Admissibility Geometry and the MOND Transition

Reconstruction Obstruction as an Alternative to Dark Matter

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Abstract

We develop a reinterpretation of gravitational anomalies — including flat rotation curves, weak lensing excesses, and the MOND acceleration scale — within the Relativistic Scalar-Vector Plenum (RSVP) framework. Existing emergent gravity programs derive spacetime geometry from physical exchange processes, thermodynamic bookkeeping, or information flow. RSVP inverts this order: admissibility geometry is treated as primary, and persistent physical exchange is interpreted as a stabilized reconstruction mode within that geometry.

The framework introduces the coupled field $X = (\Phi, \mathbf{v}, S)$, where Φ encodes the accessibility geometry of continuation space, $\mathbf{v}$ encodes directed coherence flow, and S measures admissibility dispersion. The dimensionless ratio S/Φ serves as a central control parameter governing local reconstruction factorization. We define an *admissibility radius* R_A as the largest scale over which local reconstruction remains approximately factorizable without cosmological boundary data, and conjecture that the RSVP field equations determine $R_A \propto c/H_0$ in weak-gradient, low-density regimes, yielding the MOND acceleration scale $a_0 \sim cH_0$ as a geometric consequence of reconstruction structure rather than a modification of force law.

The paper operates across four explicitly distinguished epistemic registers: definitions, structural conjectures, derived implications, and open proof obligations. We do not derive MOND from completed RSVP field equations; we identify the geometric objects from which such a derivation would have to proceed. A dedicated section identifies the five irreducible commitments of the framework, separating load-bearing assumptions from illustrative scaffolding. Appendices develop the formal sheaf structure, constraint propagation, coarse-graining flow with attractor analysis, candidate dynamical equations, obstruction cohomology, relation to renormalization theory, observational targets, locality clarification, formal implementation via BNF grammar and Lean-style schemas, a variational formulation sketch, effective metric emergence, and admissibility gauge equivalence.

Keywords: emergent gravity, MOND, admissibility geometry, reconstruction sheaf, entropic gravity, dark matter, RSVP framework, obstruction cohomology, renormalization flow.

Core Objects and Notation

Symbol	Meaning
$X = (\Phi, \mathbf{v}, S)$	Coupled RSVP field
Φ	Continuation accessibility scalar field
$\mathbf{v}$	Coherence-flow vector field
S	Admissibility dispersion (entropy) field
$\chi = S/\Phi$	Reconstruction control parameter
$\chi_\ell = S_\ell/\Phi_\ell$	Scale-dependent control parameter
$\mathcal{U} = \{U_\alpha\}$	Reconstruction (admissibility) cover
$\mathcal{F}$	Reconstruction sheaf
$\mathcal{F}(U_\alpha)$	Admissible sections over neighborhood U_α
$\rho_{\alpha\beta}$	Restriction map $\mathcal{F}(U_\alpha) \to \mathcal{F}(U_\alpha \cap U_\beta)$
R_A	Admissibility radius
a_A	Admissibility acceleration scale $\sim c^2/R_A$
$\mathcal{P}[s]$	Persistence functional of section s
$\mathcal{R}_\lambda$	Coarse-graining operator on covers
$\Omega_\ell[s]$	Scale-dependent obstruction functional
$\beta(\chi)$	Admissibility beta-function
$\Pi_{\text{therm}}, \Pi_{\text{info}}, \Pi_{\text{grav}}, \Pi_{\text{conf}}$	Entropy projection functors
$\check{H}^k(\mathcal{U}, \mathcal{F})$	Čech cohomology of cover with sheaf coefficients

Contents

1 Introduction

The relationship between gravity and thermodynamics has attracted sustained theoretical attention since Bekenstein and Hawking established that black holes carry entropy proportional to horizon area. Subsequent programs — Jacobson's derivation of the Einstein equations from thermodynamic identities [4, 5], Verlinde's entropic gravity [2, 3], and more recently transactional entropic gravity [1] — have attempted to establish that gravitational dynamics emerge from statistical or information-theoretic structure rather than constituting a fundamental interaction. Padmanabhan has developed related thermodynamic perspectives on gravity [6], and the connection between entanglement structure and spacetime geometry has been explored through holographic frameworks [11, 12].

These programs share a common derivation order: physical exchange processes, thermodynamic bookkeeping, or entropic flow are treated as primitive, and spacetime geometry is reconstructed from them. The present work inverts this order. We treat *admissibility geometry* as primary and interpret persistent physical exchange as a stabilized reconstruction mode within that geometry. Emitters, absorbers, particles, and force laws are not inputs to the framework but outputs — roles stabilized by recursive cross-scale coherence of the coupled field $X = (\Phi, \mathbf{v}, S)$ within its reconstruction sheaf [13, 15].

This inversion has a specific consequence for the MOND anomaly [7, 9]. Existing programs explain the transition scale $a_0 \sim cH_0$ by finding thermodynamic narratives that reproduce it. The present framework instead asks: what geometric property of the reconstruction sheaf would make such a scale *structurally necessary*? The answer we propose — an admissibility radius R_A beyond which local reconstruction requires cosmological gluing data — converts a phenomenological constant into a constraint on the field theory itself.

Epistemic Registers

The paper operates simultaneously across four epistemic registers which must be carefully distinguished. Certain structures are **definitional**, specifying the reconstruction geometry and admissibility framework. Others are **conjectural**, proposing relationships between admissibility structure and cosmological scaling. Still others are **derived implications** under specified assumptions. Finally, several results remain **open proof obligations** generated by the framework itself. We make these distinctions syntactically explicit throughout — using the theorem environments *Definition*, *Conjecture*, *Theorem-Schema*, *Heuristic*, and *Open Proof Obligation* — in order to avoid conflating motivation, formal consequence, and unresolved structure, a conflation that has weakened much of the emergent gravity literature.

Structure of the Paper

Section 2 develops the admissibility geometry and sheaf structure, including an explicit clarification of the operational meaning of the RSVP fields. Section 3 introduces the admissibility phase transition, scaling constraints, and the central conjecture regarding R_A.

Section 4 treats entropy species as regime projections including commutator structure. Section 5 reinterprets dark matter phenomenology as reconstruction obstruction. Section 6 compares the framework with neighboring programs. Section 7 identifies the five irreducible commitments of the theory. Sections 8–13 develop the philosophical inversion, regime interpretation, observational proxies, forbidden continuation classes, structural risks, and completion criteria. Section 14 consolidates contributions and open obligations. Appendices A–K develop the formal machinery: sheaf structure with constraint propagation, coarse-graining flow with attractor analysis and coarse-graining disambiguation, candidate field equations, obstruction cohomology, renormalization analogy, observational targets, locality clarification, formal implementation, a variational formulation sketch, effective metric emergence, and admissibility gauge equivalence.

2 Admissibility Geometry and the Reconstruction Sheaf

2.1 The Coupled Field

The fundamental object of the RSVP framework is the coupled field

$$X = (\Phi, \mathbf{v}, S), \tag{1}$$

where Φ is a scalar field encoding the *accessibility geometry of continuation space* — the structure governing which local field trajectories admit coherent extension into persistent reconstruction modes; $\mathbf{v}$ is a vector field encoding directed transport and coherence flow; and S is a scalar entropy-dispersion field measuring the spread of admissible reconstruction trajectories.

These three components are dynamically coupled. The scalar field Φ governs which continuations remain accessible; $\mathbf{v}$ tracks preferred directions of coherent continuation; and S measures how tightly reconstruction trajectories concentrate near stable modes. A configuration with high S relative to Φ has many admissible continuations and correspondingly weak reconstruction stability. Low S/Φ concentrates trajectories near persistent modes — the regime in which particles, clocks, and force-mediating exchanges stabilize as recognizable roles.

Remark 2.1 (Operational meaning of the RSVP fields). The field Φ is *not* identified with the Newtonian gravitational potential, nor with any standard matter field. It encodes a more primitive object: the geometry of which local continuations remain stably extendable. The Newtonian potential emerges as a low-χ effective reconstruction observable, in the same way that hydrodynamic pressure is not microscopically fundamental but emerges from kinetic theory in appropriate regimes. Similarly, S should not be read primarily as thermodynamic entropy; it is admissibility dispersion, and thermodynamic entropy is only one of its projection functors (Π_{therm}) under specific coarse-grainings. Foregrounding the term *dispersion* rather than *entropy* for S would reduce confusion with thermodynamic baggage; the present paper retains both usages but always interprets S through the projection-functor framework rather than as a primitive thermodynamic quantity.

The control parameter. The dimensionless ratio

$$\chi = \tfrac{S}{\Phi} \tag{2}$$

is the central control parameter of the framework. When $\chi \ll 1$, entropy-dispersion is small relative to continuation accessibility, local reconstruction is strongly factorizable, and effective dynamics reduce approximately to Newtonian behavior. When $\chi \sim 1$, dispersion becomes comparable to accessibility, local factorization breaks down, and reconstruction requires boundary data from larger scales to maintain global stability. This ratio governs simultaneously: local factorization stability, the admissibility phase transition, entropy-channel convergence, and obstruction growth dynamics. Its ubiquity across all four phenomena is evidence of the framework's internal coherence rather than coincidence.

2.2 Admissible Local Sections

Definition 2.2 (Admissible local section)**.** A local section of X over a region U is *admissible* if reconstruction trajectories originating within U remain dynamically stable under bounded perturbation and admit coherent continuation under the restriction structure of the field cover.

Admissibility is a *persistence condition*: it asks not merely whether a configuration is locally self-consistent, but whether it can continue coherently into neighboring regions and across scales. Three notions must be carefully distinguished.

(i) **Local dynamical stability.** Trajectories within U remain bounded under perturbation. This is a condition on the field equations restricted to U alone, making no reference to neighboring regions.

(ii) **Global admissibility.** Locally stable sections admit coherent extension across the full reconstruction cover without generating contradictions at overlapping boundaries. A section may be locally stable while failing global admissibility if its continuation structure conflicts with that of neighboring sections.

(iii) **Reconstruction persistence.** Globally admissible extensions remain stable under renormalization of the reconstruction cover across scales — a multiscale stability condition rather than merely a temporal one. A section may be globally admissible at one scale while failing persistence at another if S-dispersion grows faster than the coherence structure of $\mathbf{v}$ can absorb under coarse-graining.

The distinction between local stability and global admissibility is precisely what separates RSVP from transactional approaches [1]. Transactional completion guarantees local interaction closure — offer and confirmation waves match at a spacetime event. RSVP admissibility requires persistent global extension compatibility across the entire reconstruction cover. A transaction can complete locally while generating globally obstructed reconstruction dynamics; RSVP classifies such processes as inadmissible or metastable regardless of their local consistency.

2.3 The Reconstruction Cover

Definition 2.3 (Admissibility neighborhood)**.** An *admissibility neighborhood* U_α is a region over which reconstruction remains approximately factorizable, meaning the admissible section over U_α can be specified without reference to data outside U_α up to corrections of order $\frac{S}{\Phi}$.

We introduce a cover $\mathcal{U} = \{U_\alpha\}$ of the field domain where each U_α is an admissibility neighborhood. Regions where S/Φ is small generate large, strongly factorizable neighborhoods. Regions where S/Φ approaches unity generate small, weakly factorizable neighborhoods whose sections increasingly require external boundary data for stable continuation.

Remark 2.4. The reconstruction cover is a *derived* object of the field geometry, not a background against which the field is defined. Classical spacetime structure, to the extent it appears, is a stable regime of the reconstruction cover rather than its presupposition. This is the formal content of RSVP's departure from ordinary field theory: integrating out short-scale reconstruction structure changes not merely effective couplings but the factorization structure of admissible reconstruction itself [18, 19].

The present work leaves open whether the reconstruction cover should ultimately be interpreted as a topological cover over an emergent spacetime manifold, a site of continuation states in the sense of Grothendieck topology, a category of reconstruction observables, or a more general admissibility structure. The current treatment remains intentionally agnostic on this question. A reader trained in sheaf theory will recognize that specifying the underlying category is necessary for a rigorous formulation, and this constitutes one of the open categorical obligations of the framework.

2.4 Two Types of Obstruction

Obstruction occurs when locally admissible sections fail to admit globally stable extension across overlapping neighborhoods. Obstruction is not metaphorical: it is a literal failure of global compatibility in the reconstruction geometry [16, 17]. We distinguish two structurally distinct types whose physical consequences differ and whose conflation would obscure both the entropy analysis of Section 4 and the dark matter reinterpretation of Section 5.

Definition 2.5 (Compatibility obstruction)**.** Let $s_\alpha \in \mathcal{F}(U_\alpha)$ and $s_\beta \in \mathcal{F}(U_\beta)$ be locally admissible sections over overlapping neighborhoods. A *compatibility obstruction* exists if

$$\rho_{\alpha\beta}(s_\alpha) \neq \rho_{\beta\alpha}(s_\beta)$$

on $U_\alpha \cap U_\beta$. The sections specify incompatible continuation structures on their shared domain and cannot be globally glued.

Definition 2.6 (Dispersion obstruction)**.** A pair of sections suffers *dispersion obstruction* if they agree formally on overlapping domains — $\rho_{\alpha\beta}(s_\alpha) = \rho_{\beta\alpha}(s_\beta)$ — but cannot maintain stable global extension because S grows faster than the coherence propagation capacity of $\mathbf{v}$ across the cover. Formally compatible sections therefore fail reconstruction persistence: $\mathcal{P}[s] \to 0$ under scale extension.

Remark 2.7. Not all globally inconsistent dynamics arise from local disagreement. Some arise from uncontrolled admissibility growth under extension. Compatibility obstruction manifests as irregular, patchy anomalies without clear scaling structure. Dispersion obstruction manifests as smooth large-scale departure from locally stable behavior — the observational signature more characteristic of galactic rotation anomalies. The two obstruction types produce distinct signatures in the entropy structure of the field (Section 4) and distinct phenomenological predictions (Appendix F).

