Contradiction and Continuation:

A Constraint-Theoretic Derivation of Non-Classical Logic

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Abstract

This book sets out to derive Graham Priest's non-classical and dialetheic logics from a general theory of constrained representation, rather than presenting classical logic as the default and non-classical systems as deviations from it. It does not deliver the result it initially expected. Early planning aimed at a Glut–Obstruction Correspondence: some precise identification of Priest's contradictions with a geometric obstruction to assembling local evidence into a coherent whole. The completed manuscript establishes close to the opposite, and the result generalizes past the one theorem it was first stated for: **contradiction and obstruction are independent facts about the same data.** Chapter 22's Mutual Non-Determination Proposition is this claim's sharpest instance — whether a proposition is contradictory and whether its supporting evidence coheres do not determine one another, and an explicit six-cell construction proves it rather than merely failing to disprove it — but the same pattern recurs throughout the book: apparently related structures are repeatedly shown to carry independent coordinates rather than collapsing into one. Three further correspondences investigated with equal seriousness — non-normal worlds as refusal, bilattice merge as ordinary sheaf gluing, glut as a nonzero cohomology class — were rejected outright, at three different structural levels (modal, operational, geometric), after being plausible enough to have been provisionally accepted earlier in the project. The book's actual thesis, assembled from these results rather than imposed on them in advance, is that non-classical logics need not be understood as competing relaxations of one classical standard; they can be understood as responses to different failures along structural axes that turn out, on inspection, to be substantially independent of one another.

Introduction

0.1 What this book does not do

It does not treat classical logic as the ground state from which paraconsistent, intuitionistic, relevant, many-valued, and modal systems are departures requiring justification. Chapter 1 states the alternative directly: logical consequence is preservation under admissible transformation, $x \rightsquigarrow_R y$, with classical logic sitting at one specific point in the space of possible regimes R — the point combining maximal strictness on valuation (no gaps, no gluts) with unrestricted explosive propagation from contradiction under the relevant closure principles. Every other logic covered in this book is a different point in the same space — an alternative regime whose adequacy depends on the constraints governing the representational task at hand, not an equally good option regardless of task, and not a weakening of the classical one. Admissibility is precisely what lets the book discriminate among regimes; nothing here is an argument for treating all regimes as interchangeable.

By the book's own account, this way of stating things is only earned because three plausible correspondences failed to survive their chapters: non-normality is not refusal, merge is not gluing, and glut is not obstruction. A reader should not expect a book that translates every Priest concept into this framework's vocabulary and declares victory — several translations were attempted and explicitly rejected, and that outcome is treated as evidence for the book's thesis rather than as incidental failure.

0.2 The question the book actually asks

Stated once, in Chapter 1, and returned to throughout: *what must remain invariant when representations are combined, transported, refined, contradicted, or extended?* This is deliberately not the question "which logic is correct." It's a question about which structural distinctions survive contact with an actual formal system, once that system is built rather than merely proposed.

0.3 Separation before unification

The book's method, visible in retrospect across all six parts, is to take something ordinarily treated as one notion and ask whether it's actually several. Nearly every chapter that matters does this. Chapter 6 separates admissibility and accessibility: what states a regime permits, versus which of those states a given world can modally reach. Chapter 7 separates inconsistency, impossibility, and non-normality: a state containing a contradiction, a state failing to satisfy a governing regime, and a state whose evaluation rules are themselves suspended are three independent facts, not one phenomenon under three names. Chapters 9 and 10 separate positive and negative support: the two questions “is there evidence for” and “is there evidence against,” whose four possible joint answers generate Belnap and Dunn’s four truth values as a consequence rather than a stipulation. Chapter 13 separates support and constructibility: whether evidence exists, and whether it takes the specific form of an exhibitable witness, answer different questions even when the same proposition is at issue. Chapters 15 and 20–21 separate compatibility and dispersion: local evidence disagreeing outright, versus locally negligible variation that nonetheless accumulates past a threshold under coarse-graining, are different failure mechanisms with different mathematics. Chapters 20–22 separate semantic glut from geometric coherence defect: a proposition’s contradictory support status, and whether the evidence behind that status agrees with itself across overlapping contexts, do not determine one another. Chapters 20–21 separate gluing, merge, and repair: reconstructing a section from a family that already agrees, versus combining a family that doesn’t agree while retaining everything it supplied, are different operations that an earlier draft of this project once conflated under one word.

None of these separations were chosen for elegance in advance. Several were forced by a correspondence failing to hold up once its chapter was actually drafted — recorded, not smoothed over, in the correspondence ledger this book’s technical claims are checked against.

0.4 The hierarchy this produces

Six questions, asked of the same underlying material at increasing levels of structure, form the closest thing this book has to a single conceptual synopsis:

$$\text{support} \to \text{construction} \to \text{compatibility} \to \text{repair} \to \text{persistence} \to \text{continuation}.$$

Does evidence exist? Can it be exhibited? Can local descriptions coexist? If not, can they be minimally reconciled without discarding either? Does the resulting determination survive a change of scale? Does it remain identifiable as the same thing through a change of state? These are successive questions in the book’s analysis, asked in this order because each becomes askable once the last has been separated out — not because each presupposes or reduces to the one before it. Construction does not intrinsically presuppose support, and compatibility does not intrinsically presuppose construction; the book argues for their independence rather than building a reduction chain. The six parts largely track this order, though not in the sequence they were drafted.

0.5 The parameter that becomes structure

There’s a second way to see the same six parts, sharper for being purely technical: watch what happens to the single parameter R introduced in Chapter 1.

$$R \to (R, \mathcal{R}) \to (R, \mathcal{R}, \sigma^+, \sigma^-) \to \text{local support/coherence} \to \text{repair and persistence} \to R_w\text{-dependent continua}$$

Chapter 1 treats admissibility as one relation governing consequence. Chapter 6 finds that modal reasoning needs a second, independent relation, $\mathcal{R}$, accessibility. Chapters 9–10 find that a single admissibility judgment isn't enough to generate FDE's four values without splitting support into two independent coordinates, σ^+ and σ^-. Part VI finds that those coordinates, once localized across a cover, need both a support structure and a separate coherence structure to say what a family of local reports actually agrees on. Chapter 23's repair functional finds that action depends on both structures independently, via a term the earlier two-term cost function had no room for. And Chapter 7's R_w — needed to place non-normal worlds correctly — turns out, once Part V asks about identity across time, to mean that the admissibility regime itself can vary along a continuation, which is what produces this book's one genuine open problem rather than a known next step. The apparent single parameter of Chapter 1 does not stay a parameter. It progressively unpacks into structure, and the book's final open question is what happens once the last piece of that structure — R itself — stops holding still.

0.6 The methodological thesis

> Non-classical logics need not be understood as competing relaxations of one classical standard. They can instead be understood as responses to different failures or constraints along partially independent structural axes.

Paraconsistency, on this account, is what a regime looks like once it stops requiring gluts to be classically impossible. Intuitionism is what happens once construction is required in addition to support. Vagueness is what a resolution-indexed system looks like once persistence under coarse-graining, not mere agreement, becomes the relevant question. None of these needs to be motivated as a retreat from classical logic's demands; each answers a genuinely different question the classical regime never had to ask because it collapsed all these axes into one.

0.7 What this introduction promises, and no more

Every technical claim referenced above is checked against the correspondence ledger, which records status after drafting rather than before — including three correspondences the book investigated and rejected, and one genuine open problem the completed manuscript produced rather than resolved. This introduction does not promise more than that ledger backs.

Terminology Table

Pairs the book keeps distinct, and what conflating them would cost.

Distinction	If conflated	Established in
Admissibility (R) vs. accessibility ($\mathcal{R}$)	Modal necessity collapses into a single-parameter notion that can't separately vary "which states satisfy the regime" from "which states a given world can reach"	Ch.6
Inconsistency vs. impossibility vs. non-normality	A glutty state gets treated as automatically impossible, or an impossible state as automatically contradictory — neither holds	Ch.7
Positive support vs. negative support	Truth values become a stipulated alphabet rather than a consequence of two independent evidential questions	Ch.9–10
Support vs. constructibility	Non-constructive derivations and exhibitable witnesses get treated as the same accomplishment	Ch.13
Degree (many-valued) vs. evidential polarity (FDE)	Łukasiewicz-style values get read as a finer-grained version of Belnap-Dunn's four, erasing that they track different things (a truth continuum vs. two-axis evidential structure)	Ch.14

Distinction	If conflated	Established in
Compatibility obstruction vs. dispersion obstruction	Local disagreement at one resolution gets confused with locally-negligible variation accumulating across resolutions — different mechanisms, different mathematics (transitivity forbids the naive version of the second)	Ch.15, Ch.20–21
Restriction vs. gluing vs. repair	Combining a non-matching family (repair) gets described as if it were reconstructing an already-matching one (gluing)	Ch.20–21
Coherence defect ($\delta^{\pm}$) vs. glut ($\Sigma(p) = B$)	"Glut = nonzero obstruction class" gets asserted as a theorem it isn't — proven independent instead (Mutual Non-Determination Proposition)	Ch.20–22
Coherence liability vs. coherence defect	An action's cost from evidential mismatch gets treated as a fixed function of the mismatch alone, rather than depending on what the action needs from the evidence	Ch.23
Identity of states vs. continuity of histories	Two indistinguishable present states get treated as having the same continuation possibilities, ignoring that their histories may differ	Ch.16, Ch.19
State-indexed regime vs. one global regime	Non-normal worlds get explained as declining one transformation (refusal) rather than as suspending the evaluation rule that licenses transformations generally	Ch.7

Conclusion

0.8 The negative results, kept prominent rather than minimized

Three correspondences this project once found plausible enough to provisionally accept did not survive their chapters, at three different levels of the book's structure:

Three correspondences this project once found plausible enough to provisionally accept did not survive their chapters, at three different levels of the book's structure. At the modal level, non-normal worlds are not refusal (Ch.7): refusal is a targeted, constitutive decision to decline one transformation, while a non-normal world suspends the evaluation rule itself. At the operational level, bilattice merge is not sheaf gluing (Ch.20–21): gluing reconstructs a section from a family that already agrees, while merge combines a family that doesn't, retaining rather than discarding the disagreement. At the geometric level, glut is not a nonzero cohomology class (Ch.20–22): contradiction is a semantic fact about support, while the relevant obstruction machinery, once built correctly, is a fact about whether evidence coheres across overlapping contexts — proven independent of contradiction, not merely left unidentified with it.

These are evidence for the book's thesis, not embarrassments incidental to it. A project that had found ways to confirm every correspondence it initially proposed would have shown that its method could rationalize anything; a project whose method occasionally says no is one whose positive results — the Mutual Non-Determination Proposition foremost among them — are worth taking seriously precisely because the same method also produced these refusals.

0.9 The open problem this book ends on

This is not a loose end awaiting a known fix. It's the point at which the book's own framework ceases to close. The introduction's parameter arc — $R \to (R, \mathcal{R}) \to (R, \mathcal{R}, \sigma^+, \sigma^-) \to$ local support/coherence $\to$ repair and persistence $\to$ R_w-dependent continuation — traced R progressively unpacking into structure as each part of the book found the previous parameter insufficient. Completing Parts II and V together shows what happens once that unpacking reaches R itself: the thing Chapter 1 treated as fixed background turns out, once identity across time is at stake, to be capable of varying along the very trajectory it's supposed to govern.

$$x_t \xrightarrow{?} x_{t+1}, \qquad R_{x_t} \neq R_{x_{t+1}}.$$

Once admissibility is allowed to vary by state — Chapter 7's R_w, needed to correctly place non-normal worlds — trajectory-based identity (Chapter 16) needs more than a sequence of admissible states connected by $\rightsquigarrow$. It needs an account of what makes a *transition between regimes* admissible, not merely what makes each endpoint admissible on its own. Chapter 17's transition-object — treating a transition as its own kind of entity rather than a state satisfying

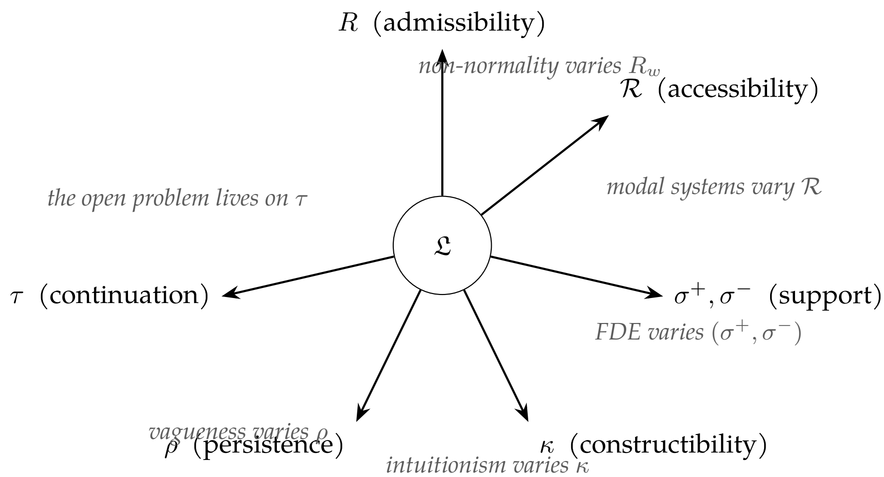

Figure 1: A schematic coordinate picture — explicitly not a formal definition — of a logical regime $\mathfrak{L} = (R, \mathcal{R}, \sigma^+, \sigma^-, \kappa, \rho, \tau)$ as a point in a space of independently variable structural dimensions. The named non-classical logics do not sit along a single axis running from classical to less classical; each alters a different coordinate, which is this book's methodological thesis made visible. The tuple is a mnemonic for the book's separations, not a definition the manuscript supplies for every symbol.

an enriched valuation — is a candidate answer, floated at exactly the point in the book where it became relevant and deliberately not promoted to a definition before it had earned that status. Whether the right structure is a fixed choice between R_{x_t} and $R_{x_{t+1}}$, some combination of the two, or a genuinely separate transition regime R_γ, is left open. This book's contribution is producing the question in a form precise enough to be wrong about, not answering it — and producing it as the place where the framework's own regress of parameters runs out, rather than as an item on a future-work list unconnected to everything before it.

0.10 What comes next

The correspondence ledger, not this conclusion, is the working document for anyone extending this material — it records status after drafting, chapter by chapter, and should be checked before any claim in this introduction or conclusion is relied upon for a further paper. Two results look most likely to warrant standalone treatment ahead of the full manuscript: the Mutual Non-Determination Proposition with its realizability construction, and the R_w/continuation problem stated above. Both are self-contained enough to be evaluated without the rest of the book's apparatus, and both are the kind of result this project was built to produce — not agreement with Priest, and not a redescription of him, but a specific, checkable place where this book's own machinery says something Priest's framework alone does not.

Part I

I: Logic as Constraint

Chapter 1 — The Myth of a Single Logic

The question this book opens with is not "which logic is correct." It's:

> What must remain invariant when representations are combined, transported, refined, contradicted, or extended?

That question doesn't presuppose an answer about how many truth values there are, whether contradictions explode, or whether every proposition is either true or false. It only presupposes that representations get combined, moved, refined, sometimes contradicted, and sometimes extended — and asks what survives those operations. Logic, on this view, is the study of what a given regime of transformation preserves.

