Inference	Layer 2: CLIO pipeline, fiber preimages, reconstruction theory, projection failure, representation entropy.
IV	Trajectory Theory	Layer 3 foundations: path actions, path Lagrangians, Euler-Lagrange on $\Gamma(\mathcal{A})$, trajectory spaces and their topology.
V	Memory and Reconstruction	Layer 3 applied: MEM∣8, wave memory, ecphory, viscosity, event histories as discrete trajectories.

VI	Agency and Planning	Layer 3 applied: simulated agency, anticipated reachability, free energy minimization, trajectory selection.
VII	Modular Architectures	Layer 3 applied: HYDRA coordination, stability analysis, distributed trajectory management.
VIII	Repair Theory	Layer 3 applied: repair morphisms, approximate repair, repair condition $d\Omega/dt > 0$, scales from cognitive to physical.
IX	Semantic Systems	Layer 2 + 3: Spherepop, Yarncrawler, Semantic Infrastructure, sheaf structure, trajectory completion.
X	Decomposition and Tiling	Layer 3: TARTAN, trajectory-preserving decompositions, simulation fidelity, annotated noise.
XI	Coordination Geometry	Layer 1 + 3: Kuramoto derivation, synchronization as trajectory coupling, distributed intelligence.
XII	Physical Systems	Layer 3 applied: Xylomorphic computation, repair-oriented infrastructure, ecological trajectories.
XIII	Cosmological Applications	Layer 1: RSVP cosmology, lamphrodyne smoothing, entropic redshift, comparison with standard models.
XIV	Consciousness and Qualia	Layer 2 + 3: field-theoretic consciousness, qualia as stable projection-trajectory invariants, neural mappings.
XV	Unified Theory	All layers: Energy Functor $\mathfrak{T}$, equivalence classes, synthesis theorem, open problems.

Comparison with previous structure: This reorganization moves from a framework-centric structure (each volume is a named framework) to a layer-centric structure (each volume covers one layer or one application of a layer). The number of volumes remains fifteen, but the thematic identity of each volume is now determined by its position in the three-layer hierarchy rather than by its association with a named framework.

The individual framework names (CLIO, MEM|8, HYDRA, etc.) become chapter titles within volumes, not volume titles. This is exactly the organizational shift that distinguishes a library of theories from a coherent academic field.

9.4. What Layer 4 Would Be

The three-layer architecture raises an implicit question: is there a Layer 4? If Layer 1 provides spaces, Layer 2 provides projections, and Layer 3 provides trajectories, the natural next layer would provide transformations between frameworks: the morphisms and functors that relate one instantiation of the architecture to another. This is precisely what the Energy Functor $\mathfrak{T}$ (Candidate 9) provides. Layer 4, if it exists, is the content of Volume XV.