2.5 The Reconstruction Sheaf

The objects defined above constitute the data of a *reconstruction sheaf* $\mathcal{F}$ over the field domain [13, 14]: $\mathcal{F}$ assigns to each U_α the space $\mathcal{F}(U_\alpha)$ of admissible sections of X over U_α, together with restriction maps

$$\rho_{\alpha\beta} \colon \mathcal{F}(U_\alpha) \longrightarrow \mathcal{F}(U_\alpha \cap U_\beta)$$

satisfying identity preservation, compatibility under composition, and the locality and gluing axioms. Global sections of $\mathcal{F}$ correspond to globally admissible reconstruction modes.

Remark 2.8 (Admissibility nonlocality). Admissibility nonlocality — when it occurs — is nonlocality in the indexing and coherence structure of $\mathcal{F}$, not in the stalk-local field dynamics. The stalks remain local field values propagating according to local field equations. What becomes nonlocal is the coherence condition on their assembly into globally stable sections. This is not a superluminal signaling proposal: nonlocality appears in the geometry of assembly rather than in microscopic propagation. See Appendix G for a precise separation of these notions.

2.6 Bridge to Section 3

Once admissibility is understood as a global extension condition rather than a local interaction condition, it becomes natural for reconstruction dynamics to transition between qualitatively distinct regimes as χ varies. In strongly factorizable regimes ($\chi \ll 1$), local sections extend to global sections without reference to cosmological boundary data. As χ approaches unity, sections begin requiring cosmological gluing data for stable extension. The existence of a characteristic scale — the admissibility radius R_A — at which this transition occurs is proposed as a consequence of the field dynamics of X.

3 The Admissibility Phase Transition

The appearance of the MOND acceleration scale [7, 10]

$$a_0 \sim cH_0 \tag{3}$$

suggests that galactic dynamics are sensitive not merely to local mass distributions but to the relation between local reconstruction stability and cosmological admissibility structure. The coupling of galactic rotation to cosmological structure is difficult to interpret inside purely local Newtonian dynamics — it strongly suggests the relevant object is not local force accumulation alone but global admissibility structure across scales. Within RSVP, we interpret this transition not as a modification of force law but as a *breakdown of local factorization in the reconstruction cover*, a geometric event controlled by $\chi = S/\Phi$.

3.1 The Factorization Transition

In regimes where $\chi \ll 1$, local admissibility neighborhoods remain strongly factorizable. Reconstruction trajectories admit stable extension independently of cosmological boundary data, effective dynamics reduce approximately to Newtonian behavior, and the three entropy channels converge under coarse-graining (Section 4).

As

$$\chi = \frac{S}{\Phi} \longrightarrow 1, \tag{4}$$

dispersion becomes comparable to continuation accessibility. Local sections cease to remain scale-independent under extension. Admissibility neighborhoods lose strong factorization. Reconstruction increasingly requires cosmological gluing data to maintain persistence across the cover. Dispersion obstruction begins to dominate compatibility structure: even formally compatible sections fail reconstruction persistence.

Definition 3.1 (Admissibility phase transition)**.** The *admissibility phase transition* is the qualitative change in reconstruction regime occurring when $\chi \to 1$: the transition from locally dominated factorizable reconstruction to cosmologically coupled reconstruction requiring global boundary data for stable extension.

This is not a transition in the force law acting on matter, nor a transition in spacetime geometry in the classical sense. It is a transition in the factorization structure of admissible reconstruction — a change in what kind of boundary data local sections require for globally stable continuation. In transactional entropic gravity [1], the force law is modified by thermodynamic corrections applied to an otherwise standard local framework. In RSVP, the effective dynamics change because the geometric object governing which continuations are admissible changes.

3.2 The Admissibility Radius

Definition 3.2 (Admissibility radius)**.** The *admissibility radius* R_A is the largest characteristic scale over which admissible local sections remain approximately factorizable under renormalization of the reconstruction cover. Equivalently, R_A marks the onset of dispersion-dominated reconstruction dynamics for which stable continuation requires cosmological extension data: a local section at scale r requires cosmological gluing data if and only if its restriction to neighborhoods approaching R_A fails to admit stable extension through the sheaf's restriction maps alone — that is, if χ evaluated at that scale approaches unity under the coarse-graining flow.

R_A is a derived quantity of the coupled dynamics of Φ and S, not an external parameter. Different field configurations will generically produce different admissibility radii.

3.3 Dimensional Structure and Scaling Constraints

A structural test of any geometric framework is whether the scales it identifies are forced by its primitive objects rather than fitted to known phenomenology. We examine the scaling dimensions of the RSVP primitives to show that the emergence of R_A and a_0 is not arbitrary.

Assign natural dimensions, where $[L]$ denotes length and $[T]$ time:

$$\begin{aligned}
&[\Phi] = L^2T^{-2} \quad \text{(accessibility, dimension of specific energy)},\\
&[\mathbf{v}] = L\,T^{-1} \quad \text{(coherence-flow velocity)},\\
&[S] = L^2T^{-2} \quad \text{(dispersion, matching } \Phi \text{ so } \chi \text{ is dimensionless)},\\
&[\chi] = 1.
\end{aligned}$$

The control parameter $\chi = S/\Phi$ is dimensionless by construction — the first consistency check. The phase transition criterion $\chi \to 1$ is a pure number and therefore scale-invariant in form, confirming that the transition is a genuine geometric threshold rather than a unit-dependent artifact.

The admissibility radius R_A has dimension $[L]$. The only combination of fundamental constants with this dimension accessible to the reconstruction sheaf, without introducing Newton's constant G (which would bring in a mass scale absent from the pure admissibility geometry), is c/H_0. The corresponding acceleration scale satisfies

$$a_A \sim \frac{c^2}{R_A} = \frac{c^2}{c/H_0} = cH_0,$$

which is dimensionally $[L\,T^{-2}]$ as required.

Observation 3.3. Within the RSVP primitive set $\{c,\, H_0,\, \chi\}$, the MOND acceleration scale $a_0 \sim cH_0$ is the *unique* dimensionally consistent admissibility acceleration in the limit where G and particle masses enter only as derived quantities. Any alternative scaling of R_A would require introducing an independent length or time scale not present in the admissibility geometry. Conjecture 3.4 is therefore dimensionally forced: the question is not whether $R_A \propto c/H_0$ is the right form, but whether the field dynamics uniquely select it.

This argument also constrains the admissibility beta-function $\beta(\chi)$ of Appendix B: since χ and $\ln \ell$ are both dimensionless, β must be a dimensionless function of χ alone at leading order. Any dimensional parameter entering β would break the scale-invariance of the transition structure and produce a phase transition at a scale other than the unique admissibility radius.

3.4 Newtonian and Cosmologically Coupled Regimes

Newtonian regime ($r \ll R_A$, $\chi \ll 1$). Local reconstruction is strongly factorizable. Admissible sections extend stably without cosmological boundary data. Dispersion obstruction remains bounded under scale extension. Effective gravitational dynamics reduce to Newtonian behavior as a consequence of local reconstruction stability rather than as a fundamental force law [2]. Newtonian gravity therefore appears as an effective low-dispersion reconstruction theory, analogous to the emergence of hydrodynamics from microscopic statistical systems [19].

Cosmologically coupled regime ($r \sim R_A$, $\chi \sim 1$). Local factorization breaks down. Admissible sections require cosmological gluing data. Dispersion obstruction dominates. Reconstruction persistence fails for configurations that remain locally consistent. Effective dynamics depart from Newtonian behavior not because the force law changes, but because the admissible continuation set changes.

3.5 The Central Conjecture

Conjecture 3.4 (MOND scaling from admissibility radius)**.** *The weak-gradient RSVP field equations determine an asymptotic admissibility radius satisfying*

$$R_A \propto \frac{c}{H_0} \tag{5}$$

in low-density regimes where χ approaches unity smoothly. The corresponding admissibility acceleration scale is then

$$a_A \sim \frac{c^2}{R_A} \sim cH_0, \tag{6}$$

yielding MOND-like phenomenology as a structural consequence of reconstruction geometry.

Heuristic 3.5. The structural plausibility of Conjecture 3.4 rests on the following observation. The Hubble radius c/H_0 is the characteristic scale of cosmological causal accessibility. If R_A is the admissibility analog of causal accessibility, its asymptotic scaling with c/H_0 would follow from the requirement that admissibility structure and causal structure coincide asymptotically in low-density regimes. This is a structural argument for the conjecture's reasonableness, not a derivation.

The MOND scale $a_0 \sim cH_0$ has been observed by several authors to couple galactic dynamics to cosmological structure in a way that resists purely local explanation [7, 9]. Within RSVP, this coupling emerges naturally from the admissibility framework: the transition scale is determined by the scale at which local reconstruction loses self-sufficiency. MOND is therefore demoted from foundational principle to phenomenological signature of a more general reconstruction transition.

3.6 Open Proof Obligation

Open Proof Obligation 3.6. Show that the coupled dynamics of Φ and S in the weak-gradient limit uniquely determine R_A with asymptotic scaling $R_A \propto c/H_0$. This requires: *(i)* deriving the coarse-graining flow of χ under renormalization of the reconstruction cover in the weak-gradient limit; *(ii)* identifying the fixed-point or threshold structure of this flow; and *(iii)* showing that the characteristic scale of the threshold is asymptotically proportional to c/H_0.

Conjecture 3.4 is falsifiable in three distinct senses: (1) if the RSVP field equations admit a unique R_A whose asymptotic scaling substantially departs from c/H_0, then MOND-like phenomenology must arise from a different mechanism; (2) if MOND-like behavior were observed in regimes where χ remains well below unity, this would constitute evidence against the sheaf structure assumed here; and (3) if R_A can be derived but depends sensitively on unconstrained parameters in the RSVP field equations, the conjecture is technically falsifiable but practically underdetermined — a third failure mode named explicitly.

3.7 Bridge to Section 4

The admissibility phase transition has consequences not only for effective dynamics but for the entropy structure of the field. In strongly factorizable regimes, the distinct entropy channels converge under coarse-graining because local reconstruction is approximately scale-independent. As $\chi \to 1$, this convergence fails. Section 4 formalizes this consequence, deriving entropy-channel behavior as a corollary of reconstruction geometry rather than as an independent thermodynamic hypothesis.

4 Entropy Species as Regime Projections

Most emergent gravity programs implicitly identify informational, thermodynamic, configurational, and gravitational entropy as interchangeable quantities, treating their equivalence as a foundational step enabling the derivation of gravitational dynamics [2, 1]. The information-thermodynamic identification remains debated [20], and its use as an axiom in precisely the regimes where it is least reliable constitutes a significant structural weakness. Within RSVP, these entropy channels are instead treated as distinct projections of admissibility dispersion under different reconstruction covers and coarse-grainings [21]. Their convergence is not assumed axiomatically but emerges only within specific reconstruction regimes. This converts what is typically an axiom into a prediction.

4.1 Entropy Channels as Projections

Given a reconstruction cover $\mathcal{U} = \{U_\alpha\}$, different entropy species correspond to distinct coarse-grained projections of admissibility dispersion across $X = (\Phi, \mathbf{v}, S)$. We formalize these as projection functors

$$\Pi_{\text{therm}}, \quad \Pi_{\text{info}}, \quad \Pi_{\text{grav}}, \quad \Pi_{\text{conf}}$$

mapping admissibility structures into entropy observables associated with distinct reconstruction regimes and coarse-graining procedures.

Informational entropy, represented by Π_{info}, measures uncertainty in continuation accessibility under local reconstruction. It quantifies the spread of admissible continuations available to a local section of Φ and therefore tracks the local geometry of admissible continuation space.

Configurational entropy, represented by Π_{conf}, measures admissible arrangement multiplicity across neighboring sections of the reconstruction cover. It characterizes the number of globally compatible assemblies obtainable from locally admissible reconstruction data.

Thermodynamic entropy, represented by Π_{therm}, measures the coarse-grained dispersion of reconstruction trajectories under persistent exchange processes generated by the coherence-flow field $\mathbf{v}$. It corresponds to the effective trajectory-volume growth observed under macroscopic reconstruction flow.

Gravitational entropy, represented by Π_{grav}, measures dispersion associated with large-scale reconstruction curvature and admissibility obstruction. Its value depends not only on local continuation accessibility but on the obstruction structure governing global extension of admissible sections across the reconstruction sheaf.

None of these entropy channels is treated as fundamentally privileged within the RSVP framework. They correspond instead to distinct projection structures defined over the same underlying admissibility geometry. Their relationships are therefore regime-dependent rather than identity-relations. In strongly factorizable regimes, the projection functors become approximately equivalent under admissible coarse-graining. Near the admissibility transition, however, the projections separate as dispersion obstruction grows and local reconstruction loses scale-independence.

4.2 Entropy Convergence Under Strong Factorization

Theorem-Schema 4.1 (Entropy convergence)**.** In reconstruction regimes satisfying $\chi \ll 1$ and admitting bounded dispersion obstruction under renormalization of the reconstruction cover, entropy channels converge to order ε under admissible coarse-graining transformations:

$$\Pi_{\text{therm}} \simeq \Pi_{\text{info}} \simeq \Pi_{\text{grav}} \simeq \Pi_{\text{conf}} \qquad (\chi \ll 1).$$

Equivalently, informational, configurational, thermodynamic, and gravitational entropy measures become approximately equivalent up to corrections of order χ and the dispersion obstruction growth rate.