Consequence as preservation under admissible transformation. Let a *state* be whatever a representational system currently commits to — a set of accepted propositions, a configuration of evidence, a model. Write $x \rightsquigarrow_R y$ for "state y is reachable from state x under the transformations a regime R permits." Logical consequence, in this vocabulary, is:

$$\Gamma \models_R \varphi \quad \text{iff} \quad \text{every } R\text{-admissible extension of a state satisfying } \Gamma \text{ also satisfies } \varphi.$$

This is not a new definition invented to be exotic. It's the same shape as the ordinary one — consequence as truth-preservation across models — with one change: which transformations count as admissible is now a parameter, R, rather than fixed in advance.

Classical logic is a point, not the origin. Classical consequence is what you get when R is set to its most stringent possible value: no tolerance for a proposition and its negation both holding (no gluts), no tolerance for a proposition holding neither way (no gaps), and unrestricted propagation — once a contradiction is admitted anywhere, every proposition becomes admissible everywhere. That last clause is *ex falso quodlibet*, and it is worth noticing that it is a substantive commitment about R, not a logical necessity. Setting R to be less strict — tolerating gluts, tolerating gaps, restricting how far a local failure propagates — doesn't produce a rival to classical logic in the sense of a competing claim about the same subject matter. It produces a different regime, and classical logic is the member of that family where the dial is turned to maximum strictness in every direction at once. Later chapters (starting in Chapter 3, and given full formal treatment in Part VI) will make this literal: the space of possible regimes is a real object, not a metaphor, and classical logic is a identifiable, describable point within it — not the space's center, and not its default.

What this buys, and what it doesn't yet. Reframing consequence this way doesn't resolve any dispute about which logic to use for a given purpose. What it does is stop the question from being framed as "classical logic, plus some deviant alternatives" — a framing that makes every non-classical system look like a patch or a concession. If classical logic is one regime among many, non-classical systems aren't deviations requiring justification; they're other points that need locating, the same way classical logic does. That relocation is this book's actual project, and it starts, in the next chapter, from further back than truth values themselves.

Chapter 2 — Distinction Before Truth

The ordinary starting point for logic takes truth values as given: propositions are things that are true or false (or, in richer systems, some fixed number of other things), and logic studies how those values combine. This chapter argues that starting point already presupposes something that itself needs an account — and that the account, once given, changes what a truth value turns out to be.

The primitive act is distinction, not valuation. Before anything can be true or false, something has to be *distinguished* — carved out as a determinate content that a truth value could even apply to. Asking whether p is true already presupposes that p names a specific distinction: this, as opposed to not-this, within some domain. That prior act — drawing the distinction — is not itself a truth-evaluable event. It's what makes truth-evaluation possible afterward.

What follows from taking this seriously. If distinction comes first, a proposition's truth value shouldn't be treated as a primitive label attached to it from outside. It should be *read off* whatever process generated the distinction and gathered evidence bearing on it. This is not a promissory note for later — it's exactly what Part VI does, in full formal detail: Chapter 19 reconstructs the four Belnap-Dunn values not as a stipulated alphabet but as the four possible outcomes of asking two independent questions — is there evidence *for* this distinction holding, and is there evidence *against* it — with N, T, F, B arising as a consequence of the support data rather than as an assumed starting vocabulary. A glut isn't a fifth, exotic truth value bolted onto a two-valued base; it's simply what "both kinds of evidence present" already means once truth values are treated as outputs rather than inputs.

Why this matters for the rest of the book. Every non-classical system covered in this book — paraconsistent, intuitionistic, many-valued, relevant, fuzzy — is usually introduced by first fixing a set of truth values and then giving rules for how they combine. That's a defensible way to present a formal system, but it obscures a question worth asking first: *why these values, and not others?* Once truth values are treated as consequences of a prior act of evidence-gathering over a distinction, that question has an answer that doesn't have to be stipulated separately for every logic in turn — the values on offer are exactly the values a given evidential structure can produce, and different logics turn out to differ in what evidential structure they're built to track, not in an arbitrary choice of alphabet.

Chapter 3 — Consistency and Closure

"Consistent" is usually treated as a single property a system either has or lacks. This chapter argues that collapsing several genuinely different notions into one word is a large part of why paraconsistency looks paradoxical from the outside, and separates out the notions that actually do the work.

Local consistency. A single region of a representational system — one patch, one context, one moment — is locally consistent with respect to a proposition p if it doesn't itself hold both p and its negation. This is a property of one piece of the system in isolation, and nothing about it says anything yet about how that piece relates to any other.

Global consistency. The whole system, taken together, is globally consistent if no proposition is jointly supported and refuted once every region's contributions are combined. This is a strictly different claim from local consistency at every region individually — a system can be locally consistent everywhere and still fail to be globally consistent, if different regions individually support and refute the same proposition without either one containing a contradiction on its own. (Part VI works this distinction out in full: Chapter 21's Glut-Free Reconstruction Obstruction is precisely the condition under which local data of this kind admits a globally consistent reading at all.)

Constraint closure. A regime R (Chapter 1's admissibility relation) is closed under a class of transformations if performing them never takes an admissible state outside admissibility. This is what actually governs whether a local failure spreads: classical explosion is the claim that R is closed under "infer anything from a contradiction" — which is one specific, and maximally permissive, choice of what counts as an admissible move once a contradiction is present. Restricting closure — refusing to admit that move — is not refusing to reason. It's choosing a different, more conservative, and equally coherent notion of what a contradiction is permitted to license.

Representational coherence. Separate again from all three above: whether the pieces contributing to a proposition's support actually *agree* with each other, not merely whether they license the same verdict. Two regions can both support p being true without their support being the same support — Chapter 20's coherence defect gives this its formal treatment, and it is a genuinely fourth notion, not a restatement of the first three. A system can be locally consistent everywhere, globally consistent, closed under a conservative propagation rule, and still be incoherent in this sense: agreeing on verdicts for the wrong reasons.

Keeping these four separate — local consistency, global consistency, closure, coherence — is not pedantry. Confusing any two of them is exactly what makes claims like "tolerating contradiction is dangerous" sound like a single, self-evident worry, when in fact the danger (if there is one) lives specifically in unrestricted closure, and neither tolerating local gluts nor tolerating global disagreement nor tolerating incoherent-but-agreeing witnesses is the same danger, or necessarily a danger at all.

Chapter 4 — Failure Without Collapse

This chapter's title is meant as a thesis, not a description: a representational system can fail — locally, even repeatedly — without that failure collapsing the system into uselessness. What follows is an account of the three different things "failure" can mean, and the condition that actually separates recoverable failure from catastrophic failure.

Three modes of failure.

Contradiction is a local matter: a proposition and its negation both supported, somewhere. On its own, per Chapter 3, this says nothing about the health of the system as a whole.

Obstruction is a structural matter, not a semantic one: local pieces of evidence that individually make sense fail to assemble into a coherent global picture, whether or not any single piece is itself contradictory. Chapter 20's coherence defect is the formal name for this; it can occur with no contradiction present at all.

Refusal is neither of the above — it's a deliberate constitutive act, the decision that some transformation is not admissible, made in advance rather than discovered as a failure after the fact. A regime that refuses a move isn't failing to reason its way past a contradiction; it's declining, as a matter of what the regime *is*, to treat that move as available.

The distinction that actually matters: inconsistency versus triviality. Classical logic's core commitment is that these three modes of failure, whenever they occur, must be treated identically — as signals that something has gone wrong that licenses anything at all to follow. That identification is the actual source of explosion, and it's worth stating as its own claim, separable from the rest of classical logic's machinery: *inconsistency* (a contradiction is present somewhere) is not the same fact as *triviality* (literally everything is now derivable). A system can be inconsistent — even irreducibly, permanently inconsistent at some point — without being trivial, provided its closure conditions (Chapter 3) don't license the move from one to the other. Priest's paraconsistent logics [7] are, in this vocabulary, exactly regimes where that licensing move has been withheld: contradiction is admitted as a fact a system can contain, while triviality remains exactly as catastrophic and exactly as avoidable as it always was.

What "failure without collapse" actually requires. Not tolerance for error, and not a weakened notion of truth. It requires a regime whose closure conditions don't route every local failure into global collapse — which is a structural property of R, specifiable and checkable, not a permissive attitude adopted toward contradiction for its own sake. Part VI gives this requirement its sharpest form: Chapter 22's repair machinery treats a glut as something with a repair cost, not as an alarm demanding total system reset, and Chapter 21's separation of contradiction from obstruction is precisely the demonstration that "something has failed" is not one fact but several, each with its own, independently manageable consequences.

0.11 Status of this draft

Part I is written to be readable on its own — a reader encountering only these four chapters gets a complete philosophical case for treating admissibility, distinction, and constrained closure as prior to truth values and classical consistency — while forward-referencing Part VI honestly rather than pretending its formal results belong here. Nothing in this draft is a new technical claim; the technical content is Part VI's, established in Chapters 19–22. What's new here is the exposition: stating the philosophical case in a form that motivates Part VI's machinery before the reader has seen it, rather than only in retrospect. One thing to watch when Parts II–V are drafted: Chapter 3's four-way consistency/closure/coherence distinction is doing real setup work for Part II's treatment of non-normal and impossible worlds, and Part V's identity-under-transport material will likely want to lean on Chapter 2's distinction-before-truth framing rather than restate it — worth checking those connections hold once those parts exist, rather than assuming they will.

Part II

II: Worlds and Admissibility

Chapter 5 — Possible Worlds as Admissible States

Take the most conservative starting point available: a **world** w is a state satisfying some governing regime R (Chapter 1) — $w \in \mathrm{Adm}(R)$. As a first approximation, modal necessity is invariance across the admissible states R reaches:

$$\Box_R p \quad \text{iff} \quad p \text{ holds throughout the } R\text{-accessible admissible states.}$$

This formula is provisional, and it's worth being honest about why before using it for anything: it makes a single relation, R, do two jobs at once — deciding which states are *admissible at all*, and deciding which admissible states are *relevant to evaluating* $\Box$ *at a given world*. Those are not obviously the same question. Whether a state satisfies the constraints governing the domain (admissibility) and whether that state is one this particular world can "see" for purposes of modal evaluation (accessibility) are different facts about a state, and a definition that quietly identifies them is buying convenience at the cost of a real distinction. Chapter 6 exists specifically to undo that conflation; this chapter's job is only to introduce $\mathrm{Adm}(R)$ itself; readers should treat the box formula above as scaffolding to be dismantled, not as this chapter's finished result.

What admissibility does and doesn't settle. $\mathrm{Adm}(R)$ gives worlds a structural role — the space of states a regime permits — without committing to what worlds metaphysically *are*. Priest's own survey of realism, actualism, and Meinongian approaches to possible worlds is a debate about the second question; this framework only needs an answer to the first one to get started, and Priest introduces possible-world semantics [8] in exactly this same methodologically modest role before taking up the ontological question separately. Nothing here forces a choice between treating admissible states as concretely existing alternatives, as actualized constructions, or as objects that need not exist in the same sense actual things do. The chapter deliberately leaves that choice open, because nothing built so far depends on it being made.

Chapter 6 — Necessity, Accessibility, and Invariance

Chapter 5's box formula conflated two relations. This chapter separates them and, in doing so, corrects something the book's own outline had gotten slightly wrong: "necessity as invariance," stated without further qualification, describes something looser than what Kripke semantics [4] actually is. Kripke necessity is not invariance across admissible states generally — it's truth throughout the states a specific accessibility relation selects, and that relation is additional structure, not a byproduct of admissibility itself.

Two parameters. Let R continue to determine admissibility — which states satisfy the governing regime — and introduce $\mathcal{R}$, independently, as the modal accessibility relation: $w\mathcal{R}v$ means v is relevant to evaluating modal claims at w. Necessity becomes

$$w \models \Box p \quad \iff \quad \forall v\, (w\mathcal{R}v \Rightarrow v \models p),$$

with no requirement that $\mathcal{R}$ be derived from, or even compatible with, R in any fixed way. A state can be $\mathcal{R}$-related to w without being R-admissible in the same sense w is (worlds under consideration for modal purposes need not all satisfy identical constraints), and a state can be R-admissible without being $\mathcal{R}$-accessible from any world in particular (an admissible state simply not under consideration from a given vantage point).

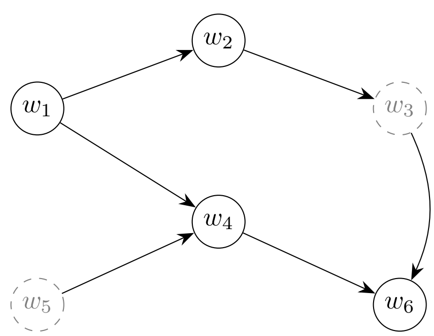

Figure 2: Admissibility and accessibility are predicates of different types. Arrows are accessibility edges $w\mathcal{R}v$; solid nodes are R-admissible states, dashed gray nodes are not. $w_2\mathcal{R}w_3$ holds even though $w_3 \notin \mathrm{Adm}(R)$ — an accessible world need not be admissible — while $w_6 \in \mathrm{Adm}(R)$ is reached only from an inadmissible state and from w_4; admissibility generates no accessibility edge of its own. The rejected single-R formulation of Chapter 5 would have made this frame inexpressible.

What this buys. Priest's various normal modal systems — the differences between K, T, S4, S5 and the rest — are differences in what structural properties $\mathcal{R}$ is required to have: reflexivity, transitivity, symmetry, and combinations of these, imposed on accessibility specifically. None of this needs to touch R at all. A system can strengthen or weaken its accessibility structure while leaving admissibility exactly as it was, or vice versa — change what counts as an admissible state without touching which admissible states are mutually accessible. Treating “changing the modal logic” and “changing the admissibility regime” as the same kind of move would have been a real error, not a harmless simplification, and separating them here is what makes the next chapter’s argument possible at all.

Chapter 7 — Non-Normal and Impossible Worlds

This chapter is Part II's centerpiece, and it starts by retracting a claim rather than building on one. An earlier working note in this project's correspondence matrix rated "a non-normal world deletes the inference rule itself from what's admissible" as a direct match to this book's refusal machinery. That rating is downgraded here, from Direct to Bridge, until this chapter actually establishes it — asserting a correspondence because the vocabulary sounds compatible is exactly the kind of move this book has tried not to make, and there's no reason to exempt this claim from that discipline.

What Priest's non-normal worlds actually do, reconstructed first. At an ordinary ("normal") world, $\Box p$ is evaluated compositionally: true iff p holds at every $\mathcal{R}$-accessible world, per Chapter 6. At a **non-normal** world, that compositional rule is suspended — the truth value of $\Box p$ at a non-normal world is not computed from what holds at accessible worlds at all; it can be assigned independently, including being false at a non-normal world even for a p that is a logical truth holding at every other world. The consequence that matters most: modus ponens itself, ordinarily a validity-preserving step everywhere, need not hold at a non-normal world, because the world's evaluation behavior is no longer tied to the mechanism that made modus ponens valid elsewhere. **Impossible worlds** generalize this further still: worlds at which the ordinary logical laws governing the rest of the framework may simply fail to apply, not merely modal ones.

What actually reproduces this, once reconstructed. The behavior above is not a targeted decision to decline one specific transformation — that would be refusal, in Chapter 4's sense, and refusal is a *Level 3* operation: a constitutive choice about which repair moves an agent or system will make. What non-normal worlds do is different in kind: the *evaluation rule itself* is locally not in force. In this book's vocabulary, that is best captured not by refusal but by treating the regime as state-indexed rather than global: R_w, a regime that can itself vary from state to state, rather than one fixed R applied everywhere with exceptions carved out afterward. A non-normal world is a state whose local R_w simply does not include the closure condition (Chapter 3) that would make $\Box$'s compositional rule, or modus ponens, admissible moves there. This is a real correspondence, but a different and more specific one than the original matrix entry claimed — and it's one this chapter has now actually earned, rather than assumed.

