[illegible]

[illegible]

[illegible]

[illegible] 13, 2026

[illegible]

[illegible] $\mathcal{S} = \mathcal{C} \times \mathcal{A} \times \mathcal{L} \times \mathcal{E}$ [illegible] $\mathcal{S}$ [illegible] $\Pi : \mathcal{S} \to \mathcal{P}$ [illegible] $\mathcal{P}$ [illegible]

[illegible] $\mathcal{P}$ [illegible] ([illegible] 180–[illegible]) [illegible] $\mathcal{P}$.

[illegible]

[illegible]

1 Introduction

Many cultural activities that seem superficially unrelated share a common deep structure: they are all instances of directed motion through a structured spatial or perceptual field, where meaning or aesthetic quality emerges from the shape of that motion rather than from its endpoints alone. A filmmaker choosing a camera angle, a painter laying down a brushstroke, a child completing a figure on a page, a typist swiping a word, and a viewer's eye jumping between salient features of a scene are all performing versions of the same cognitive and bodily operation.

This paper proposes a unified mathematical framework for these activities. The central object is a trajectory: a continuous path $\gamma(t)$ through some structured space, where the space is endowed with a geometry that determines which paths are natural, smooth, or costly. We argue that the appropriate notion of naturalness is always determined by a perceptual metric—a metric tensor on a manifold of experiential states—rather than by a fixed physical or symbolic metric.

The framework has five main components.

(1) Scenario manifold $\mathcal{S}$: the product of camera, actor, lighting, and environment configuration spaces.

(2) Perceptual manifold $\mathcal{P}$: the space of viewer experiential states, carrying a metric derived from information geometry.

(3) Rendering-and-perception map $\Pi : \mathcal{S} \to \mathcal{P}$: the operator that converts scenario states into perceptual states.

(4) Geodesic principle: cinematographic excellence corresponds to approximate geodesic motion in $\mathcal{P}$ subject to a narrative potential function.

(5) Generality: the same variational structure governs painting strokes, drawing grammars, swipe gestures, eye saccades, and musical phrasing.

Modern film production already instantiates part of this structure. Virtual production systems such as Unreal Engine represent a scene as a game state $g = (C, A, L, E)$ whose components are camera pose, actor configurations, lighting, and environment, and the camera path through that state space is literally a trajectory. But a film recorded with a physical camera also constitutes a path through a latent scenario space, whether or not that space is ever explicitly modeled. The theoretical reinterpretation proposed here treats this implicit structure as the primary object of analysis.

Section 2 defines scenario space formally. Section 3 introduces the perceptual manifold and its metric. Section 4 states the central theorem and derives the variational equations. Section 5 reinterprets classical editing conventions. Section 6 grounds the metric in information geometry. Section 7 extends the framework to painting and visual gradients. Section 8 addresses children's drawing and the grammar of spatial action. Section 9 treats **QWERTY** swipe gestures as a trajectory language. Section 10 develops topological invariants of gesture classes. Section 11 connects the [illegible] 12 [illegible] 13 [illegible] 14 [illegible] 15 [illegible] 16 [illegible]

2 [illegible]

2.1 [illegible]

[illegible]

$$\mathcal{S} = \mathcal{C} \times \mathcal{A} \times \mathcal{L} \times \mathcal{E},$$

[illegible] $\mathcal{C}$ [illegible], $\mathcal{A}$ [illegible], $\mathcal{L}$ [illegible] $\mathcal{E}$ [illegible] $\mathcal{S}$ [illegible]

[illegible] 2.1 ([illegible]). [illegible]

$$s = (C, A, L, E) \in \mathcal{S}$$

[illegible]:

- $C = (x, y, z, \theta, \phi, f) \in \mathcal{C} \cong \mathbb{R}^3 \times S^2 \times \mathbb{R}_{>0}$ [illegible];
- $A = (a_1, \ldots, a_k) \in \mathcal{A}$ [illegible] k [illegible] a_i [illegible];
- $L \in \mathcal{L}$ [illegible] ([illegible]);
- $E \in \mathcal{E}$ [illegible] ([illegible]).