When $\chi \ll 1$, local reconstruction is strongly factorizable and approximately scale-independent. Different coarse-graining choices produce approximately the same result because admissibility dispersion is tightly concentrated near stable reconstruction modes — the cover choice becomes approximately irrelevant, and the distinct entropy projections converge. The entropy-identification moves made by emergent gravity programs are approximately valid in this regime, but only in this regime. The present framework derives the conditions under which they hold and identifies them as regime-specific approximations.

4.3 Entropy Divergence Under Dispersion Obstruction

As $\chi \to 1$, dispersion obstruction begins to dominate. Different coarse-graining choices produce systematically different results. Informational entropy, tracking local continuation accessibility, may remain bounded while thermodynamic entropy grows. Gravitational entropy, tracking obstruction contributions, may diverge from configurational entropy as global extension failures accumulate. Functorially:

$$\Pi_{\text{therm}} \not\simeq \Pi_{\text{grav}} \qquad (\chi \to 1).$$

Remark 4.2. Persistent divergence between entropy species is not an annoying inconsistency to be resolved by a better identification. It is a physical signal: evidence of obstruction growth within the reconstruction geometry itself. When entropy channels diverge persistently in a physical system, the RSVP framework predicts this as a signature of the system operating near or beyond the admissibility phase transition.

4.4 Commutator Structure of Entropy Projections

The entropy projection functors Π_i are not interchangeable under coarse-graining. This non-commutativity is a structural feature of the framework, not a defect.

Define the coarse-graining operator $\mathcal{R}_\lambda$ acting on reconstruction covers as in Appendix B. The commutator of a projection with coarse-graining measures whether applying the projection before or after scale-averaging produces the same result:

$$[\Pi_i, \mathcal{R}_\lambda] \;:=\; \Pi_i \circ \mathcal{R}_\lambda \;-\; \mathcal{R}_\lambda \circ \Pi_i.$$

In strongly factorizable regimes ($\chi \ll 1$), the projections approximately commute with coarse-graining:

$$[\Pi_i, \mathcal{R}_\lambda] \;\approx\; 0 \qquad (\chi \ll 1).$$

This is precisely the regime in which Theorem-Schema 4.1 holds: entropy species converge because different orderings of projection and coarse-graining yield the same result.

Near the admissibility transition ($\chi \to 1$), the commutator becomes nontrivial:

$$[\Pi_{\text{therm}}, \mathcal{R}_\lambda] \not\approx 0 \qquad (\chi \to 1).$$

The thermodynamic projection and gravitational projection respond differently to coarse-graining near the transition because they track different aspects of admissibility dispersion — local trajectory averaging versus large-scale obstruction growth, respectively. Their divergence is therefore not a failure of the framework but a prediction: it identifies the admissibility phase transition as the point at which entropy-channel ordering becomes physically meaningful.

Remark 4.3. The non-commutativity $[\Pi_i, \mathcal{R}_\lambda] \neq 0$ near $\chi \to 1$ is the entropy- theoretic manifestation of the same breakdown captured geometrically by dispersion obstruction (Definition 2.6). Both say the same thing from different angles: coarse-graining and local projection cease to commute when reconstruction factorization fails. This convergence of geometric and entropic descriptions onto the same transition criterion is a structural consistency check.

4.5 The Kastner–Schlatter Contrast

Kastner and Schlatter [1] identify Shannon information entropy with thermodynamic entropy as a foundational step, using this identification to drive the derivation of gravitational dynamics. Within RSVP, this identification is approximately valid when $\chi \ll 1$ (Theorem-Schema 4.1) and fails as a foundational assumption precisely in the regimes of greatest physical interest: the weak-gradient, low-density regimes where MOND-like behavior appears and χ approaches unity. Founding the derivation on entropy identification in those regimes imports an approximation as an axiom exactly where the approximation breaks down.

4.6 Bridge to Section 5

The existence of persistent entropy-channel divergence without local dynamical inconsistency suggests that globally anomalous galactic behavior need not arise from missing mass distributions. Recall Remark 2.4: the reconstruction cover is a derived object of the field geometry, not a background against which the field is defined. Anomalies in the global extension structure of the cover are anomalies in the geometry itself, not in the matter content of a pre-given space.

5 Dark Matter as Reconstruction Obstruction

5.1 The Observational Problem

The observational signatures attributed to dark matter are well established empirically: flat galactic rotation curves [10, 9], weak gravitational lensing excesses, cluster dynamics requiring additional gravitational sources, and large-scale structure formation rates exceeding baryonic predictions.

Standard cosmology interprets these anomalies as evidence for additional unseen mass distributions embedded within a fixed spacetime background. Within RSVP, the anomalies are instead interpreted as *signatures of obstruction in the global extension structure of the reconstruction cover*. As established in Remark 2.4: the reconstruction cover is a derived object of the field geometry, not a background against which the field is defined. Anomalies in the global extension structure of the cover are anomalies in the geometry itself.

5.2 Compatibility and Dispersion Obstruction in Galactic Dynamics

Compatibility obstruction (Definition 2.5) would manifest as local dynamical inconsistency — irregular, patchy anomalies without clear scaling structure. The observational signatures of galactic anomalies do not have this character. Galaxies are locally stable and internally coherent; their stellar and gas dynamics are well described by standard hydrodynamics and Newtonian gravity at small radii.

The anomaly appears in the *persistence* of those dynamics under large-scale reconstruction. This is structurally closer to dispersion obstruction (Definition 2.6). Local sections remain formally compatible across neighboring regions; the failure is not disagreement but persistence failure under scale extension: as χ approaches unity near R_A, local reconstruction dynamics lose scale-independence and can no longer be extended stably without cosmological boundary data. The point is not that galaxies behave inconsistently. The point is that their local reconstruction dynamics fail to remain self-sufficient under cosmological extension.

5.3 Rotation Curves as Persistence Failure

In strongly factorizable regimes ($\chi \ll 1$), orbital dynamics are governed predominantly by local continuation accessibility, and the admissible trajectory set is closed under local extension — the regime in which Newtonian dynamics holds as an emergent approximation. As dispersion obstruction grows and χ approaches unity near R_A, local continuation sets cease to remain closed under scale extension. The admissible trajectory structure at galactic radii approaching R_A is no longer determined by local field data alone — it requires cosmological gluing data for stable continuation. The effective admissible trajectory set therefore departs from Newtonian closure even in the absence of additional local mass.

The transition criterion $\chi \to 1$ marks the onset of this regime — the same criterion governing the admissibility phase transition (Section 3) and the entropy-channel divergence (Section 4). The unification of all three phenomena under the same control parameter is the framework's strongest structural feature.

Observation 5.1. *The anomaly is a geometric phase transition.* Flat rotation curves, MOND-like scaling, and entropy-channel divergence are manifestations of the same underlying transition in the factorization structure of admissible reconstruction, controlled by $\chi \to 1$ near R_A.

5.4 Weak Lensing and Extension Geometry

Within RSVP, photon propagation probes the continuation accessibility geometry of Φ rather than merely local mass density. In strongly factorizable regimes, continuation accessibility tracks local mass distributions closely and lensing reduces approximately to standard gravitational lensing. In regimes approaching the admissibility phase transition, the continuation accessibility geometry becomes influenced by obstruction structure in the reconstruction cover.

Open Proof Obligation 5.2. Compute the effective lensing potential from the obstruction curvature of $\mathcal{F}$ in the weak-gradient limit and compare with observed convergence maps, including Bullet Cluster phenomenology.

The framework identifies this as a possible reinterpretation of lensing anomalies in terms of obstruction curvature. Whether the resulting phenomenology quantitatively reproduces observed lensing profiles remains an open problem. The Bullet Cluster remains a decisive empirical challenge: the framework must reproduce the observed lensing structure through obstruction curvature of the reconstruction geometry. The restraint here is deliberate; this target is stated honestly rather than rhetorically dismissed.

5.5 Entropy Divergence as Observational Signal

Regions dominated by dispersion obstruction should exhibit increasing divergence between informational, thermodynamic, configurational, and gravitational entropy measures. Gravitational entropy, tracking obstruction contributions, should depart from thermodynamic entropy of local baryonic processes in exactly the regimes where rotation curve anomalies appear. Standard ΛCDM does not predict entropy-channel divergence as a geometric signal; RSVP predicts that such divergence is a structural signature of the admissibility phase transition.

Open Proof Obligation 5.3. Derive explicit expressions for the gravitational entropy contribution of reconstruction obstruction in the weak-gradient RSVP limit and determine whether predicted divergence onset correlates with observed rotation curve transition radii.

5.6 Situating the Reinterpretation

The RSVP reinterpretation differs structurally from all three major competing programs. Particle dark matter models preserve local factorization while adding unseen sources. Entropic gravity programs [3, 6] preserve entropy identification while modifying effective dynamics. MOND and TeVeS [7, 8] modify the force law through fitted interpolation functions. RSVP instead interprets galactic anomalies as signatures of breakdown in scale-independent reconstruction itself — the onset of dispersion-dominated extension dynamics controlled by χ approaching unity near R_A (Observation 5.1).

6 Comparison with Neighboring Programs

6.1 Organizing Principle

The most useful axis for comparing competing approaches is derivation architecture: what object is treated as primitive, what undergoes transition, what role entropy plays, and what counts as an anomaly.

Framework	**Primitive**	**Anomaly**	**Entropy**	**Locality**
ΛCDM	Matter source	Missing source	Identified	Local field
MOND/TeVeS	Modified force	Wrong dynamics	Secondary	Modified local
Verlinde	Entropy equiv.	Incomplete thermo.	Axiom	Holographic
Trans. entropic	Exchange completion	Incomplete thermo.	Interchangeable	Local exchange
RSVP	Admissibility geometry	Phase transition	Regime projection	Sheaf-indexed

Table 1: Comparative taxonomy organized by derivation architecture.

6.2 Standard Cosmology (ΛCDM)

ΛCDM treats matter source distributions within a fixed pseudo-Riemannian background as primitive. What RSVP interprets as obstruction structure in a derived cover, ΛCDM interprets as source deficits within a fixed background. These are not empirically equivalent restatements: they predict different entropy-channel signatures, different lensing phenomenology in obstruction-dominated regimes, and different structure formation dynamics. RSVP constrains the anomaly structure through χ and R_A; the phenomenology is not freely adjustable but is derived from field dynamics.

6.3 MOND and TeVeS

MOND [7] reproduces flat rotation curves and the baryonic Tully-Fisher relation [10] with remarkable precision using a single additional parameter. Within the present framework, MOND is demoted from foundational principle to phenomenological signature. The MOND acceleration scale $a_0 \sim cH_0$ is not a new fundamental constant but a derived geometric scale — the acceleration associated with the onset of cosmologically coupled reconstruction at R_A. MOND-like dynamics appear when $\chi \to 1$ near R_A, not because the force law has been modified but because the admissible continuation set has changed.

The observed robustness of the baryonic Tully-Fisher relation [10] across galaxy types, which is difficult to understand in ΛCDM and built into MOND by construction, would in RSVP reflect universality of the admissibility phase transition under coarse-graining — an emergent property of the reconstruction geometry rather than a tuned interpolation (see Appendix E).

6.4 Verlinde's Emergent Gravity

Verlinde [2, 3] begins with entropy identification and derives effective geometry. RSVP begins with admissibility geometry and derives the conditions under which entropy species approximately converge (Theorem-Schema 4.1). Verlinde's framework inherits the entropy-identification problem [20] as a foundational commitment. RSVP converts this axiom into a theorem-schema valid in strongly factorizable regimes and failing in precisely the regimes — $\chi \to 1$ — where anomalous dynamics appear.

6.5 Transactional Entropic Gravity

Kastner and Schlatter [1] provide the most immediately adjacent comparison. Both programs reject fundamental spacetime; both treat geometric structure as emergent. The divergence is in derivation order and primitive object.

Transactional entropic gravity treats exchange process completion as primitive — spacetime links are generated by completed transactions between irreducible emitters and absorbers. RSVP treats admissibility geometry as primitive — emitters, absorbers, and exchange processes are stabilized roles within the reconstruction sheaf.

Three structural consequences follow: (1) the transactional MOND derivation depends on entropy-species identification at specific steps; the RSVP conjecture depends on a single geometric condition ($\chi \to 1$ at R_A) controlling all phenomena simultaneously; (2) transactional completion is strictly weaker than sheaf-compatible global extension — a transaction can complete locally while generating globally inadmissible dynamics; (3) entropy-species interchangeability appears in RSVP as an approximate consequence of strongly factorizable reconstruction, valid in the Newtonian regime and failing in the MOND regime.

RSVP does not currently outperform transactional entropic gravity quantitatively. The advantage claimed is architectural: fewer primitive objects controlling more phenomena through a single transition structure, with entropy convergence as a derived consequence.

6.6 The Inversion as Theoretical Commitment

The organizing inversion — admissibility geometry generates exchange structure, not vice versa — reflects a specific theoretical bet about where the explanatory primitives of gravitational physics are located. If the bet is correct, existing phenomenology should emerge as special cases of reconstruction geometry. If wrong, the transactional program has the correct derivation order and RSVP's sheaf structure is descriptive rather than explanatory. The paper makes the question precise and identifies the geometric objects on which it turns.

7 Minimal Assumptions and Irreducible Commitments

The RSVP framework has grown sufficiently rich that it is important to distinguish which assumptions are load-bearing from which are illustrative scaffolding. This section identifies the irreducible commitments of the theory — the propositions that, if abandoned, would dissolve the framework rather than merely simplify it.

(I) Admissibility geometry is primary. The reconstruction cover $\mathcal{U}$, admissibility classes, and the sheaf $\mathcal{F}$ are not derived from a more primitive physical ontology of particles, fields on fixed spacetime, or exchange processes. They are the foundational objects from which physical roles (emitter, absorber, particle, clock, force) emerge as stabilized reconstruction modes. This commitment is what distinguishes RSVP from transactional and holographic programs.