The control case Part III supplies. Before Part III existed, it would have been tempting to treat "impossible" and "contains a contradiction" as roughly the same idea. Part III makes that identification untenable. An FDE world can contain both p and $\neg p$ — a glut, in Chapter 10's sense — while its consequence behavior remains perfectly well-defined: Chapter 11's restricted propagation tells you exactly what follows from that glut and what doesn't, compositionally, with no suspension of any evaluation rule anywhere. Such a world is inconsistent without being impossible in the sense this chapter has just defined, and without being non-normal — nothing about its evaluation machinery has been switched off. Conversely, a non-normal or impossible state can fail to validate modus ponens, or fail to respect $\Box$'s compositional rule, while containing no glut at all — every atomic proposition perfectly determinate, no overdetermination anywhere, and still a state where some structural rule simply isn't in force. These are three independent axes, not one phenomenon under three names:

$$\boxed{\text{inconsistency} \neq \text{impossibility} \neq \text{non-normality}}$$

This belongs beside Chapter 3's four-way consistency/closure/coherence split as the same kind of discipline applied to modal vocabulary instead of evidential vocabulary. And it produces one sentence worth treating as foundational for the rest of this book's modal material:

> **Impossibility is indexed to a constraint regime; contradiction is indexed to support.**

A state is impossible or not relative to which R_w it's being measured against — the same state can be admissible under one regime and inadmissible under a stricter one, with nothing about its content having changed. A proposition is contradictory or not as a fact about its support, full stop, independent of any regime. Confusing these two coordinates — treating the presence of a glut as automatically making a world impossible — would undo exactly the distinction Part III spent four chapters establishing.

Chapter 8 — Conditionals as Constrained Transport

This chapter combines what earlier planning had kept as two separate chapters — strict implication and Lewis-style conditional logic [5] — because both are attempts at the same underlying question, and treating them separately obscured that.

The question. Given a state w and a supposition p — not necessarily true at w — which transformations away from w are licensed when evaluating a further claim q under that supposition? Strict implication answers this with maximum severity: p strictly implies q iff q holds at every state where p holds, full stop, which is exactly the source of strict implication's well-known paradoxes (a necessary truth is strictly implied by anything, an impossible proposition strictly implies anything) — the same over-permissive propagation problem Chapter 3 and Chapter 11 have already diagnosed in other settings, showing up again here in conditional form. Lewis and Stalnaker's similarity-sphere semantics answers it with an ordering: evaluate q at the p-worlds *most similar* to w, rather than at all of them.

A candidate reformulation. In this book's vocabulary, the question becomes: given w and p, which p-satisfying transformations of w are R-admissible? Write this as

$$p \Rightarrow_R q$$

with truth depending on admissible p-transformations of w rather than on material implication or on an externally supplied similarity metric. This reformulation has real content — it ties the conditional's behavior to the same admissibility apparatus governing everything else in this book, rather than treating conditionals as needing their own separate primitive machinery — but it should not be oversold. What it does not yet do is tell you *which* transformations count as admissible for a given supposition, which is exactly the work Lewis's similarity ordering was doing, and which this book's distinguishability geometry has been suggested, elsewhere, as a candidate replacement for.

The debt this chapter leaves open, deliberately. It would be premature to claim distinguishability geometry *replaces* Lewisian similarity spheres — the correspondence matrix marks that Bridge, and this chapter doesn't upgrade it, because nothing here has shown the similarity ordering can actually be derived rather than separately stipulated. The honest question to end on: can a similarity ordering over p-satisfying transformations be derived from the geometry of admissible transformation itself — from the same structure that already determines $\mathrm{Adm}(R)$ and R_w — rather than supplied as independent primitive structure the way Lewis's own semantics requires? If yes, conditionals stop needing a bespoke ordering relation and become a further consequence of admissibility geometry already on hand. If no, conditionals are a genuinely independent piece of structure this framework hasn't reduced, and the book should say so plainly rather than paper over it. This chapter poses that question; it does not answer it.

0.12 Status of this draft

Part II's progression — alternative states, accessibility, violations of admissibility, transport under supposition — holds together, and the chapter that mattered most going in, Chapter 7, survived contact with Part III's control case rather than needing to be softened to fit it. One real result came out of testing rather than assuming: the original correspondence-matrix claim about non-normal worlds and refusal was wrong in its specifics, even though something in that neighborhood is true — the corrected version (state-indexed regime R_w, not refusal) is more precise and, on inspection, more defensible than the claim it replaced. Chapter 8 ends with a named open debt rather than a premature claim, per instruction. Worth checking once Part V is drafted: Chapter 7's R_w move (regimes varying by state) is very close to what Part V's identity-under-transport material will need for tracking a single entity across states with potentially different local constraints — that connection wasn't planned in advance and should be verified rather than assumed to hold.

Part III

III: Gaps, Gluts, and Propagation

Chapter 9 — Gaps: Determination Without Bivalence

Chapter 2 argued that distinction precedes truth: before a proposition can be true or false, something has to be carved out as a determinate content for a truth value to apply to. This chapter takes the next step, and it's a smaller one than it might look: once truth values are read off evidence rather than assumed in advance, the possibility that evidence is simply *absent* has to be taken seriously as its own case, not folded into "false" or treated as a defect.

Two independent questions, minimally. Ask, of a proposition p: is there positive support for p — evidence, a witness, a derivation, in whatever sense support is available in the context at hand? And, separately: is there negative support for p? Nothing forces these two questions to have correlated answers. A failure to find positive support does not, by itself, hand you negative support — the absence of a reason to affirm something is not a reason to deny it. That's not a special principle needing independent defense; it's simply what it means for the two questions to be genuinely two questions rather than one question asked twice.

What underdetermination is, and isn't. When neither kind of support is present, call p **underdetermined** at that context. This is not a third truth value smuggled in alongside true and false — it's what "no answer to either question yet" already amounts to, once the two questions have been separated. Priest's own examples fit cleanly here without needing modification: a definite description with no referent ("the current King of France is bald") supplies neither positive nor negative support for predications about its subject, because there's no subject there to support anything about. A future contingent — a claim about tomorrow's sea battle, asked today — has no positive support (the battle hasn't happened) and, importantly, no negative support either (its non-occurrence hasn't happened yet), which is a different situation from a claim that has been actively falsified.

Why a gap is not an obstruction. This is the point worth stating carefully, because it will matter again once Part VI generalizes this material: underdetermination is harmless in a specific, checkable sense. An underdetermined proposition imposes no obstacle to reconstructing a larger picture around it — you can always extend an underdetermined case with more information later, in either direction, without having to retract anything you'd previously committed to, because you hadn't committed to anything yet. A gap doesn't need repairing; it needs filling in, which is a different operation with a different cost. That distinction — between a state that costs nothing to extend and a state that actively resists coherent extension — is what separates this chapter from the next one, and conflating them is the single most common way nonclassical semantics gets misread as a uniform tolerance for "weirdness," when in fact gaps and gluts are opposite kinds of case, not two flavors of the same thing.

Chapter 10 — Gluts: Contradiction as Retained Information

Chapter 9 established that absence of support in one direction doesn't force support in the other. This chapter is about what happens when both directions are actually supported at once — and argues that the right response is not automatically to treat this as an error state requiring correction.

The four statuses, from two coordinates. With both positive and negative support now genuinely in play, the same construction from Chapter 9 yields four cases rather than two: no support either way (underdetermined, Chapter 9's subject), positive support only, negative support only, and — the new case — both. Call a proposition with both positive and negative support **overdetermined**. This is Belnap and Dunn's [1, 2] fourth value, arrived at the same way the other three were: as a consequence of what the support data actually contains, not as a stipulated addition to a prior two-valued base.

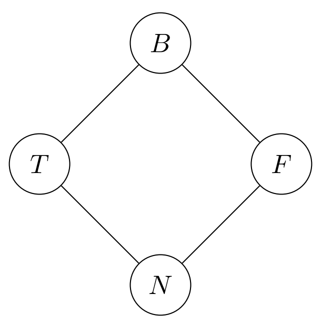

Figure 3: The Belnap–Dunn information lattice $\mathbb{F} = \{N, T, F, B\}$ under the information order $\sqsubseteq_k$. N is bottom (no support), B is top (both polarities), and T, F are incomparable. Consequently $T \vee_k F = B$: pooling positive-only and negative-only support produces a glut by construction, not by exception.

The claim that matters is stronger than "some propositions are both true and false." That phrasing makes overdetermination sound like a metaphysical oddity — a strange fact about certain propositions. The sharper claim is procedural: contradictory evidence does not need to be *deleted* merely because it is contradictory. Given two conflicting bodies of evidence, a system has a choice about what to do with the conflict, and "erase one side" is only one option among several, not the obviously correct one. Chapter 4's refusal machinery is exactly what makes the alternative precise: a regime can *refuse* the deletion — decline, as a constitutive matter, to treat "these conflict" as sufficient grounds for discarding either witness — and retain both. That refusal is not indecision or error-tolerance. It's a principled commitment to not letting a specific kind of local conflict force a loss of information that nothing yet justifies losing.

What this is not. Overdetermination is not the same epistemic situation as uncertainty.

"We don't know whether p" is Chapter 9's underdetermination — absence of support in both directions. "We have positive and negative support for p" is a different fact entirely — presence of support in both directions — and treating the two as interchangeable (as ordinary language sometimes does, calling both situations "unclear") erases a distinction the rest of this book depends on. Priest's semantic paradoxes (the Liar sentence, under a sufficiently expressive language, genuinely supports both its own truth and its own falsity by the ordinary rules governing truth predicates), and the existence of legal and normative systems that contain live, unresolved contradictions without therefore becoming useless, are both cases of overdetermination in this sense, not cases of not-yet-knowing.

The asymmetry, stated once, philosophically, ahead of its formal treatment. Chapter 9's gaps and this chapter's gluts are not mirror images of each other, whatever the symmetry of the four-value notation suggests. A gap costs nothing and obstructs nothing; it is the default, harmless case. A glut is the substantive case — the one that actually requires the rest of this book's machinery to handle responsibly. Part VI gives this asymmetry an exact formal statement; here it is enough to notice that "insufficient support" and "incompatible support" are not symmetric failures, and treating them as if they were is a mistake this book avoids from the outset rather than discovering later.

Chapter 11 — Paraconsistency: Why Contradiction Does Not Propagate

Chapter 4 separated inconsistency from triviality and left one question open: what has to be true of a system's propagation rules for the former not to become the latter? This chapter answers it, and it is the technical centerpiece of Part III.

Explosion as one specific propagation rule, not a logical necessity. Classically, from p and $\neg p$, anything follows — most directly via disjunctive syllogism: from $p \vee q$ and $\neg p$, infer q; since p is available, $p \vee q$ holds for any q whatsoever, and $\neg p$ is also available, so q follows, for arbitrary q. Every step in that derivation is individually classically valid. The question this chapter asks is not whether the derivation is classically valid — it is — but whether a system is *obligated* to make disjunctive syllogism available at a point where both p and $\neg p$ are independently supported.

Why the answer can be no, without giving up inference generally. Consider a proposition p that is overdetermined — both p and $\neg p$ supported, in Chapter 10's sense. Disjunctive syllogism, applied here, uses the *positive* support for p to license $p \vee q$, and then uses the *negative* support for p (i.e., $\neg p$) to discharge the p disjunct and leave q standing — treating the negative support as if it straightforwardly eliminated the positive support's contribution. But that is exactly the move Chapter 10 argued a system need not make: the negative support doesn't erase the positive support in an overdetermined case, by definition — that's what overdetermination *is*. Disjunctive syllogism's second step silently assumes the positive and negative supports can't coexist, which is precisely false at an overdetermined point. Blocking disjunctive syllogism at exactly these points is not an ad hoc patch to avoid an unwanted conclusion; it's the direct consequence of taking Chapter 10's retained-information picture seriously rather than only in the abstract.

The general propagation principle. Stated once, for use throughout the rest of the book: a local failure should propagate only as far as the dependency structure connecting it to other propositions actually warrants — not automatically to every proposition in the language, which is what unrestricted closure (Chapter 3) does by default. Restricting propagation at exactly the step that assumes what an overdetermined proposition specifically denies is what keeps a contradiction local. This is also why paraconsistency is not "logic with the brakes cut" — if anything, it's the more conservative regime: it declines to license an inferential step whose validity depended on an assumption (that positive and negative support are mutually exclusive) that the very situation under discussion has already falsified.

Chapter 12 — Repair and Classical Recapture

Part III closes by asking what a system should actually *do* once it has an overdetermined proposition on its hands, rather than merely tolerating its presence.

The merge. Given several independent bodies of evidence $s_1, \ldots, s_n$ bearing on p — not yet organized spatially into patches or a cover, simply distinct sources — their combination is given by the pointwise information merge

$$M(p) = \bigvee_i s_i(p),$$

the least assignment that retains everything each source individually supplied. This always exists (no combination of sources can fail to have a merge), and $M(p)$ comes out overdetermined exactly when the sources collectively supply both positive and negative support for p — which is precisely Chapter 10's notion, arrived at now as a consequence of combining evidence rather than stipulated about a single source.

Why the classical alternative genuinely fails, rather than merely being undesirable. Suppose a system insists on a glut-free reconstruction — a single, classical assignment of true-or-false to p that's consistent with everything the sources supplied. If the sources jointly supply both positive and negative support, no such assignment exists: anything siding with the positive evidence contradicts the negative evidence, and vice versa. This is a genuine non-existence result, not a preference — a classical, no-glut regime has *no* admissible reconstruction of that evidence at all. Paraconsistency's contribution here is not "tolerating an error the classical approach would have caught." It's supplying a reconstruction — $M(p) = B$ — where the classical regime supplies none. The paraconsistent regime has strictly more admissible outcomes available to it, and this is one of them being used, not an error being excused.

Classical recapture. A system that admits overdetermination everywhere it's forced to, but nowhere else, and actively minimizes how much of the language ends up overdetermined, recovers classical behavior almost everywhere while preserving exactly the local gluts the evidence actually demands — this is Priest's minimally inconsistent LP, and in this book's vocabulary it is a repair operator biased toward consistency: pay the cost of a glut only where the evidence leaves no alternative, default to classical, glut-free assignment everywhere else. This is the first appearance, at the level of a single proposition and its sources, of a idea Part VI will generalize considerably: repair is not the opposite of classical reasoning, it's classical reasoning with an escape hatch for the specific cases where classical reasoning has nowhere to go.

The arc of Part III, and what it deliberately withholds. Chapters 9–12 trace one continuous line: underdetermination, independent support, overdetermination, restricted propagation, repair. Everything in this arc is local and order-theoretic — a fact about single propositions and the evidence bearing on them, not about how evidence distributed across many regions of a larger structure does or doesn't cohere with itself. That second question — what happens once these support objects are localized over a genuine cover, and whether local overdetermination has anything to do with whether the pieces of evidence agree with each other in the first place — is Part VI's question, not this one's, and this part has deliberately not reached for it. Cohomology, coherence defects, and the distinction between a glut and an obstruction to gluing are held back on purpose: introducing them here would answer Part VI's question before its own machinery exists to ask it properly, and would blur exactly the local/global independence that turns out to be one of this book's more original claims.

0.13 Status of this draft

Written to be self-contained: a reader who stops after Part III has a complete, correct account of gaps, gluts, paraconsistency, and repair at the level of individual propositions and their evidence, without needing anything from Part VI. Deliberately withheld throughout: any mention of covers, patches, coherence defects, or cohomology — those are held for Part VI specifically so that Part III's account doesn't accidentally answer, or foreclose, the question of whether local nonclassical valuation and global structural obstruction are related. One thing to verify once Part VI is revised last (per the drafting order settled on): that Chapter 12's merge proposition and Part VI's Chapter 21 restatement of the same result are stated compatibly, since they're now the same claim appearing at two different levels of generality and any drift between the two would be worth catching before either is called final.

Part IV

IV: Construction, Vagueness, and Degree

Chapter 13 — Truth as Construction

Part III built truth values from independent positive and negative support: a proposition is true when there's evidence for it, false when there's evidence against, both or neither as the data allows. This chapter asks a question that support alone doesn't answer, and it's worth being precise about why the two are different before claiming any correspondence between them.

What support doesn't distinguish. Positive support for p, in Part III's sense, is silent about *how* that support was obtained. A proof by contradiction — assume $\neg p$, derive an absurdity, conclude p — counts as positive support exactly as much as a direct construction does; nothing in Chapters 9–12 cared about the difference. Intuitionism cares about exactly this difference, and treats it as decisive rather than incidental: on the BHK (Brouwer-Heyting-Kolmogorov) [3] reading, a proof of $p \to q$ is a specific procedure converting any proof of p into a proof of q; a proof of $p \vee q$ is a proof of one disjunct together with a specification of *which* one; and a proof of p simply is a witness — something producible, not merely something whose existence follows from the absurdity of its absence. Excluded middle, $p \vee \neg p$, fails on this reading not because intuitionists doubt that every proposition is either provable or refutable in some further sense, but because *having a proof of the disjunction* requires having a proof of one side specifically, and there are propositions for which neither is currently in hand.

A genuinely separate axis, not a restriction of Part III's. It would be a mistake to read intuitionistic truth as "Part III's positive support, but stricter" — as though constructive proof were simply positive support held to a higher standard along the same dimension. The two are asking different questions. Part III asks whether evidence exists, for or against, independent of its provenance. Intuitionism asks, given that evidence exists, whether it takes the specific form of an exhibitable construction — and this question is orthogonal to the evidential question, not a refinement of it. A piece of positive support could in principle be a valid, non-constructive derivation (existence without witness) or a genuine construction (existence with witness); Part III's apparatus doesn't distinguish these, and didn't need to for the results it was building.

Proof as reconstruction path. With that separation in place, the correspondence this book has anticipated since its opening chapters can be stated properly rather than merely gestured at: a constructive proof of p is a **reconstruction path** to p in the sense this book's reconstruction cluster already uses that phrase — a specific, traceable route by which p becomes available, as opposed to a bare guarantee that some route exists. The precise place this shows up, rather than the looser slogan it's tempting to reach for: intuitionistic logic does not in general license

$$\neg\neg p \to p,$$

because a construction that transforms every attempted proof of $\neg p$ into an absurdity is not thereby a construction of p itself — it shows $\neg p$ is unavailable, not that p is available, and those are different accomplishments. That is structurally the same refusal this book's repair machinery makes when it declines to treat "no admissible reconstruction currently known" as equivalent to "no admissible reconstruction exists": both positions insist that availability has to be demonstrated by production, not inferred from the unavailability of its negation. Stating the correspondence at the level of $\neg\neg p \to p$ specifically, rather than at the level of a general slogan about "inferring existence from absurdity," is what keeps this correspondence exact rather than approximate.

What this now makes visible. Between Part III and this chapter, the book has quietly assembled three independent coordinates rather than one: positive support, negative support, and constructibility. An assertion can carry positive support without a constructive witness (a valid non-constructive derivation); once a construction is available, it supplies a particularly strong species of positive support, but the converse doesn't hold. This isn't a result this chapter needs to develop further yet — it's simply worth naming, since it's early evidence for this book's larger claim that nonclassical logics needn't be arranged along one axis of "how far from classical," but can differ along several independent dimensions at once.

Chapter 14 — Degrees of Distinction

Kleene's and Łukasiewicz's [6] many-valued logics assign propositions values along a chain — three values, finitely many, or a continuum — rather than reading four values off two independent evidential coordinates the way Part III did. This chapter's main work is resisting an easy but wrong move: treating degree-valued logics as a generalization of FDE, with Part III's four values as a coarse special case of a finer many-valued scheme.

Why the two shouldn't be collapsed. FDE's four values arise from asking two independent yes/no questions — positive support, negative support — and reading off one of four combinations. Nothing about that construction involves degree; T and B aren't more or less true than each other, they're different combinations of a binary fact along two axes. A many-valued logic, by contrast, is committed to propositions admitting *degrees* of truth along a single ordered scale — not "how much evidence in which of two directions" but "how true, full stop," a fundamentally different kind of quantity. Forcing these into one framework — treating Łukasiewicz values as refinements of FDE's four points, say — would blur a distinction Part III spent real effort establishing: that gaps and gluts are about evidential structure, not about a proposition occupying some intermediate position on a truth continuum. Keep them as genuinely separate formal commitments, useful for different purposes, rather than one subsuming the other.

What degree actually tracks — and what's this book's addition rather than the logic's own content. It would overclaim to say many-valued semantics *means* resolution-stability. A value like 0.7, on its own, carries no built-in commitment to surviving 70% of some refinement process — depending on the system, intermediate values play quite different semantic roles (partial belief, verisimilitude, membership degree), and none of Kleene's or Łukasiewicz's own machinery forces a resolution-theoretic reading. What this chapter proposes is narrower and should be labeled as an addition, not a discovery: **resolution-stability is one interpretation of degree, proposed here to connect degree semantics to this book's distinguishability geometry** — not something many-valued logic already meant on its own.

Stated with the structure this actually requires: fix a resolution parameter $\lambda \in \Lambda$ and let $v_\lambda(p) \in [0, 1]$ be the degree assigned to p at resolution λ. Two different objects now need to be kept apart. $v_\lambda(p)$ is a single number — the degree at one fixed resolution, no more informative than an ordinary many-valued assignment. The **resolution profile**, $\lambda \mapsto v_\lambda(p)$, is a function tracking how that degree moves as resolution changes, and it is *only* the profile that carries the structure the next chapter needs. Nothing about many-valued logic itself supplies a family $\{v_\lambda\}_{\lambda \in \Lambda}$ rather than one value — that family, and the profile built from it, is this chapter's proposed addition, offered as a testable interpretation rather than presented as inherent in the semantics it's applied to.

Chapter 15 — Boundary Cases

The sorites paradox — remove one grain from a heap, and it's still a heap; repeat enough times, and it isn't — is usually treated as a puzzle about vagueness specifically, calling for its own bespoke machinery (supervaluation, degree-valued semantics, epistemicism about a sharp but unknowable cutoff). This chapter argues it's an instance of something this book already has a name for, once Chapter 14's reframing is taken seriously.

The concept that was promised earlier and now gets built. Much earlier in this project's development, a distinction surfaced between two different ways local data can fail to support a coherent global picture: *compatibility obstruction* (Part III and Part VI's subject — sections, or bodies of evidence, disagreeing with each other) and a second, distinct notion, noted at the time and deliberately not developed until a chapter existed that actually needed it. That chapter is this one.

Getting the definition right — a first version breaks on contact with transitivity. The tempting statement is: every adjacent resolution agrees exactly, and yet the extremes disagree. That doesn't work, and it's worth seeing why before stating the correct version, since the failure is instructive. If $v_\lambda = v_{\lambda'}$ for every adjacent pair in a chain of resolutions, transitivity of equality forces the two endpoints to agree as well — exact local agreement all the way down *cannot* produce global disagreement; something else in the structure would have to be non-transitive, scale-dependent, or lossy for that to happen, and "exact agreement everywhere, yet disagreement overall" is simply not a coherent way to state what sorites-style predicates are doing.

Dispersion obstruction, stated correctly. What actually happens is not exact agreement failing to propagate — it's *small* variation accumulating past a threshold. Suppose consecutive resolutions satisfy

$$d\big(v_{\lambda_i}, v_{\lambda_{i+1}}\big) \le \epsilon_i$$

at every step, where each individual ϵ_i falls beneath whatever counts as a noticeable difference at that step — but the *sum* $\sum_i \epsilon_i$ need not be small at all. Every single transition can be locally negligible while the accumulated trajectory crosses the predicate's macroscopic boundary. This is genuinely different from compatibility obstruction, not merely compatibility obstruction with a scale parameter bolted on:

> **Compatibility obstruction is failure to agree. Dispersion obstruction is failure of locally small variation to remain globally small under iteration.**

A toy example, to make the mechanism concrete. Let $v(n) = n/1000$ for n grains removed. Each single removal changes the value by exactly $|v(n) - v(n-1)| = 0.001$ — negligible by any reasonable local threshold. Traversing the entire chain gives $|v(1000) - v(0)| = 1$ — the full range. Nothing contradictory occurred at any step. Nothing failed to agree at any overlap. Tiny changes simply accumulated, which is precisely what a persistence functional needs to detect: $\mathcal{P}[s] \to 0$ under repeated coarsening should track accumulated small displacement, not the presence or absence of any single disagreement.

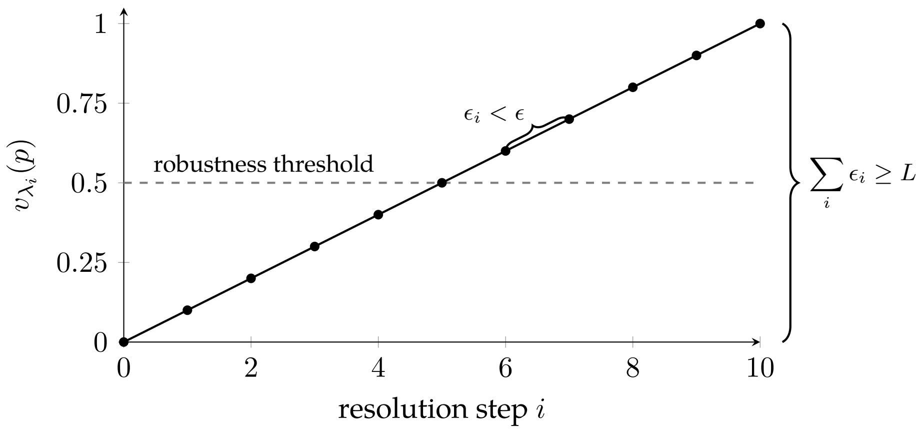

Figure 4: Dispersion under refinement. Each increment $|v_{\lambda_{i+1}}(p) - v_{\lambda_i}(p)| = \epsilon_i$ falls below any reasonable local threshold ϵ, yet the accumulated displacement $\sum_i \epsilon_i$ crosses the predicate's robustness threshold (dashed). Local smallness does not imply global persistence. No step contains a disagreement, so this failure is invisible to Chapter 21's overlap comparison — dispersion and compatibility obstruction are different mechanisms.

Why this had to wait for Chapter 14, and couldn't live in Part III or Part VI. Compatibility obstruction, as built in Part III and formalized further in Part VI, is a fact about disagreement between fixed pieces of local data at one resolution — it has no notion of *scale* built into it at all; two patches either agree or they don't, full stop. Dispersion obstruction needs a family of resolutions, a distance between values at adjacent resolutions, and a way of accumulating that distance across the whole family — precisely the resolution-profile structure Chapter 14 introduced and Parts III and VI never needed, because bare agreement or disagreement doesn't carry a magnitude the way accumulation requires. Vagueness, on this account, isn't a fourth kind of nonclassical semantics standing beside gaps, gluts, and constructive failure, and it also isn't merely "compatibility obstruction plus a scale parameter" — it's a genuinely different mathematical mechanism, a third failure mode distinguishable from the other two by what it actually requires to detect it:

$$\boxed{\text{compatibility} \neq \text{continuity} \neq \text{persistence}}$$

Compatibility (Part III, Part VI) asks whether fixed local pieces agree. Continuity, in whatever sense a topology on a space of configurations might supply, asks which configurations count as *nearby*. Persistence — this chapter's actual subject — asks whether small displacement, repeatedly accumulated, stays small. A bare topology encodes the second without automatically supplying the third: nearness alone doesn't hand you a magnitude of accumulated displacement unless something further (a metric, a uniformity, a filtration, or the $\mathcal{P}$ functional used here) is layered on top of it.

What this chapter leaves open. Whether $\mathcal{P}[s] \to 0$, defined via accumulated pairwise distance, is the right formal target isn't settled here — the choice of persistence functional is a modeling decision, not a derivation. Nor does this chapter resolve the further question of how dispersion obstruction relates to Part VI's geometric program (Chapter 23's proposed topology on the support-coherence space), though the three-way non-collapse above gives whoever revises Part VI last a specific, answerable test rather than an open-ended comparison: **does Chapter 23's proposed topology merely encode neighborhoods and continuity, or does it contain enough additional structure (a metric, uniformity, or filtration) to measure persistence under refinement?** If only the former, this chapter and Chapter 23 are not duplicates — Chapter 23 supplies the qualitative geometry, and dispersion needs the extra scale structure this chapter has now made explicit on top of it. If the topology turns out already to contain that structure, that's worth knowing too, but it should be discovered by testing Chapter 23 against this question, not assumed either way in advance.

0.14 Status of this draft

Revised once already, on substantive rather than stylistic grounds. Chapter 13's central claim is now pinned to $\neg\neg p \to p$ failing specifically, rather than a looser slogan about inferring existence from absurdity, which was directionally right but risked overstating how simple intuitionistic negation is. Chapter 14 now explicitly labels resolution-stability as this book's proposed interpretation of degree, not something many-valued semantics already meant, and introduces the resolution profile $\lambda \mapsto v_\lambda(p)$ as the object that actually carries the needed structure. Chapter 15's original definition of dispersion obstruction was mathematically broken — exact local agreement cannot produce global disagreement, by transitivity alone — and has been replaced with the correct mechanism, accumulated small variation exceeding a threshold, together with a worked toy example and the compatibility/continuity/persistence non-collapse. That non-collapse also converts the earlier open question about Chapter 23 into a specific, testable one rather than a vague "are these the same idea": does Chapter 23's topology contain enough structure to measure persistence, not just neighborhoods. All three corrections make the chapters more defensible, not less ambitious — Chapter 15 in particular went from an evocative analogy with sorites to an actual mechanism.

Part V

V: Identity, Change, and Worldhood

A numbering note before this part begins: the original planning assigned Chapter 19 to Part VI. Resolving that now rather than letting it accumulate — Part V takes Chapters 16–19, and Part VI shifts to Chapters 20–24. The renumbering of Part VI's own files is deferred to that part's final revision pass, per the drafting order already settled on; this note exists so the collision doesn't get forgotten between now and then.

Unlike Parts II through IV, this part is not aimed at synthesis. The correspondence matrix already marks identity as a Bridge and flags genuine tensions between Priest's accounts of change and motion and this book's own machinery. Part V is where that's allowed to matter: can this framework disagree with Priest without the disagreement collapsing back into a redescription of him in different words? Each chapter below presents Priest's position [7] at full strength before departing from it, and at least one departure is left as a live disagreement rather than resolved.

Chapter 16 — Identity Under Transport

Priest first. Necessary identity, contingent identity, rigid versus non-rigid designation, and the problem of tracking a single referent across possible worlds are old and difficult questions in their own right, independent of anything this book adds. Rigid designation, in particular, is a specific technical commitment: a rigid designator picks out the same individual at every world where that individual exists, full stop, with reference held fixed while everything else about the world varies. Whatever this book proposes needs to earn its place beside that account, not simply assume it's been superseded.