(II) The reconstruction cover is derived, not background. There is no pre-given manifold against which the field evolves. The topology of admissibility neighborhoods is determined dynamically by $\chi = S/\Phi$. Classical spacetime, to the extent it appears, is a stable low-χ regime of the cover, not its presupposition (Remark 2.4).

(III) Coarse-graining modifies factorization structure. Under renormalization of the reconstruction cover, what changes is not merely effective couplings (as in standard EFT) but the factorization structure of admissible reconstruction itself. This is what allows admissibility nonlocality to appear without modifying local propagation (Appendix G).

(IV) Entropy convergence is a regime consequence, not an axiom. The approximate identification of entropy species holds when $\chi \ll 1$ and breaks down as $\chi \to 1$. This is a prediction, not a foundational assumption (Theorem-Schema 4.1, Section 4.4).

(V) Obstruction is dynamical, not merely topological. The distinction between compatibility and dispersion obstruction (Definitions 2.5, 2.6) is essential: obstruction classes can grow under coarse-graining even when they are trivializable at a fixed scale. This dynamical obstruction is what the framework uses to interpret galactic anomalies.

Everything else in the paper — the candidate dynamical equations of Appendix C, the specific form of the beta-function in Appendix B, the detailed structure of the Lean schemas in Appendix H — is illustrative rather than irreducible. Those elements demonstrate that the five commitments above are compatible with coherent dynamics and formal implementation, but they do not exhaust the space of theories satisfying (I)–(V).

Remark 7.1. The minimality of commitments (I)–(V) is itself a structural feature. A framework whose central claims reduce to five propositions, from which all major results are derived, is more constrained than one requiring independent axioms for each phenomenon. The recurrence of χ, R_A, and the obstruction classes across Sections 2–5 is evidence that the commitments are not merely compatible but actively generative.

8 The Great Inversion and the Ontological Status of Physical Law

A persistent ambiguity in contemporary approaches to gravity concerns the ontological ordering between microscopic processes and macroscopic geometry. In most emergent gravity programs, local interactions, thermodynamic exchanges, or entanglement structures are treated as fundamental, with spacetime geometry reconstructed as a large-scale byproduct. The RSVP framework reverses this ordering. Admissibility geometry is not derived from physical exchange; rather, physical exchange appears only insofar as it is permitted as a stable reconstruction mode within a pre-existing geometric admissibility structure.

This inversion eliminates a hidden circularity present in many bottom-up constructions. If local exchanges are taken as primitive, one must still specify which exchanges are allowed to persist across time, and under what conditions they remain stable. These persistence conditions are often introduced implicitly through thermodynamic or informational assumptions. In the RSVP framework, such assumptions are elevated to the level of geometric constraint. The admissibility structure determines which continuations are permitted, and physical processes are those continuations that achieve stable realization.

Under this interpretation, particles are not fundamental objects in the usual sense. They are persistent resonant modes within the admissibility geometry: localized configurations where continuation accessibility and coherence flow align sufficiently to maintain stability under extension. The apparent solidity of matter reflects a regime of strong reconstruction persistence rather than a primitive ontology. Observational anomalies are therefore not interpreted as evidence for missing entities within a fixed background, but as indications that the reconstruction structure itself has entered a regime where its factorization properties fail.

Traditional physical theories typically begin by specifying primitive entities, fields, background geometry, and dynamical laws governing their interaction. The RSVP framework instead begins from admissible continuation, persistence under extension, compatibility of reconstruction, and obstruction constraints governing global coherence. This constitutes a shift from ontology-first physics toward constraint-first physics. Physical law becomes increasingly interpretable as constraint structure governing globally persistent continuation rather than as a description of material substance.

The recurring mathematical objects of the framework — χ, R_A, Ω_ℓ, $\mathcal{F}$, and the entropy projection functors — all function primarily as constraints on admissible extension rather than as descriptions of substance. Sheaf theory is fundamentally concerned with conditions under which local consistency extends to global coherence, and that is precisely the structural role admissibility plays within RSVP. The framework therefore increasingly resembles a generalized consistency geometry: its central objects are not material constituents but admissibility relations, extension constraints, persistence criteria, and obstruction structures governing which reconstructions can stabilize across scale.

9 The Control Parameter and the Emergence of Physical Regimes

The dynamical behavior of the framework is governed not by the individual fields Φ, $\mathbf{v}$, or S in isolation, but by the dimensionless ratio $\chi = S/\Phi$. This ratio functions as a regime-defining parameter that determines the structural character of admissible reconstruction. Its role is analogous to that of an order parameter, but it governs the factorization properties of reconstruction rather than the phase of a material system.

When $\chi \ll 1$, admissibility dispersion is negligible relative to continuation accessibility. Admissible trajectories concentrate around stable modes, and local sections extend without requiring information from larger scales. The resulting structure is strongly factorizable: local dynamics close under extension, and the behavior of physical systems can be determined from local data alone. Classical Newtonian dynamics and general relativity emerge as effective descriptions precisely in this regime. Their apparent universality reflects the dominance of accessibility over dispersion, not a fundamental status.

As χ approaches unity, this structure degrades. The dispersion of admissible continuations becomes comparable to the accessibility itself, and local reconstruction loses closure under extension. The breakdown is structural rather than dynamical: the governing equations do not change, but the conditions under which local solutions extend consistently are no longer satisfied. This transition explains the empirical breakdown of classical dynamics at galactic scales without requiring any modification of the underlying force laws. The laws remain intact; the regime in which they provide a closed description has ceased to apply.

This reframes the MOND phenomenology. Rather than representing a modification of Newtonian dynamics, the MOND transition marks the boundary between regimes of admissibility geometry. The apparent change in behavior reflects a transition in the structure of admissible continuations rather than a change in the dynamical equations themselves. The acceleration scale $a_0 \sim cH_0$ is not a new constant of nature but the scale associated with the breakdown of local reconstruction closure — a geometric threshold determined by the admissibility structure of Φ and S.

10 Observable Proxies for Reconstruction Geometry

A persistent difficulty for any reconstruction-theoretic framework is the distinction between formally meaningful geometric objects and operationally measurable quantities. The RSVP program introduces objects such as admissibility dispersion, persistence obstruction, and reconstruction curvature whose mathematical role is structurally clear while their observational realization remains incomplete. This section distinguishes sharply between primitive reconstruction objects and their possible empirical proxies, and in doing so identifies the operational bridge that a successful completion of the theory must eventually construct.

The coupled field $X = (\Phi, \mathbf{v}, S)$ is not assumed to be directly observable. Observations access only coarse-grained effective projections of admissibility structure. Consequently, the relevant empirical question is not whether Φ, χ, or Ω_ℓ are directly measurable, but whether stable observational signatures exist whose behavior systematically tracks their reconstruction dynamics.

The control parameter $\chi = S/\Phi$ is interpreted operationally as a measure of reconstruction instability under scale extension. Since neither S nor Φ is independently observable in the present formulation, χ must be inferred indirectly through signatures of factorization breakdown. Candidate observational proxies include the onset radii of rotational persistence anomalies, mismatch between baryonic predictions and large-scale orbital stability, environment-sensitive deviations from MOND interpolation structure, and entropy-channel divergence between thermodynamic and gravitational reconstruction measures.

The obstruction functional $\Omega_\ell[s]$ should not be interpreted as directly measurable cohomological data. Instead, Ω_ℓ represents an effective obstruction intensity whose observable manifestations may include weak lensing residuals after baryonic subtraction, persistence instability under environmental perturbation, anomalous scaling behavior near galactic transition radii, and cluster-scale reconstruction mismatch.

The framework therefore predicts not merely modified dynamics but systematic correlations between χ, Ω_ℓ, entropy-channel divergence, and persistence stability. The existence or nonexistence of these correlations constitutes an empirical test of the reconstruction interpretation.

Remark 10.1. The present framework does not yet provide a completed observational dictionary mapping reconstruction geometry to astrophysical observables. The purpose of this section is narrower: to identify the operational bridge that a successful completion of the theory must eventually construct. A framework that can specify what would count as empirical support or falsification, even before the measurement is possible, is structurally more mature than one whose observational content remains entirely implicit.

11 Admissibility Selection Principles and Forbidden Continuation Classes

One of the principal risks of generalized reconstruction frameworks is excessive permissiveness. If every sufficiently smooth local continuation can be rendered globally admissible through appropriate reinterpretation of the reconstruction cover, then obstruction loses predictive force and the framework degenerates into descriptive topology. The RSVP program therefore requires a distinction between formal constructibility and persistent admissibility.

Not every mathematically definable reconstruction corresponds to a physically persistent reconstruction mode. A continuation structure must satisfy increasingly restrictive admissibility criteria under extension and coarse-graining flow. We therefore distinguish the following hierarchy: constructible configurations form the broadest class; locally admissible configurations must additionally satisfy bounded perturbation stability; globally extendable configurations must also cohere across the full reconstruction cover without compatibility obstruction; and persistent configurations must furthermore survive renormalization of the cover without dispersion obstruction growth exceeding coherence transport capacity.

Definition 11.1 (Forbidden continuation class)**.** A continuation class is *forbidden* if no admissible coarse-graining flow preserves reconstruction persistence under extension beyond finite scale. Equivalently, every admissible extension flow eventually drives the persistence functional $\mathcal{P}[s]$ to zero.

Forbidden continuation classes represent geometries of reconstruction that cannot stabilize as globally persistent physical regimes. The framework predicts that certain classes of admissibility structure are dynamically excluded regardless of their local formal consistency. Candidate forbidden classes include runaway dispersion geometries with unbounded entropy-channel divergence, obstruction cascades whose growth exceeds coherence transport capacity, non-factorizable continuation structures lacking stable attractor behavior, and reconstruction flows possessing no bounded admissibility fixed point.

This exclusion structure is essential. Mature physical theories derive much of their predictive strength not from explanatory flexibility but from impossibility structure. The RSVP program therefore increasingly shifts from ontology-generation toward admissibility exclusion. The transition from descriptive admissibility geometry to exclusion geometry marks the most important maturation step available to the framework: a successful reconstruction theory must eventually identify not only what can persist, but what cannot.

12 Known Structural Risks and Failure Modes

The RSVP framework remains incomplete and structurally vulnerable in several important respects. Naming these risks explicitly is important both for conceptual clarity and for preventing the framework from expanding into unconstrained abstraction. A framework that understands its own fragility structure is structurally more mature than one that presents only its successes.

The most significant risk is *semantic overextension.* The framework employs a common vocabulary — admissibility, reconstruction, persistence, obstruction, continuation, and projection — across gravitational, thermodynamic, informational, and geometric domains. This unification is potentially powerful, but creates a risk that the same concepts become sufficiently elastic to redescribe phenomena without generating unique constraints. The framework therefore lives or dies by whether the same small set of structures — χ, R_A, Ω_ℓ, and the reconstruction sheaf — repeatedly generate nontrivial predictions across independent domains rather than functioning as a universal explanatory solvent.

A related risk is *universality inflation.* The paper repeatedly suggests that distinct microscopic reconstruction dynamics may flow toward common large-scale admissibility behavior under coarse-graining. While this possibility is motivated by ordinary renormalization theory, the present work does not yet derive the relevant admissibility operator algebra, identify the relevant versus irrelevant operators governing admissibility flow, or prove the existence of universality classes for reconstruction dynamics. The universality hypothesis therefore remains speculative, and a critic asking why admissibility flow should possess universality classes at all has not yet been given a satisfying answer.

The framework also faces *observational opacity.* Many central objects remain operationally indirect, and the theory currently lacks a completed observational dictionary relating admissibility quantities to uniquely measurable astrophysical observables. Without such a dictionary, the framework risks remaining geometrically expressive but empirically underdetermined. The proxies identified in Section 10 are a first step toward resolving this, but they remain candidate mappings rather than completed operational definitions.

Coarse-graining ambiguity represents a fourth structural vulnerability. The admissibility coarse-graining operator $\mathcal{R}_\lambda$ is not yet rigorously defined. The framework draws analogies from Wilsonian renormalization while modifying the factorization structure of admissible continuation itself, but has not yet specified what degrees of freedom are integrated out, how reconstruction neighborhoods evolve under flow, or what invariants are preserved. The admissibility coarse-graining operation is not assumed to coincide with ordinary Wilsonian integration over momentum shells — it acts instead on the factorization structure of admissible continuation itself — but until the operator is rigorously constructed, the beta-function formalism of Appendix B remains heuristic.

Insufficient exclusion structure is a fifth risk. If admissibility conditions remain too permissive, the framework risks collapsing into generalized reconstruction semantics in which every phenomenon can be redescribed in admissibility language after the fact. The introduction of forbidden continuation classes in Section 11 is intended as a step toward resolving this, but the relevant exclusion theorems remain undeveloped.

Finally, the *categorical status* of the reconstruction sheaf remains unresolved. It is currently unclear whether the admissibility cover should ultimately be interpreted as a topological cover over emergent spacetime, a continuation-state site, a reconstruction category, or a more general higher-categorical admissibility structure. The present framework remains intentionally agnostic on this question, but a reader trained in sheaf theory will eventually demand an answer to the question of what category these sheaves actually live over.

Remark 12.1. These risks should not be interpreted merely as weaknesses. They identify the exact structural obligations whose resolution would determine whether RSVP matures into a genuine physical reconstruction theory or remains a descriptive geometric framework. A framework that can name the conditions under which it would fail is structurally more credible than one whose explanatory architecture can indefinitely absorb anomaly through reinterpretation.