The reversal, stated carefully. Rather than beginning with two objects a, b and asking whether $a = b$ persists across states, begin with a history — an admissible continuation

$$x_{t_0} \rightsquigarrow x_{t_1} \rightsquigarrow \cdots \rightsquigarrow x_{t_n}$$

— and ask what invariant, if any, warrants treating the endpoints as stages of a single continuing entity rather than as a sequence of merely resembling but numerically distinct objects. This gives a specific, appropriately limited thesis:

> Identity under transport is not primitive equality repeated across states; it is an invariant of admissible continuation.

What this is not yet claiming. It would overreach to say this *derives* Priest's rigid designation — nothing here shows rigid designation is somehow reducible to continuation-invariance, and this chapter doesn't attempt that reduction. Rigid designation becomes, instead, the comparison case: it holds reference fixed while worlds vary, treating fixity as primitive; this book's account asks what structure would make holding reference fixed a *legitimate* move in the first place, rather than an unexplained stipulation. Those are different projects that happen to bear on the same phenomenon, and keeping them distinct is more honest than claiming an unearned reduction.

Reusing Chapter 2, explicitly. Chapter 2 argued that distinction precedes truth — a proposition presupposes a prior act of carving out a determinate content. The same move applies here directly: before asking whether two appearances are stages of one identity, the system already has to have determined what counts as a distinguishable continuation at all, as opposed to two independent, coincidentally similar objects. This isn't a new principle invented for this chapter; it's Chapter 2's principle, applied one level up, from propositions to persisting things.

An open connection to Chapter 7, surfaced rather than resolved here. Chapter 7 introduced R_w, a regime that can vary state by state rather than one fixed R governing everywhere. If R_w genuinely varies across a continuation $x_{t_0} \rightsquigarrow \cdots \rightsquigarrow x_{t_n}$, this chapter's talk of an "admissible continuation" is underspecified: admissible relative to which regime — the source state's, the target state's, some rule governing the transition itself? This chapter doesn't answer that, and it shouldn't be answered by quietly picking one option without noticing the choice was made. It's a live question for whoever next develops this material, and it connects suggestively to Chapter 17's move of giving transitions their own type distinct from the states they connect: if a transition step needs its own admissibility condition rather than inheriting one from either endpoint, that would mirror Chapter 17's ontological move rather than merely resembling it by coincidence. Left open here, not assumed either way.

Chapter 17 — The Instant of Change

Priest at full strength. If something changes from A to $\neg A$, what characterizes the instant of transition itself? Assigning that instant wholly to the A side, or wholly to the $\neg A$ side, looks arbitrary either way — nothing distinguishes the transition instant from any other instant on the side it's assigned to, yet something has to be different about it, since it's specifically the boundary. Priest's dialetheic proposal is that the transition instant genuinely instantiates both: at that instant, and only that instant, A and $\neg A$ both hold. This is not a cheap move — it's a serious answer to a genuinely hard problem, and it deserves to be stated at this strength before anything else is proposed.

The rival answer. Introduce a continuation-interface as a distinct kind of object, rather than enriching the valuation of the boundary state itself:

$$A \mid \partial(A, \neg A) \mid \neg A.$$

The essential move is that the boundary object $\partial(A, \neg A)$ need not share the same type as the states it connects. A and $\neg A$ are states; $\partial(A, \neg A)$ is a transition, a different kind of entity, not a third state squeezed between the other two and not a state that happens to satisfy both. This is a genuinely deeper disagreement than "Priest says both hold; this book says there's a boundary object" — Priest's account enriches the *valuation* available to states (a state can satisfy more than one of A, $\neg A$); this book's account proposes instead enriching the *ontology of transitions* (adding a new kind of object the formalism didn't have room for), leaving state valuation exactly as constrained as it always was.

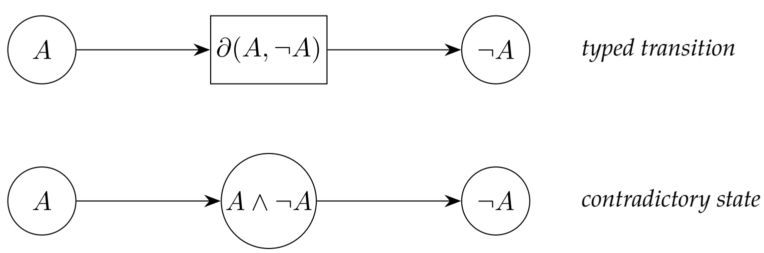

Figure 5: Two representations of the instant of change. Above: this book's continuation-interface, where the middle object is a *transition* — a different type from the states it connects (drawn as a different shape, because it is one). Below: Priest's dialetheic account, where the middle object is a *state* satisfying both A and $\neg A$. The disagreement is not over which truth value the middle point receives; it is over whether the transition should be represented as a state at all. Per this chapter's qualifier, neither representation is claimed to be correct for every change.

If this holds formally, the payoff is sharp — and needs an equally sharp qualifier. Should this distinction survive further scrutiny, it suggests: a contradiction attributed to a state may sometimes indicate that the formalism lacks an object representing the transition between states, rather than indicating that the state itself is genuinely overdetermined. But "sometimes" is doing real work in that sentence and cannot be dropped. Part III spent four chapters establishing that genuine gluts — states legitimately supporting both a proposition and its negation — are not errors requiring elimination. This chapter's proposal must not be read as a general strategy for explaining away every glut as "really" a missing transition object; that would undo Part III's central result rather than complementing it. The honest claim is narrower: *some* contradictions attributed to states may be artifacts of a state-only vocabulary lacking a transition object, and identifying which contradictions are which — genuine overdetermination versus a missing-object artifact — is exactly the kind of question this chapter raises and does not settle.

Chapter 18 — Motion Without Contradictory Location

This chapter puts RSVP to genuine argumentative work rather than treating it as a source of illustrative vocabulary, and the correspondence stays marked **Tension**, as the matrix already has it — this chapter doesn't try to resolve that rating.

Two rival representations of motion. Priest's Hegelian treatment characterizes motion through contradictory occupancy: at the relevant instant, a moving object is both at a location and not at that location, which is how motion gets represented at all within a framework built from static positional predicates evaluated instant by instant. RSVP offers a different starting point entirely — it already has a vector field v, so motion doesn't need to be reconstructed from a sequence of static positions in the first place; it's represented dynamically from the start:

$$(\Phi, v, S)_t \longmapsto (\Phi, v, S)_{t+\Delta t}.$$

The question this actually raises. Not "which account is correct," but something more specific: do contradictory positional predicates reveal something genuinely ontological about motion — that motion really does involve a kind of instantaneous self-contradiction — or do they reveal something merely *inadequate* about representing motion using static positional predicates alone, a limitation of the vocabulary rather than a discovery about the world? RSVP's vector field doesn't answer this question by fiat; it demonstrates that a coherent, contradiction-free representation of motion is available once the representation is allowed to include velocity as a primitive rather than trying to derive motion from position sequences after the fact. Whether that shows Priest's contradiction is dispensable, or merely shows one particular vocabulary happens to avoid it, is the tension this chapter leaves standing.

The backward connection to Chapter 16. This sharpens rather than merely echoes Chapter 16's thesis. If states are projections of histories — if what's available at an instant is a slice through a continuation rather than a self-sufficient object — then a position-only representation at a single instant has *already* discarded exactly the information (velocity, direction, rate) needed to distinguish "is located here" from "is moving through here." Priest's contradictory occupancy may be less a fact about motion and more a symptom of asking a state-only vocabulary a question it was never equipped to answer, given that the same vocabulary was already shown, in Chapter 16, to be too thin to carry identity across time either.

Chapter 19 — Time, Memory, and Irreversibility

What Chain of Memory is not being used for. CoM formalizes reasoning trajectories — sequences of latent states an inferential process passes through — and this chapter does not derive a theory of physical time from it. That would overreach in exactly the way this book has tried to avoid elsewhere: CoM's trajectories are about how a reasoning process gets from one cognitive state to another, not about the structure of time itself, and treating the two as the same theory would be a category error, not a correspondence.

What CoM is used for instead: a narrower, real distinction.

$$\text{instantaneous state} \neq \text{history-indexed state}.$$

Two states that are, at a given moment, indistinguishable in every respect that a snapshot could capture can nonetheless have arrived by different histories — and those different histories can license different continuations going forward, even though nothing about the present moment, examined on its own, shows this. Memory, on this account, is not simply extra information carried inside an otherwise complete state. It can encode *the path by which the state became reachable*, which is a fact about the state's history, not a fact addable to its instantaneous content without changing what kind of object a "state" is taken to be.

A route from Priest toward this book's irreversibility work, stated as a route rather than an equivalence. Priest's Spread Hypothesis and his broader treatment of temporal flow are themselves attempts to give content to something like "the present moment is thicker than an instant." This chapter's history-indexed-state distinction is a different, narrower device aimed at a related target — not offered as the same theory in different notation, and not offered as a replacement for Priest's account, but as a route toward this book's own irreversibility material that happens to pass near Priest's territory without claiming to arrive at the same destination.

Where the Leibniz Continuity Condition tension belongs, finally. A discrete repair operation, $A_t \to A_{t+1}$, can look like a discontinuous jump at one descriptive level — nothing between the two states, an instantaneous replacement — while being implemented, at a finer level of description, by a continuous underlying trajectory that the discrete description simply doesn't resolve. This chapter's job is to ask, honestly, whether that distinction actually reconciles the Leibniz Continuity Condition (change is continuous, no jumps) with this book's discrete repair events, or whether it's merely a redescription that doesn't touch the real disagreement. If a finer-grained continuous trajectory really does underlie every discrete repair this book has posited, the tension dissolves without either side having to give ground. If it doesn't — if some repairs are genuinely discontinuous even under arbitrarily fine redescription — the tension should be preserved exactly as the correspondence matrix has kept it, rather than talked away by an appeal to description-level relativity that hasn't actually been shown to apply.

0.15 The arc of Part V, and what it hands to Part VI

Four chapters, one continuous progression: identity requires continuation across states (Chapter 16); change introduces a transition object that states alone don't contain (Chapter 17); motion requires dynamical information absent from position predicates taken instant by instant (Chapter 18); memory distinguishes histories that terminate in otherwise identical states (Chapter 19). Each chapter, in turn, challenges the sufficiency of an instantaneous state a little further, and the four together support a thesis Part V can claim as its own:

> A state does not contain all the structure required to explain its own continuation.

This is not merely a summary — it's exactly what Part VI needs before its machinery can be trusted. Once Part VI moves into local sections, restriction maps, reconstruction, and gluing, there is now a stated reason not to assume a section's pointwise values exhaust its semantics: Part V has spent four chapters showing that assumption fails even for single, non-localized states, well before any cover or gluing condition enters the picture.

Two debts to preserve explicitly for Part VI's final pass, not to resolve here. First, Part IV established three genuinely different failure modes — glut (overdetermination), incompatibility (compatibility obstruction), and dispersion (persistence failure under coarse-graining) — and this book should resist deciding whether dispersion is "really" topology until Part VI's own machinery is on the table to test against. If Chapter 23/24's proposed topology turns out unable to distinguish refinement-instability from ordinary incompatibility, that's evidence the topology needs more structure, not a license to identify the two phenomena by definitional fiat. Second, and more general: the final Part VI pass should function less as an occasion to add machinery and more as a type-check across the whole book — every occurrence of support, construction, admissibility, accessibility, compatibility, repair, persistence, transition, and obstruction, checked against every other part, to make sure two notions this book worked to keep separate haven't quietly collapsed back together somewhere along the way.

0.16 Status of this draft

Written to let disagreement stand where it's earned. Chapter 17's central result is qualified as carefully as its payoff is stated — "sometimes," not "always," and the reason that qualifier can't be dropped is spelled out rather than left implicit. Chapter 18 keeps its Tension rating rather than resolving it in either direction. Chapter 19 poses the LCC question as a genuine either/or rather than assuming the reconciling answer. Nothing in this part quietly redescribes Priest in this book's vocabulary without either agreeing with him outright or stating a specific, checkable point of departure — which was the actual test this part was supposed to be.

Part VI

VI: Local Logic and Global Obstruction

Chapter 20 — Local Support and Logical Regimes

A proposition is usually introduced as something that is simply true or false, with the four-valued and gap-tolerant systems arriving later as modifications to that starting point — extra machinery bolted onto a prior notion of truth value. This chapter takes the opposite route. Truth values are not primitive here. They are read off something more basic: what evidence is actually available for and against a claim.

Support, not truth value, as the starting object. Fix a proposition p and a region U of whatever domain is under discussion — a context, an observational patch, a tile in the sense already established elsewhere in this book. Rather than asking whether p is true at U, ask two separate questions: is there positive evidence for p at U, and is there negative evidence for p at U? These questions are independent of one another — nothing forces an answer to one to constrain the answer to the other — and that independence is the whole of what makes four values rather than two arise naturally, without stipulation.

Formally, let $\mathcal{E}^+(U)$ and $\mathcal{E}^-(U)$ be free abelian groups generated by the distinct evidential witnesses — observations, derivations, testimony — available for and against p within U respectively. A single patch's support is then the pair $(e^+, e^-) \in \mathcal{E}^+(U) \times \mathcal{E}^-(U)$, and the **support map**

$$q(e^+, e^-) = \begin{cases} N & e^+ = 0,\ e^- = 0 \\ T & e^+ \neq 0,\ e^- = 0 \\ F & e^+ = 0,\ e^- \neq 0 \\ B & e^+ \neq 0,\ e^- \neq 0 \end{cases}$$

reads off one of Belnap and Dunn's [1, 2] four values — Neither, True, False, Both — as a consequence of the support data, not as a starting assumption. $\mathbb{F} = \{N, T, F, B\}$, ordered by the information order $N \sqsubseteq T \sqsubseteq B$, $N \sqsubseteq F \sqsubseteq B$ with T, F incomparable, is a complete lattice, and this matters later; for now it is enough to note that q is a genuine function of the underlying evidence, and that a glut is simply what you get when both kinds of evidence happen to be present. Nothing about tolerating contradiction has been stipulated. The four values are a description of four possible evidential situations, and Both is one of them, on exactly the same footing as the other three.

What this chapter does not yet ask. Everything above is stated for a single patch U in isolation. It says nothing about what happens when two overlapping patches each report support for the same p — whether their evidence agrees, whether it should be expected to, or what follows if it doesn't. That is a genuinely separate question, deferred entirely to Chapter 21. Keeping it separate is not merely tidy exposition; the distinction between what a single patch supports and what multiple patches agree on turns out to be the load-bearing distinction of this part of the book, and blurring it — treating "every patch says T" and "every patch says T

for the same reason" as the same fact — is precisely the mistake this book's formal apparatus is built to make impossible to state by accident.

Chapter 21 — Compatibility and Gluing

Chapter 20 fixed what a single patch supports. This chapter asks whether neighboring patches' support for the same proposition is the *same* support — and gets the answer from a much simpler place than it might appear to require.

The question, precisely. Let $\{U_i\}$ be a cover, and suppose each patch U_i reports local evidence $e_i^+ \in \mathcal{E}^+(U_i)$ for p's positive case (symmetrically for e_i^-). Where two patches overlap, restriction maps ρ_{U_{ij},U_i} carry each patch's evidence down to what's visible on the shared region $U_{ij} = U_i \cap U_j$. The question is whether $\rho_{U_{ij},U_i}(e_i^+)$ and $\rho_{U_{ij},U_j}(e_j^+)$ agree, as elements of $\mathcal{E}^+(U_{ij})$.

It is tempting, and was in fact this book's own first attempt, to reach here for Čech cohomology and ask whether some class is nonzero in H^1. That reach is wrong, for a reason worth stating once, precisely, so it need not be revisited: for any specific chosen tuple of local sections $s = (s_i)$, the comparison δs — the collection of pairwise differences on overlaps — is by construction a coboundary, and therefore trivial in cohomology, whether or not the s_i actually agree. Asking "do these particular witnesses match" is answered directly by whether δs itself is the zero cochain, not by any cohomology class built from it. The right tool is simpler than the wrong one, not more elaborate, and this chapter uses only the tool the question actually needs.

Coherence defect. Define

$$\delta^+(p) = \left(\rho_{U_{ij},U_j}(e_j^+) - \rho_{U_{ij},U_i}(e_i^+)\right)_{ij}$$

as an element of the Čech 1-cochains of $\mathcal{E}^+$ relative to the cover, and $\delta^-(p)$ symmetrically. Say the positive (respectively negative) evidence for p is **polarity-coherent** across the cover when $\delta^+(p) = 0$ (respectively $\delta^-(p) = 0$): every pair of overlapping patches is contributing the *same* witnesses, not merely witnesses that happen to license the same verdict.

Two different senses of restriction. The restriction maps used above carry a modeling choice worth making explicit rather than leaving implicit, since the two readings genuinely differ. Restriction can mean plain domain restriction — the same assignment, viewed on a smaller region, nothing lost. Or it can mean projection with possible information loss — evidence available across the whole of U_i need not remain reconstructible from $V \subset U_i$ alone, if the witnessing particulars live outside V's own resources. The second reading is the one under which the coherence defect is doing real epistemic work rather than bookkeeping, and it is the reading this book adopts going forward: restriction is a genuine act of observation-with-possible-loss, not a notational convenience. Spatial restriction (the morphism ρ_{VU}) and epistemic refinement (an ordering of increasing information, as in Kripke-style forcing [4]) are different things, and treating them as interchangeable was an earlier error in this construction's history worth naming so it does not recur silently.

What this chapter still does not settle. Coherence, as defined here, says nothing about which FDE value the patches end up reporting. A proposition can be perfectly polarity-coherent

Gluing

$$\begin{tikzcd} s_1 \arrow[dr] & & s_2 \arrow[dl] \\ & s_1|_{U_{12}} = s_2|_{U_{12}} \arrow[d, Rightarrow] & \\ & s \in \mathcal{A}(U_1 \cup U_2) & \end{tikzcd}$$

Information-preserving merge

$$\begin{tikzcd} T \arrow[dr] & & F \arrow[dl] \\ & B & \end{tikzcd}$$

Figure 6: Gluing (left) requires a matching family: local sections that already agree on the overlap combine into a unique global section. Information-preserving merge (right) requires no such agreement: an FDE join can combine *disagreeing* reports (positive-only and negative-only) into B, retaining rather than discarding what conflicts. These are different operations, not two names for one — see the retracted correspondence discussed below.

and still be a glut (every patch agrees, using the same witnesses, that p is Both). A proposition can be badly incoherent and still show no glut at all — every patch might independently report T, using witnesses that don't match each other in the slightest. Whether that combination is even possible, and what it would mean, is Chapter 22's question, not this one's — this chapter has deliberately stopped at the evidence layer and gone no further.

Chapter 22 — Contradiction Is Not Obstruction

Chapters 20 and 21 built two structures over the same local data: what a patch supports (Chapter 20), and whether neighboring patches' support coheres (Chapter 21). This chapter's claim is that these are not two views of one fact. They are genuinely separate facts, capable of varying independently — and that independence, rather than any single dramatic case of contradiction, is the central result of this part of the book.

The merge. Given a family of local reports $\{q_i(p)\}$, define the pointwise merge

$$\Sigma(p) = \bigvee_i q_i(p),$$

the join in $\mathbb{F}$'s information order. Because $\mathbb{F}$ is a complete lattice, this join always exists — there is no family of local reports the merge fails to combine. Two propositions about it settle what "merging" produces:

> **Support-Merge Proposition.** $\Sigma(p) = B$ if and only if some patch in the family reports positive support and some patch (possibly the same one) reports negative support. $\Sigma(p) = N$ if and only if every patch touching p reports N.
>
> **Glut-Free Reconstruction Obstruction.** A pointwise glut-free common extension of the family into $\{N, T, F\}$ exists if and only if no proposition collects both positive and negative support across the family — that is, iff $\Sigma(p) \neq B$ for every p.

Both are purely combinatorial statements about q_i-values; neither mentions the coherence defect from Chapter 21. That is not an oversight — it is the point. And a genuine asymmetry falls out of the merge that is worth stating on its own: N merges with anything to return that thing unchanged ($N \vee x = x$), so absence of evidence never itself obstructs reconstruction. Only actually incompatible polar evidence does. Gluts and gaps are not mirror images of one another, whatever FDE's fourfold notation suggests at a glance: **a glut is incompatible polar evidence; a gap is merely insufficient polar evidence.**

Mutual Non-Determination Proposition. Neither the coherence status $(\delta^+(p), \delta^-(p))$ from Chapter 21 nor the merged value $\Sigma(p)$ from this chapter determines the other. The proof is a four-cell construction, and its economy is the whole argument: a two-patch cover with overlap U_{12} populates all four combinations directly.

A note on continuity with Part III. This is the same construction Chapter 12 built at the level of distinct evidence sources rather than cover-indexed patches — $M(p) = \bigvee_i s_i(p)$ there, $\Sigma(p) = \bigvee_i q_i(p)$ here. The two aren't merely analogous; Σ *is* M, specialized to the case where the sources s_i happen to be indexed by a genuine cover with overlaps, which is exactly what licenses asking the coherence question ($\delta^\pm$) that Chapter 12 had no apparatus for and deliberately didn't attempt. The name change from M to Σ marks that shift in structure, not a change in what the operation does.

	δ coherent	δ incoherent
$\Sigma(p) = T$	(a) both patches report T via the *same* witness	(c) both patches report T via *different*, non-matching witnesses
$\Sigma(p) = F$	(a′) both patches report F via the *same* witness	(c′) both patches report F via *different*, non-matching witnesses
$\Sigma(p) = B$	(b) both polarities individually coherent, genuinely in conflict	(d) conflicting *and* internally incoherent

The T/F rows are genuinely separate cells, not mirror images assumed free — (a′) sets $e_1^- = e_2^- = v$ (a shared negative witness, no positive evidence anywhere), giving $\delta^- = 0$; (c′) sets $e_1^- = v_1 \neq v_2 = e_2^-$, giving $\delta^- \neq 0$ while both patches still report the same verdict F. Both are built the same way (a) and (c) were, not inferred from them. The B row already covers both polarities by construction — a glut requires both positive and negative support present somewhere in the family, so (b) and (d) don't need separate positive- and negative-side versions the way the non-glut rows did. Six cells, all built, none left to symmetry. That every cell is populated is the entire proof — neither axis constrains the other, because both are demonstrably free to vary while the other is held fixed.

merged value $\Sigma(p)$	*δ coherent*	*δ incoherent*
	(a) $\Sigma = T$ coherent	(c) $\Sigma = T$ incoherent
	(a′) $\Sigma = F$ coherent	(c′) $\Sigma = F$ incoherent
	(b) $\Sigma = B$ coherent	(d) $\Sigma = B$ incoherent

Figure 7: The six realizability witnesses proving the Mutual Non-Determination Proposition. Reading across a row: the same merged value $\Sigma(p)$ occurs under both coherent and incoherent evidence. Reading down a column: the same coherence status occurs under all three merged values. Neither axis is a function of the other. Case (c) is the one Priest's semantics alone cannot see: a classical-looking verdict resting on evidence that does not actually agree with itself.

Case (b) is Priest's glut in its cleanest form: two well-founded, internally coherent positions in genuine disagreement — the positive case coheres into one witness, the negative case coheres into a different one, and they simply conflict. Call this the **clean glut**. Case (d) is the same conflict compounded by internal incoherence on at least one side — part of what looks like cross-polarity disagreement may really be several non-agreeing positive (or negative) sub-cases masquerading as one. Call this the **confounded glut**. Both are legitimate uses of the word "glut," and this book will keep them distinguished rather than treating every $\Sigma(p) = B$ as the same kind of event.

Case (c) deserves the most attention of the four, precisely because it is the least dramatic. Nothing here looks unusual under FDE: both patches say T, no contradiction anywhere, nothing for a paraconsistent logic to even notice. And yet the agreement is hollow in a specific, checkable sense — the patches are not agreeing about the same thing, only about the same verdict. This is the case that carries the rest of Part VI, and it recurs through Chapters 21 to 23 for that reason: it is the demonstration that the support layer built in Chapters 20 and 21 is not decorative machinery sitting underneath an otherwise complete four-valued logic. It is detecting something four-valued logic alone cannot see at all.

The headline result. Contradiction, in Priest's sense, is a fact about $\Sigma(p)$. Obstruction to coherent reconstruction, in the sense this book has been building since Chapter 21, is a fact about $\delta^{\pm}(p)$. This chapter's title is meant literally: **contradiction is not obstruction.** A glut can be perfectly coherent. A non-glut can be badly incoherent. Priest was right that gluts needn't explode; this chapter's addition is that whether something needs repairing at all cannot be read off $\Sigma(p)$ in isolation — which sets up exactly the question Chapter 23 has to answer.

Chapter 23 — Repair and Reconstruction

The progression through this part of the book has been

support (Ch. 19) $\to$ coherence (Ch. 20) $\to$ non-determination (Ch. 21) $\to$ repair (this chapter).

Chapter 22 established that support and coherence are independent facts. This chapter asks what a repair operator does with both of them at once — and the answer is that it needs a third term it didn't previously have.

Why case (c) cannot be folded into existing repair cost. This book's repair theory already has a cost formulation, $E(a) + \lambda R(a)$, where E is the ordinary cost of an action a and R is the cost of repairing a polarity conflict — a glut. Case (c) from Chapter 22 has $R(a) = 0$, exactly and correctly: there is no glut, no polarity conflict of any kind, both patches agree p is T. Whatever case (c) costs, it is not a repair cost in the sense R was built to capture, and treating it as one would erase the very distinction Chapters 21 and 22 exist to establish.

The third term. Repair cost needs

$$J(a) = E(a) + \lambda R(a) + \mu C(a),$$

where $C(a, p)$ is **coherence liability** — the cost an action incurs from depending on evidence that failed to cohere at Chapter 21's level, even when Chapter 22's merge saw nothing wrong. Crucially, C cannot be a fixed function of the coherence defect alone:

$$C(a, p) = \rho_a\big(\delta^+(p), \delta^-(p)\big),$$

action-relative by construction. The coherence defect is an objective structural fact about the evidence configuration, unchanged by anyone's plans; ρ_a is where the actual liability lives, and it depends on what the action in question needs from the evidence.

A bridge reported open by camera evidence in one context and by traffic-authority report in another is a witness mismatch — but for an action like *drive across*, that mismatch is corroboration, not incoherence: two independent reasons for the same conclusion. The same structural fact, applied to an action like *use this credential* where the two contexts' positive evidence for "the key is valid" trace to two different underlying keys, is disqualifying — the action needs the witnesses to be identical, not merely to agree on a verdict. Same $\delta^+(p) \neq 0$, opposite consequence, depending entirely on what the action depends on. This is why ρ_a cannot be dispensed with in favor of a single fixed penalty function: the liability is a fact about the action's relationship to the evidence, not a fact about the evidence alone.

What this buys. Under the old two-term functional, an action relying on case (c)'s proposition is indistinguishable from one relying on case (a) whenever their ordinary costs are equal — neither has a glut, so $R = 0$ for both, and $E + \lambda R$ has nothing left to compare. Under the three-term functional, a witness-sensitive action pays through C where a witness-insensitive one does not, even though $\Sigma(p) = T$ identically for both:

> Two states indistinguishable under the FDE polarity merge can induce different action preferences once evidential coherence enters the decision functional.

Which gives this part of the book its actual thesis, stated plainly: **logical agreement does not imply evidential coherence, and rational action may depend on both independently.** It is case (c), not the dramatic glut cases, that earns the extra machinery its place — a classical-looking, contradiction-free agreement can still be the wrong thing to act on, for reasons a four-valued semantics alone has no way to express.

A deliberate incompleteness. This chapter does not commit to a canonical form for ρ_a beyond the minimal binary flag — sensitive or insensitive — used in the bridge/credential contrast above. That is a choice, not a gap: establishing that an action-relative coherence liability distinguishes configurations invisible to the old functional is the result this chapter needs, and it holds regardless of which particular metric or coefficient structure eventually fills in ρ_a. Committing to a graded, numerical version now would answer a question this chapter hasn't yet earned the right to ask — whether ρ_a should depend on *which* witness-identity condition an action needs, and how C should aggregate across actions depending on several propositions at once, are left open for exactly that reason.

0.17 Status of this draft

Written sequentially, as a test of whether the five revisions of scratch work compose without the correction history as scaffolding. They do. The mirror-case gap flagged in the first pass — $\Sigma(p) = F$ under each coherence status, previously asserted by symmetry rather than built — is now closed with explicit constructions (a′) and (c′) added to Chapter 21's table. The progression from support to coherence to non-determination to repair holds together as stated, case (c) does the work across three chapters that it was meant to, and Chapter 23's deliberate incompleteness around ρ_a is the honest state of that question rather than a placeholder for something already decided.

Chapter 24 — The Geometry of Logical Regimes

Chapters 20 through 23 built three things over each proposition and each cover: a support configuration (Chapter 20), a coherence defect on that configuration (Chapter 21), and a repair cost that depends on both at once (Chapter 23), with Chapter 22 establishing that the first two vary independently. What hasn't been asked yet is what kind of *space* all of this lives in — and whether calling it a geometry, rather than merely a list of cases, is earned or aspirational. This chapter argues it's earned, in one precise sense, and proposes the rest as a program rather than claiming more than has been built.

0.18 1. What's actually rigorous: T, F, B, N as fibers, not points

Fix a cover $\{U_i\}$ and a proposition p. Define the **support-coherence space** for p,

$$\mathcal{S}(p) = \Big\{ \big(e_i^+, e_i^-\big)_i \;\Big|\; e_i^+ \in \mathcal{E}^+(U_i),\ e_i^- \in \mathcal{E}^-(U_i) \Big\},$$

the set of all possible support assignments across the cover — the full data Chapter 20 works with, before anything is merged or compared. There is a projection

$$\pi : \mathcal{S}(p) \to \mathbb{F}, \qquad \pi\big((e_i^+, e_i^-)_i\big) = \Sigma(p) = \bigvee_i q(e_i^+, e_i^-),$$

Chapter 22's merge. Chapter 22's central result, restated in this chapter's terms, is that $\pi^{-1}(T)$ is not a point — it contains both case (a) (coherent) and case (c) (incoherent), which are different elements of $\mathcal{S}(p)$ that happen to share an image under π. The same holds for every fiber. This is what it means, precisely rather than suggestively, to say that T, F, B, N are **coarse projections of regions in a richer space**, not native points of the space itself. Nothing here needed new machinery — it's Chapter 22's result, restated as a fact about a map between two spaces rather than as a fact about two separately-computed quantities. That restatement is the whole of what's rigorous in this chapter; everything past this section is proposed rather than proven.

0.19 2. Why a taxonomy isn't yet a geometry

A taxonomy sorts $\mathcal{S}(p)$ into labeled bins — clean glut, confounded glut, coherent-T, incoherent-T, and so on, exactly as Chapter 22's six-cell table does. A geometry needs more: a notion of which configurations are *near* which others, and what it costs to move from one to another. Without that, "the space of logical regimes" is a figure of speech for a partition, and the chapter's title would be false advertising. Two candidate structures are worth taking seriously, and this is where TARTAN stops being a correspondence-matrix entry from an earlier chapter and becomes load-bearing.