13 Criteria for a Successful RSVP Completion

Speculative frameworks often fail to define what successful completion would actually look like, allowing the program to defer resolution indefinitely into abstraction. The RSVP program therefore requires explicit completion criteria that give the framework a visible finish line.

A successful reconstruction-theoretic completion of RSVP would minimally require a derived admissibility beta-function from explicit RSVP field dynamics; a rigorous definition of admissibility coarse-graining specifying what degrees of freedom are integrated out and what invariants are preserved; operational observables corresponding to χ and obstruction structure; a derivation of $R_A \propto c/H_0$ from weak-gradient reconstruction flow without parameter fitting; recovery of Einstein-like behavior in strongly factorizable regimes as an emergent consequence rather than a postulate; quantitative lensing predictions from obstruction curvature capable of distinguishing the framework from dark matter models on Bullet Cluster scales; proof or refutation of entropy-channel convergence under strong factorization; classification of forbidden continuation classes with accompanying exclusion theorems; and demonstration that the framework generates unique empirical constraints rather than post-hoc reinterpretations.

More broadly, the framework must eventually answer three foundational questions. Which reconstruction geometries persist? Which reconstruction geometries are forbidden? Which observable structures uniquely distinguish admissibility geometry from ordinary matter-source explanations? The success or failure of the RSVP program should ultimately be judged not by the breadth of its interpretive vocabulary but by its ability to constrain continuation structure, generate exclusion geometry, and produce empirically distinguishable predictions.

Remark 13.1. A framework that can state the conditions under which it would fail is structurally more mature than one whose explanatory architecture can indefinitely absorb anomaly through reinterpretation. The completion criteria above are not aspirational rhetoric but genuine falsifiability conditions: if the framework cannot eventually satisfy them, the reconstruction interpretation should be abandoned in favor of programs with stronger exclusion structure.

14 Conclusion

14.1 The Central Reinterpretation

The present work develops a reinterpretation of gravitational anomalies organized around a single foundational inversion. Existing emergent gravity programs treat physical exchange, thermodynamic bookkeeping, or information flow as primitive and derive spacetime geometry from them. RSVP treats admissibility geometry as primary and interprets persistent physical exchange as a stabilized reconstruction mode. Emitters, absorbers, particles, force laws, and thermodynamic entropy are not inputs to the framework but outputs — roles stabilized by recursive cross-scale coherence of the coupled field $X = (\Phi, \mathbf{v}, S)$ within its reconstruction sheaf.

The framework's explanatory structure is organized by a small set of recurring mathematical objects: the coupled field X; the control parameter $\chi = S/\Phi$; the reconstruction cover $\mathcal{U}$; the admissibility radius R_A; the compatibility and dispersion obstruction classes;

and the entropy projection functors $\Pi_{\text{therm}}, \Pi_{\text{info}}, \Pi_{\text{grav}}, \Pi_{\text{conf}}$. The control parameter χ governs simultaneously: Newtonian gravitational stability, the MOND-like transition, entropy-channel convergence, and galactic obstruction phenomenology. This unification under a single order parameter is the strongest evidence of internal coherence.

14.2 What the Paper Claims and Does Not Claim

The paper does not present: a completed derivation of MOND from RSVP field equations; a quantitative alternative to ΛCDM; or a solved lensing model capable of reproducing Bullet Cluster phenomenology.

The paper does present: a reconstruction-geometric reinterpretation of gravitational anomalies organized by a stable conceptual hierarchy; formal definitions of admissibility geometry, reconstruction cover, and obstruction types; a constrained and explicitly falsifiable conjecture (Conjecture 3.4) with three named failure modes; Theorem-Schema 4.1 converting an axiom of neighboring programs into a derived consequence; and a comparative taxonomy organized by derivation architecture (Table 1).

> The present work does not derive MOND or dark matter phenomenology from completed RSVP field equations; it identifies the geometric objects from which such derivations would have to proceed.

14.3 Open Problems as Evidence of Structural Coherence

Most speculative frameworks accumulate explanatory flexibility. The present framework instead generates constrained open problems whose resolution would confirm or falsify its central commitments:

(1) **Admissibility radius derivation** (Obligation 3.6). Show that the weak-gradient RSVP equations uniquely determine $R_A \propto c/H_0$.

(2) **Universality class structure.** Determine whether distinct admissibility geometries flow toward the same weak-gradient reconstruction behavior under coarse-graining, making MOND phenomenology geometrically necessary.

(3) **Lensing derivation target** (Obligation 5.2). Compute the effective lensing potential from obstruction curvature of $\mathcal{F}$ and compare with observed convergence maps.

(4) **Entropy divergence signature** (Obligation 5.3). Identify observational contexts where gravitational entropy departs measurably from thermodynamic entropy of local baryonic processes.

(5) **Obstruction flow analysis.** Determine whether compatibility and dispersion obstruction produce distinguishable galactic signatures varying systematically with galaxy morphology, surface brightness, or environment.

These problems are obligations generated by the framework's own internal structure. Their existence as constrained derivation targets — rather than free parameters or post-hoc interpretive choices — is evidence that the framework has crossed the threshold where speculative architecture begins behaving as a research program. A framework that knows the shape of its own incompleteness is structurally more mature than one that knows only the shape of its successes.

14.4 Final Philosophical Position

If the RSVP program is correct, then gravity, inertia, dark matter phenomenology, and entropy equivalence are not independent physical ingredients assembled into a cosmological model. They are stable regimes and failure modes of reconstruction geometry — consequences of the factorization structure of admissible continuation under a derived cover whose topology is controlled by a single dimensionless parameter.

Whether this inversion reflects physical reality remains unresolved. The present work aims only to make the inversion mathematically explicit, empirically constrained, and structurally falsifiable.

A Formal Structure of the Reconstruction Sheaf

This appendix develops the formal sheaf-theoretic structure underlying the reconstruction framework. The purpose is not to introduce new physical claims but to make explicit the mathematical objects already used operationally throughout Sections 2–5.

A.1 Presheaf and Sheaf Structure

Let $\mathcal{R}$ denote the reconstruction domain and $\mathcal{U} = \{U_\alpha\}$ a cover by admissibility neighborhoods. The *reconstruction presheaf* $\mathcal{F}$ assigns to each U_α a set $\mathcal{F}(U_\alpha)$ of admissible sections of X over U_α, together with restriction maps

$$\rho^U_V \colon \mathcal{F}(U) \longrightarrow \mathcal{F}(V), \qquad V \subseteq U,$$

satisfying $\rho^U_U = \mathrm{id}$ and $\rho^V_W \circ \rho^U_V = \rho^U_W$ for $W \subseteq V \subseteq U$.

$\mathcal{F}$ is a *sheaf* if it satisfies the following for every open cover $\{U_i\}$ of any U:

(i) *Locality:* if $\rho^U_{U_i}(s) = \rho^U_{U_i}(t)$ for all i, then $s = t$.

(ii) *Gluing:* if $\rho^{U_i}_{U_i \cap U_j}(s_i) = \rho^{U_j}_{U_i \cap U_j}(s_j)$ for all i, j, then there exists $s \in \mathcal{F}(U)$ with $\rho^U_{U_i}(s) = s_i$.

Compatibility obstruction (Definition 2.5) corresponds to failure of condition (ii): sections cannot be glued because they disagree on overlaps. Dispersion obstruction (Definition 2.6) is subtler: sections satisfy (ii) formally but the glued section fails reconstruction persistence under coarse-graining because admissibility dispersion increases faster than coherence propagation.

A.2 Admissible Sections and Persistence

An admissible local section is not merely a field restriction but a structured pair consisting of: a local restriction of the coupled field

$$X = (\Phi, \mathbf{v}, S)$$

to an admissibility neighborhood U_α, together with a continuation structure specifying the admissible extension behavior of reconstruction trajectories originating within U_α.

Formally, a section

$$s_\alpha \in \mathcal{F}(U_\alpha)$$

is admissible only if two conditions are simultaneously satisfied. First, local perturbations of the section must preserve bounded continuation behavior, so that small deformations of initial reconstruction data do not generate uncontrolled trajectory divergence. Second, admissible extensions of the section must remain stable under renormalization of the reconstruction cover. The section must therefore persist not only locally but under scale-dependent restructuring of admissibility neighborhoods induced by coarse-graining flow.

This second condition distinguishes reconstruction persistence from ordinary local field consistency. A section may satisfy all local field equations and remain formally compatible on overlaps while nevertheless failing persistence under extension because admissibility dispersion grows faster than coherence propagation under cover renormalization.

A global reconstruction mode corresponds to a global section

$$s \in \mathcal{F}(\mathcal{R})$$

whose restrictions agree coherently across the full reconstruction cover and whose continuation structure remains stable under admissible coarse-graining flow. Persistence is therefore a multiscale extension property of the reconstruction sheaf rather than merely a statement of local dynamical consistency.

A.3 Persistence Functional

We define a *persistence functional*

$$\mathcal{P}[s] \;\in\; [0,1]$$

measuring stability of admissible extensions under renormalization of the reconstruction cover. A section satisfies reconstruction persistence if $\mathcal{P}[s]$ remains bounded away from zero under scale extension. Dispersion obstruction occurs when $\mathcal{P}[s] \to 0$ under coarse-graining, even for sections satisfying the standard gluing conditions.

A.4 Dynamical Cover and Gluing

Unlike ordinary differential-geometric frameworks in which the manifold is fixed and fields evolve upon it, the RSVP framework treats the effective reconstruction topology as dependent upon admissibility flow. In RSVP, gluing conditions are dynamically weighted by admissibility dispersion and continuation accessibility rather than determined solely by overlap consistency. The resulting structure combines: local field dynamics, scale-dependent admissibility flow, and global extension stability into a single reconstruction structure.

The following commutative diagram summarizes the restriction structure for a triple cover:

$$\begin{tikzcd}
\mathcal{F}(U_\alpha) \arrow[r, "\rho_{\alpha\beta}"] \arrow[dr, "\rho_{\alpha\gamma}"'] & \mathcal{F}(U_\alpha \cap U_\beta) \arrow[d, "\rho_{\alpha\beta,\gamma}"] \\
 & \mathcal{F}(U_\alpha \cap U_\beta \cap U_\gamma)
\end{tikzcd}$$

Consistency of this diagram for all triples is the cocycle condition whose failure generates compatibility obstructions classified by $\check{H}^1(\mathcal{U}, \mathcal{F})$.

A.5 Constraint Propagation and Reconstruction Stability

Admissibility is defined locally but must persist globally. A key missing link between the sheaf language and the field dynamics is an account of how local admissibility violations propagate under coarse-graining. We sketch the required structure here.

Define a *reconstruction constraint functional*

$$\mathcal{C}[X] \;=\; 0$$

whose vanishing characterizes admissible field configurations. In the candidate dynamics of Appendix C, this constraint is approximately realized when $\chi \ll 1$ and reconstruction trajectories are strongly localized near stable modes. The constraint is violated as $\chi \to 1$.

Define *propagation operators* at scale ℓ:

$$\mathcal{T}_\ell \colon \mathcal{F}(U_\ell) \;\longrightarrow\; \mathcal{F}(U_{\lambda\ell}), \qquad \lambda > 1,$$

describing how admissible sections at scale ℓ map to sections at the coarser scale $\lambda\ell$. Reconstruction stability requires that the constraint propagates covariantly:

$$\mathcal{T}_\ell\big(\mathcal{C}[X]\big) \;=\; 0$$

whenever $\mathcal{C}[X] = 0$ at the finer scale. Failure of this condition — a section satisfying $\mathcal{C} = 0$ locally but $\mathcal{T}_\ell(\mathcal{C}) \neq 0$ at the coarser scale — is precisely dispersion obstruction: local admissibility that does not propagate stably under renormalization of the cover.

The propagation operators $\{\mathcal{T}_\ell\}$ form a directed system. Their composition law

$$\mathcal{T}_{\lambda\ell} \circ \mathcal{T}_\ell \;=\; \mathcal{T}_{\lambda^2\ell}$$

is a consistency condition analogous to the cocycle condition for restriction maps. A complete theory of constraint propagation would derive the $\mathcal{T}_\ell$ from the field equations of Appendix C and show that constraint violation first appears at $\ell \sim R_A$ — formalizing the admissibility phase transition as a constraint-propagation threshold rather than merely a qualitative change in reconstruction regime.

Open Proof Obligation A.1. Derive the propagation operators $\mathcal{T}_\ell$ from the coupled dynamics of Φ, $\mathbf{v}$, and S and show that $\mathcal{T}_\ell(\mathcal{C}) \neq 0$ first appears at $\ell \sim R_A \propto c/H_0$ in weak-gradient regimes. This would convert the admissibility phase transition from a geometric threshold into a propagation-failure theorem.

B Coarse-Graining Flow and the Admissibility Order Parameter

The main text introduced $\chi = S/\Phi$ as the central control parameter. This appendix develops a schematic coarse-graining framework and clarifies the sense in which the admissibility phase transition may possess universality structure analogous to ordinary critical phenomena [18, 19].

B.1 Scale-Dependent Fields and Order Parameter

Let $\mathcal{U}_\ell$ denote the reconstruction cover at coarse-graining scale ℓ. Integrating out short-scale reconstruction modes modifies both continuation accessibility and admissibility dispersion:

$$\Phi \;\longrightarrow\; \Phi_\ell, \qquad S \;\longrightarrow\; S_\ell.$$

The scale-dependent order parameter is

$$\chi_\ell \;=\; \frac{S_\ell}{\Phi_\ell}.$$

The Newtonian regime ($\chi_\ell \ll 1$) corresponds to strongly factorizable reconstruction; the admissibility transition occurs when $\chi_\ell \to 1$.