0.20 3. Neighborhoods from refinement — the proposed topology

$\mathcal{S}(p)$ as defined above is relative to one fixed cover $\{U_i\}$. TARTAN's recursive tiling gives a *family* of covers, ordered by refinement — each tile decomposable into finer sub-tiles, with the TARTAN Admissibility Theorem guaranteeing that global admissibility reduces to tile-local admissibility at every level of that refinement, not just at one chosen resolution. This suggests treating $\mathcal{S}(p)$ not as fixed to one cover but as assembled from the whole refinement diagram — coarser covers give coarser support-coherence spaces, finer covers give finer ones, and the refinement maps between them are exactly what a topology on the assembled space would need to be built from: two configurations count as *near* each other if they agree at some coarse level of tiling and only differ once the tiling is refined past some threshold.

This is proposed, not built. What would need proving: that refining a cover is compatible with the coherence defect in the right way (does refining a tile ever *manufacture* an apparent coherence defect that wasn't real at the coarser resolution, or only ever reveal genuine ones that coarseness had hidden?), and that the resulting limit or colimit construction is well-behaved enough to actually deserve the name topology rather than merely a directed system of unrelated spaces. Both are real technical questions, not merely items to gesture past.

Running Part IV's test against this proposal. Chapter 15 posed a specific question for whoever built this topology to answer, rather than leaving the relationship between dispersion obstruction and this chapter's program unresolved indefinitely: does the topology sketched above contain enough structure to measure *persistence* under refinement, or only *neighborhoods*? As stated, it's the latter — "near" here means agreeing at some coarse level and differing only past a refinement threshold, which is a qualitative nearness relation with no attached magnitude. Nothing in §3 as written could represent an accumulation of small displacements the way Chapter 15's $\sum_i \epsilon_i$ does; there's no metric, uniformity, or filtration here, only a directed system of covers. So the answer to Chapter 15's test is settled, at least for this proposal in its current form: **dispersion obstruction and this section's topology are not duplicates.** This section, if built, would supply the qualitative geometry — which configurations are near which, at what refinement depth. Dispersion needs the further scale structure Chapter 15 introduced on top of that, not in place of it. Whether this topology should be *extended* with that structure, or whether dispersion is better left as a separate functional layered over a topology that stays qualitative, is a design choice for whoever builds this section properly — this note only settles that the two aren't secretly the same thing.

0.21 4. Distance from repair cost — the proposed metric

Chapter 23's functional $J(a) = E(a) + \lambda R(a) + \mu C(a)$ was built to price an action's dependence on a given support-coherence configuration, not to measure distance between two configurations directly. But the ingredients are close enough to a metric to name the gap precisely: for two configurations $s, s' \in \mathcal{S}(p)$, define a candidate transition cost as the minimal repair expenditure ($\lambda R + \mu C$ terms, stripped of the action-specific E) needed to move from one to the other — turning a confounded glut into a clean one, or eliminating a coherence defect entirely, has some cost, and nearby configurations should be the ones reachable cheaply.

This is also proposed rather than built, and for a specific, nameable reason: J as defined in Chapter 23 is a cost *of an action*, relative to what that action needs from p. A genuine distance on $\mathcal{S}(p)$ would need a notion of repair cost that doesn't route through any particular action — something closer to $\inf_a$ over all actions, or a repair cost defined directly on configurations rather than borrowed from $C(a, p)$. Whether that infimum is well-behaved, and whether it satisfies anything like a triangle inequality, is exactly the kind of question this chapter can pose but not yet answer.

0.22 5. What the geometry would buy, if built

Supposing both of §3 and §4 could be made rigorous, the payoff is the thing the whole book has been organized around: **logical regimes become regions of $\mathcal{S}(p)$, not a list of separately-specified formal systems.** The classical regime is the region where $\delta^{\pm} = 0$ everywhere and $\Sigma(p) \neq B$ for every p — Chapter 22's Glut-Free Reconstruction condition, read as a subset rather than a proposition. The paraconsistent regime is the region admitting $\Sigma(p) = B$ without requiring $\delta = 0$ to accompany it — a strictly larger region, containing the classical one as a sub-region rather than standing beside it as an alternative. A relevant-logic-flavored regime would be the region where $C(a, p) > 0$ matters for enough actions that propagation needs the ρ_a-sensitivity machinery from Chapter 23 to behave sensibly at all — not a separate logic bolted on, but a description of where in $\mathcal{S}(p)$ an agent's actions live.

If this works, it inverts the usual presentation this book set out to avoid: instead of "here is classical logic, and here are its non-classical rivals," the picture becomes one space, with the historical logics showing up as its more heavily-traveled or more easily-described regions — classical logic isn't a rival to paraconsistency, it's the sub-region of $\mathcal{S}(p)$ where paraconsistency's extra room happens not to be needed.

0.23 6. Honest status

Section 1 is proven, in the sense that it restates an already-established result in geometric language rather than adding new content. Sections 3 and 4 are a program, stated as specifically as this draft can manage without doing the actual work: two genuine technical questions (does refinement only ever reveal coherence defects, never manufacture them; does an action-independent repair distance exist and satisfy a triangle inequality) stand between "proposed" and "built." Section 3 additionally now carries a settled answer to the question Chapter 15 posed about it: as proposed, this topology is qualitative-only and doesn't duplicate dispersion obstruction, which needs scale structure this section doesn't yet have. Section 5 is the vision the rest of the book has been implicitly working toward, stated plainly so it can be checked against, not claimed as delivered. This chapter does what it was supposed to do on the plan set out earlier: it was generated by the pressure of the chapters before it, asks questions those chapters make it possible to ask precisely, and answers only the one Chapter 15 handed it a decision procedure for.

Part VII

Mathematical Appendices

FDE Support, Merge, and Glut-Free Reconstruction

This appendix collects the finite support calculus used in Parts III and VI in one place. It adds no new philosophical claim; its purpose is to make explicit the definitions and elementary propositions on which Chapters 10, 12, 20, and 22 rely.

Signed support and the four values

Fix a proposition p. Let positive and negative support be represented by two indicators

$$\sigma^+(p), \sigma^-(p) \in \{0, 1\}.$$

Define the support-value map $q : \{0,1\}^2 \to \mathbb{F}$ by

$$q(0,0) = N, \qquad q(1,0) = T, \qquad q(0,1) = F, \qquad q(1,1) = B.$$

Equip $\mathbb{F} = \{N, T, F, B\}$ with the information order

$$N \sqsubseteq T \sqsubseteq B, \qquad N \sqsubseteq F \sqsubseteq B,$$

with T and F incomparable. Thus $\mathbb{F}$ is a finite complete lattice and, in particular,

$$T \vee F = B, \qquad N \vee x = x, \qquad B \vee x = B.$$

The same construction may be made over evidential witness groups. If $\mathcal{E}^+(U)$ and $\mathcal{E}^-(U)$ are the positive and negative evidence groups on a region U, define

$$q(e^+, e^-) = \begin{cases} N, & e^+ = 0,\ e^- = 0, \\ T, & e^+ \neq 0,\ e^- = 0, \\ F, & e^+ = 0,\ e^- \neq 0, \\ B, & e^+ \neq 0,\ e^- \neq 0. \end{cases}$$

The four-valued semantics is therefore read from two independent support coordinates rather than taken as primitive.

Information merge

Let $s_1, \ldots, s_n$ be independent support reports about p. Define

$$M(p) = \bigvee_{i=1}^{n} s_i(p).$$

When the reports are indexed by patches of a cover, Chapter 22 writes the same operation as

$$\Sigma(p) = \bigvee_i q_i(p).$$

Hence $\Sigma = M$ after specialization to cover-indexed sources.

Proposition A.1 (Support–Merge). $M(p) = B$ if and only if the family collectively contains positive support and negative support for p. Moreover, $M(p) = N$ if and only if every report is N.

Proof. Write each report as a pair (σ_i^+, σ_i^-). Join in the information order is coordinatewise Boolean disjunction, so

$$M(p) = q\left(\bigvee_i \sigma_i^+,\ \bigvee_i \sigma_i^-\right).$$

The result is B exactly when both displayed coordinates are 1, and is N exactly when both are 0. □

Glut-free reconstruction

Let $\mathbb{F}_{\neg B} = \{N, T, F\}$. A glut-free common extension of a family $\{s_i(p)\}$ is an element $u \in \mathbb{F}_{\neg B}$ such that $s_i(p) \sqsubseteq u$ for every i.

Proposition A.2 (Glut-Free Reconstruction Obstruction). A glut-free common extension exists if and only if $M(p) \neq B$.

Proof. If $M(p) \neq B$, then $M(p) \in \mathbb{F}_{\neg B}$ and, by the defining property of a join, every $s_i(p) \sqsubseteq M(p)$. Conversely, if $M(p) = B$, the family contains both polarities. Any common upper bound must therefore dominate both T and F, whose least upper bound is B. No element of $\mathbb{F}_{\neg B}$ can do so. □

This proposition makes the gap/glut asymmetry exact. N is not an obstruction: $N \vee x = x$. A gap records insufficient polar evidence. A glut records jointly retained positive and negative support, and becomes an obstruction only relative to a reconstruction regime that excludes B.

Covers, Coherence Defects, and Mutual Non-Determination

This appendix isolates the cover-level apparatus of Chapters 20–23. Its main purpose is to prevent three operations from being conflated:

$$\text{restriction} \neq \text{gluing} \neq \text{merge/repair}.$$

Evidence presheaves and restriction

Let X be a domain with cover $\mathcal{U} = \{U_i\}$. For each open region U, let $\mathcal{E}^+(U)$ and $\mathcal{E}^-(U)$ be abelian groups generated by positive and negative evidential witnesses. For $V \subseteq U$, let

$$\rho^{\pm}_{V,U} : \mathcal{E}^{\pm}(U) \to \mathcal{E}^{\pm}(V)$$

be restriction maps satisfying identity and composition. The intended reading in Part VI permits information loss under restriction: evidence available on U need not remain reconstructible from V alone.

For a chosen family $e_i^{\pm} \in \mathcal{E}^{\pm}(U_i)$, define the overlap defect

$$(\delta^{\pm} e)_{ij} = \rho^{\pm}_{U_{ij},U_j}(e_j^{\pm}) - \rho^{\pm}_{U_{ij},U_i}(e_i^{\pm}), \qquad U_{ij} = U_i \cap U_j.$$

The family is polarity-coherent when $\delta^{\pm} e = 0$.

Remark. δe is a Čech 1-cochain and, for a chosen 0-cochain e, is a coboundary by construction. Consequently its cohomology class cannot diagnose whether these particular witnesses agree: $[\delta e] = 0$ automatically whenever the ordinary cochain complex is defined. The relevant test here is the cochain-level equation $\delta e = 0$, not nonvanishing of H^1.

Ordinary gluing versus information merge

Ordinary sheaf gluing begins with a matching family,

$$\rho_{U_{ij},U_i}(e_i) = \rho_{U_{ij},U_j}(e_j),$$

and produces a global section restricting exactly to each local section. By contrast, the FDE information merge

$$\Sigma(p) = \bigvee_i q_i(p)$$

can combine reports that do not match. If one patch reports T and another F, the merge returns B; it does not produce a section whose restrictions are simultaneously the original T and F reports. Merge is therefore an information-preserving repair operation, not ordinary sheaf gluing.

Mutual non-determination

Let the semantic coordinate be the merged value $\Sigma(p) \in \{N, T, F, B\}$ and the coherence coordinate be whether $(\delta^+(p), \delta^-(p))$ vanishes.

Proposition B.1 (Mutual Non-Determination). Neither the merged FDE value nor the coherence status determines the other.

Proof by realizability. A two-patch cover suffices. For $\Sigma = T$, choose either a shared positive witness on the overlap (coherent) or two distinct positive witnesses that restrict differently (incoherent). The same construction with negative witnesses realizes coherent and incoherent cases with $\Sigma = F$. For $\Sigma = B$, choose positive and negative evidence whose respective witnesses agree across the overlap to obtain a coherent glut, or choose a mismatch in at least one polarity to obtain an incoherent glut. Thus fixing Σ does not fix coherence. Conversely, coherent and incoherent configurations occur at several distinct merged values, so coherence does not fix Σ. □

The proposition is the formal core of the book's claim that contradiction and obstruction are different coordinates. A glut can be coherent, and a classical-looking verdict can be evidentially incoherent.

Action-relative coherence liability

Chapter 23 extends the repair functional from

$$E(a) + \lambda R(a)$$

to

$$J(a) = E(a) + \lambda R(a) + \mu C(a), \qquad C(a, p) = \rho_a\big(\delta^+(p), \delta^-(p)\big).$$

The dependence on a is essential: witness mismatch can be corroborative for an action insensitive to witness identity and disqualifying for an action that requires identity of the underlying witness. Hence coherence defect is structural, while coherence liability is action-relative.

Resolution Profiles and Dispersion

Part IV separates degree-valued semantics from FDE's evidential polarity and then adds resolution as an explicit parameter. This appendix records the minimal mathematical structure needed for that move.

Resolution profiles

Let $(\Lambda, \preceq)$ index resolutions or scales, and let

$$v_\lambda(p) \in V$$

be the value assigned to proposition p at resolution λ, where V carries a distance or other quantitative comparison d. The value $v_\lambda(p)$ and the full profile

$$\lambda \longmapsto v_\lambda(p)$$

are distinct objects. Many-valued semantics alone supplies the former; interpreting the latter as resolution behavior is an additional modeling choice of this book.

For a finite refinement or coarsening chain

$$\lambda_0 \preceq \lambda_1 \preceq \cdots \preceq \lambda_n,$$

define local increments

$$\epsilon_i = d\big(v_{\lambda_i}(p), v_{\lambda_{i+1}}(p)\big)$$

and accumulated variation

$$D_{0:n}(p) = \sum_{i=0}^{n-1} \epsilon_i.$$

Proposition C.1 (Local Smallness Does Not Imply Global Persistence). For every $\varepsilon > 0$ and every target displacement $L > 0$, there are sufficiently long chains for which each $\epsilon_i < \varepsilon$ while $D_{0:n}(p) \geq L$.

Proof. Choose $n > L/\varepsilon$ and a profile with constant increment L/n. Then $L/n < \varepsilon$ while the accumulated variation is $n(L/n) = L$. □

The proposition captures the mechanism used in Chapter 15's sorites analysis. No adjacent step need contain a contradiction or a coherence failure. Dispersion is instead the accumulation of individually small changes into macroscopic displacement.

Persistence and topology

A persistence functional $\mathcal{P}[s]$ may be chosen to measure how much of a verdict survives along a refinement/coarsening trajectory. The manuscript deliberately leaves its canonical form open; one possible family of models takes $\mathcal{P}$ to decrease with accumulated variation $D_{0:n}$.

This requires more structure than a bare topology. A topology can specify neighborhoods and continuity, but does not by itself assign magnitudes to successive displacements or provide an additive accumulation law. Consequently the qualitative topology proposed in Chapter 24 and the dispersion mechanism of Chapter 15 are distinct but related: the former can organize nearness under refinement, while the latter additionally requires a metric, uniformity, filtration, persistence functional, or comparable quantitative structure.

The resulting separation is

$$\boxed{\text{compatibility} \neq \text{continuity} \neq \text{persistence.}}$$

Compatibility asks whether fixed local data agree. Continuity asks which configurations are nearby. Persistence asks whether repeated locally small displacement remains globally small.