B.2 Admissibility Beta-Function

The coarse-graining operation in RSVP is not assumed to coincide with ordinary Wilsonian integration over momentum shells. Wilsonian coarse-graining integrates out short-wavelength field modes while preserving the background spacetime manifold. Informational coarse-graining aggregates over microstates while preserving statistical ensemble structure.

Admissibility-cover renormalization instead acts on the factorization structure of admissible continuation itself: it merges reconstruction neighborhoods and asks whether the resulting coarser cover still admits globally persistent sections. These three operations are structurally distinct and should not be conflated. The admissibility beta-function introduced below governs the third operation, not the first two.

The coarse-graining flow of the reconstruction geometry may be represented schematically by the scale evolution equation

$$\frac{d\chi_\ell}{d\ln\ell} = \beta(\chi_\ell), \tag{7}$$

where

$$\chi_\ell = \left.\frac{S}{\Phi}\right|_\ell$$

denotes the scale-dependent control parameter evaluated at coarse-graining scale ℓ, and β is an *admissibility beta-function* governing the renormalization flow of reconstruction stability.

The function β is not specified phenomenologically but constrained by the structural requirements of admissible reconstruction developed throughout the main text. In strongly factorizable regimes,

$$\chi \ll 1,$$

the framework requires

$$\beta(\chi) < 0,$$

so that admissibility dispersion decreases under coarse-graining flow and local reconstruction remains stable under scale extension. This corresponds to the Newtonian regime in which reconstruction neighborhoods retain approximate factorization and admissible sections persist without cosmological gluing data.

As χ approaches unity, the flow enters a crossover region in which dispersion obstruction begins competing with coherence propagation. The qualitative structure of admissible extension changes: local sections cease to remain scale-independent, and persistence under renormalization becomes increasingly sensitive to global reconstruction geometry. The admissibility phase transition introduced in the main text corresponds to this change in flow structure near

$$\chi \sim 1.$$

Beyond the transition, the framework admits two structurally distinct possibilities. Either the flow approaches a new large-scale stable reconstruction regime governed by cosmologically coupled admissibility structure, or local factorization ceases to remain dynamically stable altogether and reconstruction persistence fails beyond finite scale. Which possibility is realized depends on the weak-gradient dynamics of the RSVP field equations and constitutes part of the open derivation program identified in Obligation 3.6.

The admissibility beta-function therefore plays a role analogous to renormalization group flow equations in statistical field theory. It governs not merely numerical coupling evolution but qualitative transitions in the factorization structure of admissible reconstruction itself.

B.3 Universality Classes

Suppose multiple microscopic RSVP field dynamics produce distinct local reconstruction behavior while nevertheless generating the same asymptotic coarse-graining flow for χ_ℓ. Then the weak-gradient galactic regime would exhibit universal phenomenology independent of microscopic reconstruction details — analogous to universality of critical exponents in condensed matter systems [19]. This possibility is particularly important for understanding the empirical robustness of MOND-like scaling relations [10, 9] across diverse galactic systems. Within RSVP, such robustness would emerge as a universality property of admissibility flow near the reconstruction transition rather than a tuned interpolation.

B.4 Admissibility Radius from Flow

The admissibility radius R_A is the characteristic scale at which $\chi_\ell \to 1$ under the coarse-graining flow. Obligation 3.6 is therefore equivalent to showing that flow equation (7), derived from the weak-gradient RSVP field equations, admits a threshold at scale ℓ^* satisfying $e^{\ell^*} \propto c/H_0$.

B.5 Attractor Structure and Stability of Newtonian Reconstruction

The Newtonian regime ($\chi \ll 1$) is not merely a parameter region; it should be an *attractor* of the admissibility flow. This subsection formalizes that requirement.

If the beta-function satisfies $\beta(\chi) < 0$ for $\chi \ll 1$, then the fixed point $\chi^* = 0$ is stable: perturbations of χ away from zero are driven back under coarse-graining. Newtonian reconstruction is therefore an infrared attractor of the admissibility flow in strongly factorizable regimes. This is the geometric meaning of Newtonian gravity's empirical robustness at small scales.

The admissibility phase transition at $\chi \sim 1$ corresponds to the boundary of this attractor basin. For $\chi < 1$, the flow returns to the Newtonian attractor. For $\chi \geq 1$, the flow either enters a new cosmologically coupled basin or diverges, depending on the sign of β beyond the transition.

A minimal consistent beta-function with this attractor structure takes the form:

$$\beta(\chi) \;=\; -\mu\,\chi\,(1-\chi) \;+\; \mathcal{O}(\chi^3)$$

for some $\mu > 0$. This function satisfies:

- $\beta(0) = 0$: the Newtonian fixed point $\chi^* = 0$ is a fixed point.
- $\beta'(0) = -\mu < 0$: the fixed point is stable (attractor).
- $\beta(1) = 0$: the transition point $\chi = 1$ is an unstable fixed point (the admissibility phase boundary).

- $\beta(\chi) < 0$ for $0 < \chi < 1$: the flow drives χ toward zero in the Newtonian regime.

This schematic form is consistent with all qualitative requirements established in the main text. Deriving the actual beta-function from the RSVP field equations — including the value of μ and the structure of higher-order terms — constitutes part of Obligation 3.6.

Remark B.1. If the attractor structure is correct, the universality of Newtonian gravity at small scales is not a fundamental fact but a consequence of reconstruction flow. Deviations from Newtonian behavior at large scales (galactic rotation anomalies) are then not anomalies requiring new matter but signatures of crossing the attractor boundary — the transition from the $\chi^* = 0$ basin to the cosmologically coupled regime.

C Candidate Dynamical Equations for the RSVP Fields

This appendix sketches candidate dynamical equations for the coupled fields $X = (\Phi, \mathbf{v}, S)$. These equations are heuristic and incomplete — candidate structures consistent with the admissibility framework, not finalized physical laws. The purpose is architectural: to show that the reconstruction-geometric interpretation can plausibly be associated with a coherent field-dynamical system.

C.1 Continuation Accessibility Field Φ

A minimal admissibility-flow equation for Φ:

$$\partial_t \Phi \;=\; D_\Phi \nabla^2 \Phi - \lambda_\Phi \Phi S + \gamma_\Phi \nabla \cdot \mathbf{v} + N_\Phi[\Phi, \mathbf{v}, S] \tag{8}$$

where D_Φ is an accessibility diffusion coefficient, λ_Φ governs suppression of continuation accessibility by dispersion growth ($-\lambda_\Phi \Phi S$ captures the central geometric tension: increasing S suppresses stable Φ), γ_Φ couples accessibility to coherence flow divergence, and N_Φ denotes higher-order nonlinear terms.

C.2 Coherence Flow Field v

$$\partial_t \mathbf{v} \;=\; D_v \nabla^2 \mathbf{v} - (\mathbf{v} \cdot \nabla)\mathbf{v} + \alpha_v \nabla \Phi - \lambda_v S \mathbf{v} + N_v[\Phi, \mathbf{v}, S] \tag{9}$$

The gradient term $\alpha_v \nabla \Phi$ drives coherence flow toward high continuation accessibility. The suppression term $-\lambda_v S \mathbf{v}$ models degradation of coherent transport under increasing dispersion. The nonlinear transport term resembles fluid advection but its interpretation is geometric: reconstruction coherence propagates directionally through admissibility structure.

C.3 Entropy-Dispersion Field S

$$\partial_t S \;=\; D_S \nabla^2 S + \sigma_S |\nabla \Phi|^2 - \mu_S S \Phi - \nabla \cdot (S \mathbf{v}) + N_S[\Phi, \mathbf{v}, S] \tag{10}$$

The source term $\sigma_S|\nabla\Phi|^2$ generates dispersion through accessibility curvature gradients. The suppression term $-\mu_S S\Phi$ stabilizes dispersion in regions of high continuation accessibility. The advection term $-\nabla\cdot(S\mathbf{v})$ transports dispersion through coherence flow.

C.4 Control Parameter Dynamics

Together, equations (8)–(10) determine the evolution of the control parameter:

$$\partial_t\chi \;=\; \frac{\Phi\,\partial_t S - S\,\partial_t\Phi}{\Phi^2}.$$

Equation (10) is of particular interest: the source term $\sigma_S(1-S/\Phi)S = \sigma_S(1-\chi)\chi\Phi$ drives χ toward unity when $\chi < 1$ and away when $\chi > 1$, producing the admissibility transition at $\chi = 1$ as a dynamical fixed point.

C.5 Epistemic Status

These equations are schematic. Many alternative systems may generate the same admissibility phenomenology; if the universality hypothesis of Appendix B is correct, weak-gradient reconstruction behavior may be largely insensitive to microscopic dynamical details. A completed RSVP field theory would require: a variational principle, covariant generalization, explicit reconstruction observables, well-posed initial value structure, and quantitative comparison with data.

D Obstruction Cohomology and Global Reconstruction Failure

This appendix sketches a cohomological interpretation of reconstruction obstruction [16, 17, 14]. The purpose is to demonstrate mathematical directionality, not claim a finished cohomological theory.

D.1 Čech Cohomology of the Reconstruction Cover

Let $\check{C}^\bullet(\mathcal{U},\mathcal{F})$ denote the Čech cochain complex associated with $\mathcal{U}$ and $\mathcal{F}$. A family of local sections $\{s_\alpha\}$ defines a 0-cochain. A compatibility obstruction corresponds to a non-trivial 1-cocycle $g_{\alpha\beta} \in \check{C}^1(\mathcal{U},\mathcal{F})$ measuring the failure of sections to agree on overlaps. Globally admissible reconstruction requires trivialization of this cocycle:

Definition D.1 (Compatibility obstruction class)**.** A *compatibility obstruction class* is a nontrivial element of

$$\check{H}^1(\mathcal{U},\mathcal{F}),$$

corresponding to the impossibility of globally gluing locally admissible sections. Vanishing $\check{H}^1$ indicates that all compatible local sections admit global extension without compatibility obstruction.

D.2 Two Obstruction Notions

Within RSVP, obstruction possesses an additional dynamical feature absent from ordinary static sheaf theory: the reconstruction cover evolves under admissibility coarse-graining. This motivates two notions:

Definition D.2 (Static obstruction)**.** A *static obstruction* is an ordinary compatibility obstruction at a fixed reconstruction scale — a nontrivial class in $\check{H}^1(\mathcal{U}_\ell, \mathcal{F})$ for some fixed ℓ.

Definition D.3 (Persistence obstruction)**.** A *persistence obstruction* occurs when an obstruction class fails to be trivialized under coarse-graining of the reconstruction cover, even if it is trivializable at some initial scale. Formally, there exists a coarse-graining flow $\mathcal{R}_\lambda$ such that the induced map

$$\mathcal{R}_\lambda^* \colon \check{H}^1(\mathcal{U}_{\lambda_1}, \mathcal{F}) \longrightarrow \check{H}^1(\mathcal{U}_{\lambda_2}, \mathcal{F}), \quad \lambda_2 > \lambda_1,$$

does not preserve triviality.

D.3 Scale-Dependent Obstruction Functional

Introduce a scale-dependent obstruction functional:

$$\Omega_\ell[s] \;\in\; \mathbb{R}_{\geq 0}$$

whose flow under coarse-graining satisfies:

$$\frac{d\Omega_\ell}{d\ln\ell} \;=\; \Gamma(\chi_\ell, \Omega_\ell) \tag{11}$$

where $\chi_\ell = S_\ell/\Phi_\ell$. In strongly factorizable regimes ($\chi_\ell \ll 1$), obstruction flow remains bounded and global reconstruction modes persist stably. Near the admissibility transition ($\chi_\ell \to 1$), dispersion obstruction may drive $\Omega_\ell \to \infty$ even in formally compatible local systems.

D.4 Galactic Anomalies as Cohomological Failure

The galactic anomaly reinterpretation of Section 5 may be reformulated cohomologically: flat rotation curves and lensing anomalies correspond to growth of persistence obstruction in the global reconstruction geometry rather than to missing matter sources. This clarifies why anomalies appear primarily in large-scale extension behavior rather than in local dynamical inconsistency.

A completed reconstruction-cohomology program would require: explicit admissibility-chain complexes, construction of scale-dependent obstruction operators, definition of persistence cohomology under admissibility renormalization, and derivation of observational signatures associated with obstruction growth.

E Relation to Renormalization and Effective Field Theory

This appendix clarifies the relation between RSVP's reconstruction geometry and ordinary Wilsonian effective field theory [18, 19]. The purpose is not to identify RSVP with standard EFT but to explain where the analogy is structurally useful and where the two frameworks diverge.

E.1 Where the Analogy Holds

In ordinary EFT, short-scale degrees of freedom are integrated out, producing scale-dependent effective couplings governing long-scale behavior. Both frameworks study persistence of effective structure under scale transformation. The analogy motivates the beta-function framework of Appendix B and the universality class hypothesis.

E.2 Where the Analogy Breaks

In ordinary EFT, the background geometry remains fixed throughout renormalization; only couplings evolve. In RSVP, coarse-graining modifies not merely effective couplings but the factorization structure of admissible reconstruction itself. This is the formal content of Remark 2.4: the reconstruction cover is a derived object of the field geometry, not a background.

Ordinary EFT assumes that locality survives coarse-graining even as effective couplings change. RSVP instead allows the factorization structure supporting effective locality to become unstable. The notion of universality acquires a modified meaning: not that many microscopic systems flow toward the same macroscopic effective behavior, but that distinct admissibility geometries may flow toward the same reconstruction-transition structure under coarse-graining.

E.3 Newtonian Gravity as Effective Reconstruction Theory

Newtonian dynamics emerges not as a fundamental interaction but as an effective low-dispersion reconstruction theory — analogous to hydrodynamics emerging from microscopic statistical systems [19]. Hydrodynamic laws are not fundamental microscopic truths; they are stable large-scale descriptions arising from strong universality under coarse-graining. Within RSVP, Newtonian dynamics plays the same role.

E.4 Entropy Equivalence as Emergent Universality

Entropy convergence (Theorem-Schema 4.1) is interpreted as an effective coarse-grained phenomenon analogous to emergent thermodynamic equations of state. Entropy equivalence is not foundational; it is a low-dispersion universality property of strongly factorizable reconstruction. Divergence of entropy channels near the admissibility transition is then unsurprising: universality breaks down because reconstruction factorization itself weakens under renormalization flow.

F Observational and Experimental Targets

This appendix organizes the empirical targets generated by the framework. It does not claim observational confirmation; its purpose is operational — to identify concrete empirical obligations and failure conditions.

F.1 Rotation Curve Transition Structure

The framework predicts that transition from Newtonian to MOND-like behavior should correlate with reconstruction-factorization breakdown ($\chi \to 1$) rather than solely with local acceleration magnitude. This differs subtly from ordinary MOND: RSVP predicts that transition behavior should correlate not only with acceleration but with indicators of reconstruction persistence, including local coherence structure, environmental reconstruction density, and scale-dependent entropy divergence. Low-surface-brightness galaxies are especially important because they probe weak-gradient regimes where $\chi \to 1$ over large spatial domains [9].

F.2 Entropy-Channel Divergence

The most distinctive RSVP prediction: in strongly factorizable regimes ($\chi \ll 1$), entropy channels converge approximately. Near the admissibility transition ($\chi \sim 1$), the framework predicts increasing divergence between channels due to dispersion obstruction growth. Standard ΛCDM and ordinary entropic gravity do not predict entropy-channel divergence as a geometric signal. Operationally, galactic regions exhibiting strong rotational anomalies should exhibit measurable mismatch between local thermodynamic entropy indicators and large-scale gravitational reconstruction structure.

F.3 Weak Lensing Signatures

Within RSVP, lensing anomalies should correlate with obstruction curvature of the reconstruction cover rather than uniquely with inferred dark matter distributions. Systems with similar baryonic matter distributions but different reconstruction persistence structure may exhibit different lensing behavior. The Bullet Cluster remains a decisive challenge: the framework must reproduce its lensing structure through obstruction curvature geometry (Obligation 5.2).

F.4 Cosmological Structure Formation

Large-scale structure formation depends not merely on matter density perturbations but on admissibility persistence under cosmological extension. The admissibility radius may evolve dynamically during cosmological history as reconstruction dispersion changes under expansion-like coarse-graining flow. These possibilities remain speculative; the framework currently lacks quantitative cosmological simulations capable of testing them.

F.5 Observational Failure Modes

The framework generates clear failure conditions: (1) if weak-gradient galactic behavior cannot be associated with coherent admissibility-transition structure, the geometric reinterpretation fails; (2) if cluster-scale lensing cannot be reproduced through obstruction curvature without reintroducing hidden matter, the framework loses explanatory advantage over ΛCDM; (3) if entropy-channel divergence fails to correlate with anomalous galactic regimes, one of RSVP's most distinctive predictions is falsified; (4) if R_A cannot be derived without arbitrary parameter tuning, the central conjecture becomes structurally underdetermined.

G Locality, Causality, and Admissibility Nonlocality

The framework employs concepts such as cosmological gluing data, admissibility nonlocality, and global extension dependence. These risk misunderstanding if interpreted as superluminal influence or hidden signaling. This appendix specifies precisely what type of nonlocality RSVP permits and what type it explicitly rejects.

G.1 The Central Distinction

The framework does *not* propose superluminal communication, violation of relativistic causal structure, or hidden instantaneous signaling between local field degrees of freedom. The central distinction introduced by RSVP is instead the distinction between locality of propagation and locality of admissibility.

The stalk-local dynamics of the reconstruction sheaf remain local. Local field values propagate according to local dynamical equations governing the coupled field

$$X = (\Phi, \mathbf{v}, S),$$

and admissible perturbations evolve continuously through neighboring reconstruction regions. In this sense, RSVP preserves ordinary local propagation structure at the level of stalk dynamics.

What becomes nonlocal is not propagation itself but the coherence conditions governing extension and assembly of local sections into globally persistent reconstruction modes. A local section may satisfy all local dynamical equations while nevertheless failing global admissibility because its continuation structure cannot be extended coherently across the reconstruction cover under renormalization flow.

Most ordinary field theories implicitly identify locality of propagation with locality of admissibility. RSVP separates these notions. Locality of propagation is a property of stalk dynamics. Locality of admissibility is a property of global extension structure in the reconstruction sheaf.

Admissibility nonlocality therefore appears not in microscopic signaling but in the indexing, restriction, and gluing structure of admissible reconstruction itself. The dependence on cosmological gluing data discussed throughout the main text is not a proposal for instantaneous causal influence across spacetime. It is a statement that persistence of admissible continuation may depend on global reconstruction coherence conditions extending beyond the local neighborhood of a given section.

This distinction becomes especially important near the admissibility transition

$$\chi = \frac{S}{\Phi} \sim 1,$$

where local reconstruction ceases to remain strongly factorizable under coarse-graining. In strongly factorizable regimes,

$$\chi \ll 1,$$

local admissibility appears effectively self-contained because global extension conditions reduce approximately to local continuation structure. Near the transition, however, persistence becomes increasingly sensitive to large-scale reconstruction geometry even while local propagation remains relativistically local.

RSVP therefore weakens locality of admissibility without abandoning locality of propagation. The framework modifies the geometry of globally admissible extension, not the local causal structure of field evolution.

G.2 Admissibility Nonlocality Defined

Admissibility nonlocality exists in: the indexing structure of the cover, the gluing conditions, and the persistence structure of extension under renormalization. It does *not* exist in the stalk-local dynamical equations themselves. Nonlocality appears in the geometry of assembly rather than in microscopic propagation.

A local section may satisfy all local dynamical equations while nevertheless failing to admit globally stable extension because admissibility persistence depends upon large-scale coherence of the reconstruction cover. The dependence is *extension dependence*, not *dynamical signaling dependence.*

G.3 Analogy with Gauge Theory and General Relativity

This distinction is analogous in spirit to global constraint structure in gauge theory and general relativity. In electromagnetism, local electric-field measurements are constrained globally by Gauss-law structure. In general relativity, local geometry is constrained by global consistency conditions on spacetime slicing. RSVP generalizes this: local reconstruction trajectories remain locally dynamical, but admissibility of their persistent extension depends upon global reconstruction geometry.

G.4 Interpretation of “Cosmological Gluing Data”

The term does not refer to signals propagating instantaneously from cosmological distances. It refers to the fact that globally persistent continuation may require compatibility with large-scale reconstruction structure. The admissible continuation set of a local trajectory depends upon the global extension geometry of the reconstruction cover — an extension condition, not a propagation condition.

Near the admissibility transition ($\chi \to 1$), local factorization weakens. Local continuation trajectories cease to remain closed under extension independently of cosmological admissibility structure. The resulting dependence may appear nonlocal from the perspective of ordinary local field theory even though no superluminal propagation occurs.

G.5 Formal Statement

Admissibility nonlocality occurs in the indexing and coherence structure of $\mathcal{F}$ rather than in the stalk-local field dynamics. A completed RSVP field theory would require rigorous proof that admissibility nonlocality remains compatible with relativistic causal structure under all admissible reconstruction flows. The present appendix establishes only the conceptual separation necessary for the framework to avoid immediate conflict with ordinary causal locality. This separation follows directly from the central inversion: persistent physical exchange is generated by admissibility geometry rather than vice versa. If admissibility geometry is primary, then locality of admissibility and locality of propagation need not coincide.

H Formal Implementation: Grammar, Typed Structures, and Proof Schemas

This appendix outlines possible formal implementation directions for the RSVP admissibility framework. The guiding principle is that the framework should eventually be expressible as a typed formal system in which admissibility, obstruction, entropy-channel convergence, and reconstruction persistence can be checked, simulated, or proven under specified assumptions.

H.1 BNF Grammar for Reconstruction Objects

A minimal formal language must represent fields, regions, covers, sections, restriction maps, obstruction classes, entropy channels, and scale transformations.

Listing 1: BNF grammar for RSVP reconstruction theory

```
<Theory>      ::= <FieldDecl> <CoverDecl> <SheafDecl>
                  <DynamicsDecl> <ObstructionDecl> <EntropyDecl>

<FieldDecl>   ::= "field" <Name> "=" "(" <Phi> "," <v> "," <S> ")
   "
<Phi>         ::= "Phi" ":" <Domain> "->" Real
<v>           ::= "v"   ":" <Domain> "->" Vector
<S>           ::= "S"   ":" <Domain> "->" Real

```

```
<Domain>     ::= "ReconDomain" | <Region>
<Region>     ::= "U" <Index> | <Region> "cap" <Region>
                            | <Region> "cup" <Region>

<CoverDecl>  ::= "cover" <Name> "=" "{" <RegionList> "}"

<SheafDecl>  ::= "sheaf" <Name> ":" <CoverName> "->" <
   SectionSpace>
<SectionSpace> ::= "Sections" "(" <FieldName> "," <Region> ")"

<RestrMap>   ::= "restrict" <Section> "from" <Region> "to" <
   Region>

<Admiss>     ::= "admissible" "(" <Section> ")"
               | "stable"     "(" <Section> ")"
               | "persistent" "(" <Section> ")"

<Chi>        ::= "chi" "=" <S> "/" <Phi>

<ObsDecl>    ::= "obstruction" <ObsType> "(" <Section> "," <
   Section> ")"
<ObsType>    ::= "compatibility" | "dispersion"

<EntropyDecl>::= "entropy" <EntType> "(" <Section> ")"
<EntType>    ::= "informational" | "configurational"
               | "thermodynamic" | "gravitational"

<DynDecl>    ::= "flow" <Name> ":" <Scale> "->" <TheoryState>

<Formula>    ::= <Admiss> | <ChiRel> | <EntRel> | <ObsRel>
               | <Formula> "and" <Formula>
               | <Formula> "implies" <Formula>

<ChiRel>     ::= "chi << 1" | "chi ~ 1" | "chi -> 1"
<EntRel>     ::= "converges" "(" <EntTypeList> ")"
               | "diverges"  "(" <EntTypeList> ")"
<ObsRel>     ::= "bounded" "(" <ObsType> ")"
               | "grows"   "(" <ObsType> ")"
```

H.2 BNF for Epistemic Status of Claims

Listing 2: BNF grammar for claim classification

```
<Claim> ::= <Definition>
          | <Conjecture>
          | <TheoremSchema>
          | <ProofObligation>
```

```
          | <PhenomInterp>

<Definition>     ::= "define"    <Object> "as" <FormalExpr>
<Conjecture>     ::= "conjecture" <Statement> "under" <
   Assumptions>
<TheoremSchema>  ::= "theorem-schema" <Statement> "provided" <
   Conditions>
<ProofObligation>::= "prove"     <Statement> "from" <Defs> "and"
    <Dynamics>
<PhenomInterp>   ::= "interpret" <Observation> "as" <
   FrameworkObject>

<Object>    ::= "AdmissibilityRadius"    | "CompatObstruction"
              | "DispersionObstruction"  | "EntropyChannel"
              | "ReconstructionCover"    | "
                 AdmissPhaseTransition"

<Relation>  ::= "determines" | "converges_to" | "diverges_from"
              | "scales_as"  | "obstructs"    | "extends_to"
```

This grammar prevents category errors: “MOND is derived” is not yet a theorem; it is a conjectural derivation target. “The admissibility radius is the relevant geometric object” is a definitional and structural claim. “Entropy channels converge under strong factorization” is a theorem-schema requiring proof under specified dynamics.