Part VIII

Correspondence Ledger

Priest-Flyxion Correspondence Ledger (rev. 4)

The matrix's function changes here. It no longer records what looked formally alike before drafting; it records what the finished six-part draft actually established, chapter by chapter, against the manuscript files themselves (all six parts are drafted and were checked directly for this revision, not reconstructed from progress summaries). Four columns: the proposed correspondence, where it was tested, its final status, and the exact qualification or open obligation attached to that status.

Status vocabulary, revised (rev. 4) to separate three things "Bridge" was doing at once: - **Direct — established**: the chapter actually supplied the translation, not just a resemblance. - **Bridge**: the manuscript develops a substantive connection but stops short of derivation, without pinning down one specific missing step — the gap is real but not yet nameable as a single result. - **Open obligation**: the manuscript develops the connection far enough to expose one specific, nameable missing result — a proof, a derivation, or a technical question stated precisely enough that answering it would close the gap. Stronger than Bridge because the debt is specific, not diffuse. - **Tension — retained**: a genuine disagreement, kept rather than resolved, and not to be papered over later. - **Rejected**: a plausible-sounding correspondence was investigated and found to conflate two genuinely different structures. Stronger than "None" — this is a negative result, not an absence of one. - **Distinct but related**: two structures address the same territory without being the same object; neither Direct nor Rejected fits. - **None**: no useful correspondence found; rare enough not to need its own section. - **Unpursued** (excluded from the tables below, listed separately at the end): proposed during planning, but no chapter ever tested it. This is not a status the finished manuscript can be scored against — the manuscript simply doesn't contain an attempt. Keeping these in the main tables alongside Bridge and Open obligation would misstate what the ledger claims to record: status *after drafting*, for material that was actually drafted.

C.1 Part I — Logic as Constraint

Correspondence	Tested in	Status	Qualification / open obligation
Classical consequence ↔ admissibility at maximal strictness	Ch.1	Direct — established	Classical logic is R turned to maximal strictness in every direction, not a default.
Logical consequence as truth-preservation ↔ $x \rightsquigarrow_R y$	Ch.1	Direct — established	Same shape as the ordinary definition, with admissibility as the varying parameter.
Explosion ↔ unrestricted propagation closure	Ch.1, Ch.4	Direct — established	Explosion is a specific, substantive closure choice, not a logical necessity.
Lewis's argument for explosion ↔ repair cost formulation	Ch.1	Open obligation	Named missing result: a formal derivation from $E(a) + \lambda R(a)$, not yet supplied — currently gestured at only.

C.2 Part II — Worlds and Admissibility

Correspondence	Tested in	Status	Qualification / open obligation
Possible worlds ↔ admissible states $w \in \mathrm{Adm}(R)$	Ch.5	Direct — established	Deliberately ontology-neutral; realism/actualism/Meinongian debate left open on purpose.
Kripke necessity ↔ single-parameter invariance	Ch.5 (provisional), Ch.6 (corrected)	**Rejected as originally framed**, replaced	The single-R box formula conflates admissibility and accessibility; Ch.6 splits them into $(R, \mathcal{R})$. The rejection is of the *conflation*, not of necessity-as-invariance generally.

Correspondence	Tested in	Status	Qualification / open obligation
Normal modal systems (K, T, S4, S5) $\leftrightarrow$ structural constraints on $\mathcal{R}$	Ch.6	Direct — established	Reflexivity/transitivity/symmetry imposed on $\mathcal{R}$ specifically, leaving R untouched.
Non-normal worlds $\leftrightarrow$ refusal	Ch.7	**Rejected**, replaced by non-normal worlds $\leftrightarrow$ state-indexed regime R_w	Refusal is a targeted, Level-3 decline of one transformation; non-normal worlds suspend the compositional evaluation rule itself — a different structural level. The original Direct rating was vocabulary-matching, not a derivation.
Impossible worlds $\leftrightarrow$ locally varying admissibility	Ch.7	Direct — established, with a downstream open obligation	R_w formalizes this cleanly at the level of single states; the obligation surfaces one part later (see Part V/VI section below), not here.
Inconsistency / impossibility / non-normality as one phenomenon	Ch.7, using Part III as control case	Rejected as a collapse; three independent axes established	"Impossibility is indexed to a constraint regime; contradiction is indexed to support" — stated as this part's foundational sentence.
Strict implication, conditional logics $\leftrightarrow$ admissibility transport $p \Rightarrow_R q$	Ch.8	Open obligation	Named missing result: the transformation-selection rule Lewis's similarity ordering supplied has no replacement yet.

Correspondence	Tested in	Status	Qualification / open obligation
Distinguishability geometry ↔ Lewisian similarity spheres	Ch.8	Open obligation	Chapter ends on a precisely stated question (can similarity be derived from admissibility geometry) rather than claiming the replacement — a nameable research question, not a diffuse gap.

C.3 Part III — Gaps, Gluts, and Propagation

*(Editorial note: the working title "Gaps, Gluts, and Obstructions" is retired for this part. Part VI earned the right to use "obstruction" as a technical term distinct from contradiction; using the same word in Part III's own heading, before that distinction exists, would quietly reinstate the association the later chapters worked hard to separate. If the manuscript's actual Part III title still reads "...and Obstructions," it should be changed to match.)

Correspondence	Tested in	Status	Qualification / open obligation
Truth-value gaps ↔ underdetermination (absence of both polarities)	Ch.9	Direct — established	Gap is not a third truth value bolted on; it's what "no answer yet" already means once the two support questions are separated.
Truth-value gluts ↔ overdetermination (retained conflicting evidence)	Ch.10	Direct — established	Sharper than "some propositions are both true and false" — contradictory evidence need not be deleted merely for conflicting.
FDE's four values ↔ independent σ^{+}, σ^{-}	Ch.9–10, sharpened again in Ch.20	Direct — established	Became the entry point into Part VI's sheaf machinery, not merely a tableaux gloss.

Correspondence	Tested in	Status	Qualification / open obligation
Disjunctive syllogism failure ↔ restricted propagation	Ch.11	Direct — established, technical centerpiece	DS's second step is shown to silently assume positive/negative support are mutually exclusive — false exactly at overdetermined points. Not an ad hoc patch.
LPm / classical recapture ↔ repair biased toward consistency	Ch.12	Direct — established	Pays the cost of a glut only where evidence leaves no alternative; classical elsewhere.
FDE merge ↔ glut-free reconstruction obstruction	Ch.12, restated at cover level Ch.23	Direct — established, order-theoretic	Deliberately not cohomological in Part III; the cover-level generalization was checked for compatibility during Part VI's final pass and confirmed to be the same operation, not merely analogous (see Ch.23 row below).
Relevant logics (ternary relation) ↔ propagation licensing	Ch.11 gestures at it via the DS mechanism	Open obligation	Named missing result: expressive equivalence to the ternary relation, neither shown nor disproven.

C.4 Part IV — Construction, Vagueness, and Degree

Correspondence	Tested in	Status	Qualification / open obligation
Intuitionistic construction ↔ reconstruction path	Ch.13	Direct — established, sharpened	Pinned specifically to $\neg\neg p \to p$ failing, not a looser "existence from absurdity" slogan — the earlier looser version would have overstated the correspondence.
Support vs. constructibility as one axis vs. two	Ch.13	Rejected as one axis; established as two orthogonal coordinates	A non-constructive derivation and a genuine construction can both count as positive support; Part III's apparatus doesn't (and didn't need to) distinguish them.
Many-valued/fuzzy degree ↔ resolution-stability	Ch.14	Bridge, explicitly labeled as this book's added interpretation	Corrected from an earlier draft that stated this as though many-valued semantics already meant it. $v_\lambda(p)$ vs. the resolution profile $\lambda \mapsto v_\lambda(p)$ now kept distinct.
Sorites / vagueness ↔ dispersion obstruction	Ch.15	Bridge, developed	Corrected definition: accumulation of small variation, not exact-agreement-everywhere ($\sum_i \epsilon_i$ need not be small even when each ϵ_i is negligible). The earlier "N and B both fail to fit {T,F}" framing this traces back to is now understood to have been too coarse; superseded, not merely refined.

Correspondence	Tested in	Status	Qualification / open obligation
Dispersion obstruction ↔ Ch.24's proposed topology	Ch.15, tested directly against Ch.24 §3 during Part VI's final pass	**Distinct but related**	Ch.24's topology is qualitative-only (neighborhoods, no metric); dispersion needs the accumulation structure Ch.15 introduced. Confirmed by direct test, not assumed either way.

C.5 Part V — Identity, Change, and Worldhood

Correspondence	Tested in	Status	Qualification / open obligation
Necessary/contingent identity, rigid designation ↔ trajectory-first ontology	Ch.16	Bridge	Explicitly not claimed to derive rigid designation — rigid designation is the comparison case, not a target reduced to this book's account.
Identity ↔ invariant of admissible continuation	Ch.16	Direct — established, with one open dependency	The thesis itself is stated cleanly; but see the R_w/continuation row below — this chapter's own notion of "admissible" is underspecified once regimes vary by state.

Correspondence	Tested in	Status	Qualification / open obligation
Priest's contradictory transition instant ↔ continuation-interface object	Ch.17	Tension — retained, as a productive alternative	Explicitly qualified "sometimes, not always" — must not become a general argument against dialetheism, given Part III's four chapters on genuine gluts. Not a derivation of Priest's account; a rival with a named scope limit.
Hegelean motion ↔ RSVP field dynamics	Ch.18	Tension — retained	Not upgraded despite RSVP doing real argumentative work; the ontological-fact-vs-inadequate-vocabulary question is left open on purpose.
Leibniz Continuity Condition ↔ discrete repair events	Ch.19	Tension — retained, given a specific test	"Does a continuous trajectory underlie every discrete repair at some finer description" is posed as a real either/or, not assumed to reconcile.
Spread Hypothesis / temporal flow ↔ Chain of Memory	Ch.19	Bridge, deliberately narrowed	Explicitly not a theory of physical time — narrowed to instantaneous-state ≠ history-indexed-state. Overselling this was an identified risk and avoided.

Correspondence	Tested in	Status	Qualification / open obligation
R_w **(Ch.7) ↔ admissible continuation (Ch.16)**	Ch.16, surfaced during Part VI's final pass	**New open research problem — not resolved during reconciliation, on purpose**	$x_t \xrightarrow{?} x_{t+1}$ with $R_{x_t} \neq R_{x_{t+1}}$: "admissible continuation" doesn't yet say relative to which regime a transition step is judged — R_{x_t}, $R_{x_{t+1}}$, an intersection/union, a transport of one into the other, or a distinct transition regime R_γ. Ch.17's transition-as-its-own-type move suggests the last option but was correctly not promoted to a definition. This is a genuine result of completing all six parts, not a loose end to patch quietly.

C.6 Part VI — Local Logic and Global Obstruction

Correspondence	Tested in	Status	Qualification / open obligation
Local valuations ↔ local admissible sections $s_i \in \mathcal{A}(U_i)$	Ch.20	Direct — established	

Correspondence	Tested in	Status	Qualification / open obligation
Compatibility of local descriptions ↔ Čech coboundary vanishing	Ch.21	Direct — established, corrected apparatus	Not literal H^1: for any chosen 0-cochain, δs is a coboundary by construction, so its class in H^1 is trivially zero regardless of agreement. Renamed **coherence defect** $\delta^{\pm}(p) \in \check{C}^1$, a cochain-level check.
Failure of global coherent determination ↔ nonzero H^1 / glut	Ch.20–22	**Rejected as originally stated**, replaced by the Mutual Non-Determination Proposition	"Glut $= [g] \neq 0$" is not a valid identification — glut is semantic (dual support), H^1-type obstruction is geometric. The corrected result: coherence status and the FDE merge $\Sigma(p)$ determine neither each other, proven by an explicit six-cell realizability table (Ch.22), not asserted by analogy.
Bilattice join ↔ ordinary sheaf gluing	Ch.20–21	**Rejected**	Join is an information-preserving merge/repair of a non-matching family; gluing requires a family that already agrees on overlaps. Different operations under the same informal word "combine." Terminology fixed going forward: restriction ≠ gluing ≠ repair.

Correspondence	Tested in	Status	Qualification / open obligation
TARTAN ↔ local-to-global logical regime assembly	Ch.24 §3 (proposed)	Open obligation	Two named missing results: does refinement only reveal genuine coherence defects or manufacture spurious ones; is the resulting limit/colimit well-behaved.
Ch.12's merge M ↔ Ch.23's merge Σ	Ch.23, checked directly during Part VI's final pass	Direct — identified	$\Sigma = M$ under cover indexing; the name change marks the structural generalization (bare sources → cover-indexed patches), not a different operation.
T, F, B, N ↔ fibers of a projection, not points of a richer space	Ch.24 §1	Direct — established	A restatement of the Mutual Non-Determination Proposition in geometric language, not new content — the only fully rigorous claim in Ch.24.
Repair cost as a metric on $\mathcal{S}(p)$	Ch.24 §4 (proposed)	Open obligation	Two named missing results: whether an action-independent repair distance exists, and whether it satisfies a triangle inequality.

C.7 Unpursued correspondences

Two entries inherited from the original planning-stage matrix never had a chapter written for them. They are not weak Bridges or exposed obligations — the manuscript contains no attempt to test them at all, and listing them in the main tables above would misstate what this ledger claims to record (status *after drafting*, for material the finished six parts actually contain).

Correspondence	Original proposal	Why it stayed unpursued
Priest's semantic closure (self-referential truth predicate) ↔ Spherepop's history-based operational semantics	A self-referential object language and a computation model that can refer to its own prior states looked structurally similar	No chapter in Parts I–VI was assigned this topic; the six-part outline never routed through it
Priest's naive/paraconsistent set theory (material, relevant, and model-theoretic strategies) ↔ obstruction as candidate propagation rules	Priest's three strategies for handling set-theoretic paradoxes looked like candidate instances of this book's propagation-restriction machinery	Same reason — no chapter's scope included it

This isn't a defect in the finished book. A twenty-four-chapter argument built around one central hierarchy doesn't need to absorb every topic Priest covers across two books, and both of these are now more approachable than they would have been before this project's framework existed — they're candidates for separate future work precisely because the machinery to attempt them properly (the support/coherence/repair apparatus, the admissibility regime vocabulary) now exists in a form mature enough not to require inventing it from scratch alongside the comparison itself.

C.8 What changed, as a record rather than a silent overwrite

Three correspondences were investigated and found to conflate distinct structures, and are marked Rejected rather than quietly dropped: glut as a nonzero cohomology class, bilattice join as ordinary sheaf gluing, and non-normal worlds as refusal. All three were plausible enough to have been rated Direct or treated as settled before the relevant chapter was actually drafted. That they didn't survive contact with the chapters is evidence the drafting process constrained its own conclusions rather than rationalizing whatever correspondence was proposed first.

One new open research problem — the R_w/continuation transition rule — was generated by completing Parts II and V, not resolved by their completion. It's recorded here deliberately unresolved, per instruction, since fixing it during this reconciliation pass would erase evidence that finishing the book found a real gap rather than merely filling in a pre-drawn outline.

This ledger is now the right basis for the book's introduction and conclusion, and for identifying which individual results (the Mutual Non-Determination Proposition and its realizability proof, most likely, along with the R_w/continuation problem) might warrant a standalone paper ahead of the full manuscript.

This revision's specific change: split the undifferentiated Bridge category into Bridge (diffuse gap) and Open obligation (one nameable missing result), reclassifying five rows accordingly, and moved the two correspondences the manuscript never actually tested (semantic closure/Spherepop, naive set theory) out of the main tables into their own section. The ledger's claim to record what the finished manuscript established was slightly inconsistent with carrying two rows the manuscript never attempted; that inconsistency is now fixed rather than carried forward again.

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