H.3 Typed Structures in Lean Style

Listing 3: Lean-style typed structures for RSVP objects

```
structure ReconDomain where
  points : Type

structure Region (R : ReconDomain) where
  carrier : Set R.points

structure RSVPField (R : ReconDomain) where
  Phi : R.points -> Real
  v   : R.points -> Vector
  S   : R.points -> Real

-- Control parameter (well-defined when Phi > 0)
def PositiveAccess {R : ReconDomain}
    (X : RSVPField R) : Prop :=
  ∀ x : R.points, X.Phi x > 0

def chi {R : ReconDomain}
    (X : RSVPField R) (h : PositiveAccess X)
```

```
    (x : R.points) : Real :=
  X.S x / X.Phi x

-- Admissibility predicates
structure LocalSection {R : ReconDomain}
    (X : RSVPField R) (U : Region R) where
  field_restr : U.carrier -> RSVPField R
  continuation_data : Type
  bounded_stability : Prop

def StronglyFactorizable {R : ReconDomain}
    (X : RSVPField R) (h : PositiveAccess X)
    (U : Region R) (δ : Real) : Prop :=
  ∀ x ∈ U.carrier, chi X h x < δ

def NearTransition {R : ReconDomain}
    (X : RSVPField R) (h : PositiveAccess X)
    (U : Region R) (ε : Real) : Prop :=
  ∀ x ∈ U.carrier, |chi X h x - 1| < ε
```

H.4 Reconstruction Sheaf Structure

Listing 4: Lean-style reconstruction sheaf

```
structure ReconSheaf (R : ReconDomain) (X : RSVPField R) where
  sections : Region R -> Type
  restrict :
    ∀ {U V : Region R},
      V.carrier ⊆ U.carrier ->
      sections U -> sections V
  identity_law     : Prop  -- restrict id = id
  composition_law  : Prop  -- restrict (V ⊆ U) after (W ⊆ V) =
                            --   restrict (W ⊆ U)
  gluing_law       : Prop  -- compatible sections glue
```

H.5 Obstruction Formalization

Listing 5: Compatibility and dispersion obstruction in Lean

```
def CompatObstruction {R : ReconDomain} {X : RSVPField R}
    (F : ReconSheaf R X)
    (U V : Region R)
    (sU : F.sections U) (sV : F.sections V) : Prop :=
  -- restrictions to overlap disagree
  ¬(F.restrict (overlap_le U V) sU =
     F.restrict (overlap_le' U V) sV)
```

```

structure Scale where
  ell : Real
  pos : ell > 0

-- Persistence functional placeholder
def Persistence {R : ReconDomain} {X : RSVPField R}
    (F : ReconSheaf R X) (ℓ : Scale) (s : Type) : Real :=
  0 -- to be derived from field equations

def DispersionObstruction {R : ReconDomain} {X : RSVPField R}
    (F : ReconSheaf R X) (ε : Real) (s : Type) : Prop :=
  ∃ ℓ : Scale, Persistence F ℓ s < ε
```

H.6 Theorem Schemas in Lean Style

Listing 6: Central theorem schemas as Lean proof obligations

```
-- Entropy convergence schema
theorem entropy_convergence
    {R : ReconDomain} (X : RSVPField R)
    (h : PositiveAccess X)
    (F : ReconSheaf R X)
    (U : Region R) (δ ε : Real)
    (hFact : StronglyFactorizable X h U δ)
    (hBound :  dispersion obstruction bounded  True) :
     entropy channels converge to order (*ε *)
    True := by
  sorry  -- proof obligation: requires field equation dynamics

-- Entropy divergence schema
theorem entropy_divergence
    {R : ReconDomain} (X : RSVPField R)
    (h : PositiveAccess X)
    (F : ReconSheaf R X)
    (U : Region R) (ε : Real)
    (hTrans : NearTransition X h U ε) :
     entropy channels diverge under coarse-graining
    True := by
  sorry  -- proof obligation: requires beta-function derivation

-- Central conjecture as proof obligation
-- (RA proportional to c/H_0 in weak-gradient limit)
theorem RA_scaling_conjecture
    (field_eqs : True) :
    True := by
  sorry  -- principal open problem of the paper
```

The sorry markers are not weaknesses: they identify proof obligations. A Lean formalization would force every informal claim in the paper to become either a definition, theorem, conjecture, or unresolved proof target — precisely the discipline the present paper maintains in prose.

H.7 Algorithmic Reconstruction Procedure

A computational implementation of RSVP reconstruction obstruction analysis:

Listing 7: RSVP obstruction analysis algorithm

```
Algorithm: RSVP_Obstruction_Analysis

Input:
  X = (Phi, v, S)          -- coupled RSVP field
  R                         -- reconstruction domain
  U = {U_alpha}             -- initial admissibility cover at
     l_min
  l_min, l_max              -- scale range
  epsilon                   -- stability threshold
  delta                     -- factorization threshold

Output:
  R_A                       -- admissibility radius
  compat_obs_map            -- compatibility obstruction map
  disp_obs_map              -- dispersion obstruction map
  entropy_divergence_profile -- entropy channel divergence

Procedure:
  1. Initialize cover U at scale l = l_min.
  2. For each U_alpha:
       compute chi = S / Phi
       if chi < delta: classify as strongly factorizable
  3. For each overlap U_alpha cap U_beta:
       restrict local sections to overlap
       test compatibility agreement
       if disagreement: record compatibility obstruction
  4. Coarse-grain cover to next scale l.
  5. Update Phi_l, v_l, S_l.
  6. Recompute chi_l = S_l / Phi_l.
  7. Track persistence of previously admissible sections.
  8. If compatible sections fail persistence: record disp.
     obstruction.
  9. If chi_l -> 1 or persistence < epsilon:
       record R_A = current scale
  10. Compute entropy projections Pi_therm, Pi_info, Pi_grav,
     Pi_conf.
  11. Measure entropy-channel convergence or divergence.
```

```
12. Repeat steps 4-11 until l = l_max.
13. Return obstruction and transition profiles.
```

H.8 Formalization Roadmap

A rigorous implementation program would proceed in stages:

(1) Define the typed syntax and semantics of RSVP reconstruction objects.

(2) Implement the reconstruction sheaf in Lean 4 or Agda.

(3) Formalize compatibility obstruction through Čech cocycle data.

(4) Define dispersion obstruction as scale-instability of persistence.

(5) Implement coarse-graining flow for χ_ℓ.

(6) Prove conditional theorems linking strong factorization to entropy convergence.

(7) Prove or refute the central conjecture: weak-gradient RSVP determines $R_A \propto c/H_0$.

(8) Build numerical simulations computing obstruction maps and entropy-channel divergence profiles from candidate field equations.

The purpose of formalization is not merely aesthetic rigor. It is a guardrail against the main failure mode of speculative gravity frameworks: sliding between definitions, metaphors, conjectures, and derivations without marking the transition. A Lean implementation would force every RSVP claim to declare its type.

I Toward a Variational Formulation

The candidate dynamical equations of Appendix C appear phenomenological. A variational formulation would dramatically increase mathematical maturity and reveal whether the RSVP admissibility structure has a natural Lagrangian origin. This appendix sketches the form such a formulation would take without claiming to complete it.

I.1 Candidate Action

Introduce a candidate action functional over the reconstruction domain $\mathcal{R}$:

$$\mathcal{S}[X] \;=\; \int_{\mathcal{R}} \mathcal{L}(\Phi,\, \mathbf{v},\, S,\, \nabla\Phi,\, \nabla\mathbf{v},\, \nabla S)\; d\mu, \tag{12}$$

where $d\mu$ is a measure on $\mathcal{R}$ derived from the admissibility geometry rather than a fixed background volume form.

A minimal candidate Lagrangian density consistent with the dimensional analysis of Section 3.3 and the qualitative dynamics of Appendix C is:

$$\mathcal{L} \;=\; \frac{1}{2}(\nabla\Phi)^2 \;-\; V(\chi)\,\Phi^2 \;+\; \frac{1}{2}\,\mathbf{v}\cdot\mathbf{v} \;-\; \mathcal{W}(S,\Phi,\mathbf{v}), \tag{13}$$

where $V(\chi) = V(S/\Phi)$ is a potential encoding the admissibility phase structure and $\mathcal{W}$ is a coupling term generating the interaction between dispersion growth and coherence transport.

The potential $V(\chi)$ must satisfy:

- $V(0) = 0$: no potential energy at the Newtonian fixed point.
- $V'(\chi) > 0$: potential increases toward the transition, disfavoring high-dispersion configurations.
- A local minimum near $\chi = 0$ and a saddle or maximum near $\chi = 1$, consistent with the attractor structure of Appendix B.5.

I.2 Admissibility Persistence as Modified Stationarity

The central insight distinguishing the RSVP variational formulation from standard Euler–Lagrange theory is that admissibility persistence is *not* equivalent to stationarity of $\mathcal{S}$.

Standard field theory requires:

$$\frac{\delta\mathcal{S}}{\delta X} \;=\; 0.$$

RSVP admissibility persistence additionally requires that the stationary configuration remain stable under renormalization of the reconstruction cover:

$$\frac{\delta\mathcal{S}}{\delta X} = 0 \quad \text{and} \quad \mathcal{P}[\delta X/\delta\ell] > \varepsilon \quad \text{for all } \ell < R_A.$$

The second condition filters out stationary configurations that are classically extremal but fail reconstruction persistence under scale extension — the field-theoretic analog of dispersion obstruction.

This distinction reinforces the paper's central inversion: the relevant selection criterion for physical configurations is not classical stationarity but admissibility persistence under coarse-graining of the reconstruction cover.

Open Proof Obligation I.1. Derive the coupled field equations of Appendix C as Euler–Lagrange equations of a specific action of the form (12), and show that the admissibility persistence condition is compatible with, but strictly stronger than, classical stationarity.

J Effective Metric Emergence

Spacetime geometry is discussed philosophically in the main text but not yet mathematically. This appendix sketches how an effective metric structure might emerge from the admissibility geometry of the coupled field $X = (\Phi, \mathbf{v}, S)$ without presupposing a background metric. The effective metric construction developed here is heuristic and should be interpreted as an emergent reconstruction observable rather than a completed relativistic metric theory. Writing $g_{\mu\nu}^{\text{eff}}$ immediately raises questions of covariance, constraint algebra, stress-energy conservation, hyperbolicity, and equivalence principle; these are not resolved here, and the appendix should be read as identifying the structural direction rather than claiming completion.

J.1 Effective Metric from Admissibility Structure

In strongly factorizable regimes ($\chi \ll 1$), the continuation accessibility field Φ and coherence-flow field $\mathbf{v}$ together define a preferred local geometry. A schematic effective metric may be constructed as:

$$g_{\mu\nu}^{\text{eff}} \;=\; G_{\mu\nu}(\Phi, \mathbf{v}, S), \tag{14}$$

where $G_{\mu\nu}$ is a symmetric tensor built from the gradients of the RSVP fields. The simplest candidate consistent with dimensional constraints is:

$$g_{\mu\nu}^{\text{eff}} \;=\; f(\chi)\,\eta_{\mu\nu} \;+\; h(\chi)\,\frac{\partial_\mu\Phi\,\partial_\nu\Phi}{\Phi^2},$$

where $\eta_{\mu\nu}$ is a flat reference metric, f and h are functions of χ satisfying $f(0) = 1$ and $h(0) = 0$ (recovering flat geometry in the Newtonian limit), and corrections from h encode the admissibility curvature generated by Φ-gradients.

J.2 Null Structure and Effective Geodesics

Within this framework, null geodesics — paths along which $g_{\mu\nu}^{\text{eff}}\,dx^\mu dx^\nu = 0$ — would correspond to admissible photon continuation trajectories. In the Newtonian regime ($f \approx 1$, $h \approx 0$), these reduce to ordinary null geodesics of flat spacetime. Near the admissibility transition ($\chi \to 1$), the h-term introduces curvature driven by accessibility gradients

$\nabla\Phi$, producing lensing effects without requiring a background metric curvature sourced by matter.

This provides a schematic geometric basis for the lensing reinterpretation of Section 5: lensing anomalies arise from admissibility curvature in $g_{\mu\nu}^{\text{eff}}$ rather than from hidden mass distributions.

J.3 Relationship to General Relativity

The effective metric (14) is not postulated to satisfy the Einstein field equations. Instead, the Einstein equations would appear as an emergent approximation valid in regimes where the RSVP field dynamics produce a metric satisfying the vacuum or sourced Einstein equations to leading order. This would require showing that the Ricci tensor of $g_{\mu\nu}^{\text{eff}}$ satisfies $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \approx 8\pi G\, T_{\mu\nu}^{\text{eff}}$ in appropriate regimes. This constitutes an open derivation target.

Open Proof Obligation J.1. Derive the explicit form of $G_{\mu\nu}(\Phi, \mathbf{v}, S)$ from the RSVP field equations, compute the resulting Riemann curvature tensor, and determine in which reconstruction regimes the effective geometry approximates solutions of the Einstein equations.

K Gauge Redundancy and Admissibility Equivalence

Since RSVP increasingly resembles a geometric theory rather than a collection of fields, the question of gauge redundancy arises naturally: do distinct field configurations X and X' represent the same physical reconstruction geometry?

K.1 Admissibility Equivalence

Define an *admissibility equivalence* relation on field configurations:

$$X \;\sim\; X'$$

if and only if all entropy projection functors agree:

$$\Pi_i(X) \;\simeq\; \Pi_i(X') \quad \text{for all } i \in \{\text{therm}, \text{info}, \text{grav}, \text{conf}\},$$

and the obstruction classes of the associated reconstruction sheaves are identical:

$$\check{H}^k(\mathcal{U}_X, \mathcal{F}_X) \;\cong\; \check{H}^k(\mathcal{U}_{X'}, \mathcal{F}_{X'}) \quad \text{for all } k.$$

Two configurations that are admissibility-equivalent produce the same entropy-channel structure, the same obstruction classes, and therefore the same physical predictions. The physical state space of RSVP is the quotient:

$$\mathcal{M}_{\text{phys}} \;=\; \{X\} \,/ \sim.$$

K.2 Reconstruction-Preserving Transformations

A *reconstruction-preserving transformation* is a map $\phi : X \mapsto X'$ such that $X \sim X'$. These transformations form a group $\mathcal{G}_{\mathrm{adm}}$ — the admissibility gauge group — whose structure encodes the redundancy of the field description.

In the Newtonian regime ($\chi \ll 1$), $\mathcal{G}_{\mathrm{adm}}$ likely reduces to familiar diffeomorphism-like redundancy: smooth reparametrizations of reconstruction neighborhoods that preserve admissibility classes. Near the admissibility transition, the gauge group structure may change, reflecting the breakdown of factorization.

K.3 Relationship to Diffeomorphism Invariance

General relativity's diffeomorphism invariance is the statement that the physical content of a spacetime metric is invariant under smooth coordinate changes. In RSVP, the analogous invariance is admissibility equivalence: physical content is invariant under reconstruction-preserving transformations that preserve the sheaf structure and entropy projections.

This suggests that RSVP's gauge group generalizes diffeomorphism invariance: in strongly factorizable regimes, the two should approximately coincide, while near the admissibility transition, the gauge group may become larger, encoding the additional freedom in how cosmological boundary data is incorporated into local reconstruction.

Open Proof Obligation K.1. Classify the admissibility gauge group $\mathcal{G}_{\mathrm{adm}}$ in both the Newtonian regime ($\chi \ll 1$) and near the admissibility transition ($\chi \to 1$), and determine whether RSVP possesses a well-posed constraint surface analogous to the Gauss-law and diffeomorphism constraints of general relativity.

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