[illegible] ([illegible]) [illegible]

2.2 [illegible]

[illegible] 2.2 ([illegible]). [illegible]

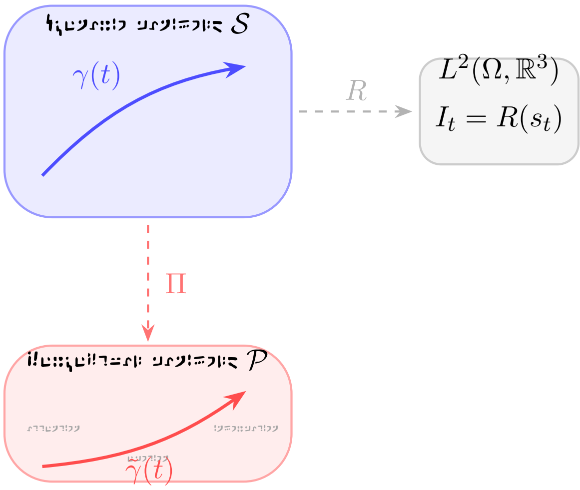
$$\gamma : [0, T] \to \mathcal{S}.$$

[illegible] $s_t = \gamma(t)$. [illegible] t [illegible]:

$$I_t = R(s_t), \qquad R : \mathcal{S} \to L^2(\Omega, \mathbb{R}^3),$$

[illegible] $\Omega \subset \mathbb{R}^2$ [illegible] $L^2(\Omega, \mathbb{R}^3)$ [illegible] Ω. [illegible] $(I_t)_{t\in[0,T]}$, [illegible] R.

[illegible] 2.3. [illegible] $R : \mathcal{S} \to L^2(\Omega, \mathbb{R}^3)$ [illegible] $\mathcal{S}$ [illegible] R [illegible] R [illegible]



[illegible] 1: [illegible] $\mathcal{S}$, [illegible] R, [illegible] $\mathcal{P}$. [illegible] $\gamma(t)$ [illegible] $\mathcal{S}$; [illegible] $\tilde{\gamma}(t)$. [illegible] $\mathcal{P}$ [illegible]

3 [illegible]

[illegible] $\mathcal{S}$ [illegible] $\mathcal{P}$.

3.1 [illegible]

[illegible] 3.1 ([illegible]). [illegible]

$$p = (a, r, o, e, q) \in \mathcal{P} \subset \Delta(\mathcal{X}) \times [0, 1]^4$$

[illegible] $\Delta(\mathcal{X})$ [illegible] $\mathcal{X}$ [illegible]:

- $a \in \Delta(\mathcal{X})$ [illegible];

- $r \in [0, 1]$ [illegible]
- $o \in [0, 1]$ [illegible]
- $e \in [0, 1]$ [illegible]
- $q \in [0, 1]$ [illegible]

[illegible] a [illegible] $\mathcal{P} \subset \Delta(\mathcal{X}) \times [0, 1]^4$ [illegible] 6.

3.2 [illegible]

[illegible] $\mathcal{P}$ [illegible] $g_{ij}(p)$ [illegible]

$$ds^2 = \sum_{i,j} g_{ij}(p)\, dp^i\, dp^j.$$

[illegible] 3.2 ([illegible]). [illegible]

$$\Pi : \mathcal{S} \to \mathcal{P}$$

[illegible] $\gamma(t)$ [illegible] $\mathcal{S}$ [illegible]

$$\tilde{\gamma}(t) = \Pi(\gamma(t)) \in \mathcal{P}.$$

[illegible] $\mathcal{P}$ [illegible] $\mathcal{S}$ [illegible] $D\Pi_s : T_s\mathcal{S} \to T_{\Pi(s)}\mathcal{P}$:

$$(D\Pi^* g)_s(u, v) = g_{\Pi(s)}(D\Pi_s \cdot u,\ D\Pi_s \cdot v), \qquad u, v \in T_s\mathcal{S}.$$

[illegible] Π [illegible]

4 [illegible]

4.1 [illegible]

[illegible] 4.1 ([illegible]). [illegible] $\gamma : [0, T] \to \mathcal{S}$ [illegible] $\tilde{\gamma} = \Pi \circ \gamma$ [illegible] $(\mathcal{P}, g)$. [illegible] $\tilde{\gamma}(0)$ [illegible] $\tilde{\gamma}(T)$, [illegible]

[illegible]

$$\mathcal{A}[\tilde{\gamma}] = \frac{1}{2}\int_0^T g_{ij}(\tilde{\gamma}(t))\,\dot{\tilde{\gamma}}^i(t)\,\dot{\tilde{\gamma}}^j(t)\,dt.$$

[illegible]

$$\ddot{\tilde{\gamma}}^k + \Gamma^k_{ij}\,\dot{\tilde{\gamma}}^i\,\dot{\tilde{\gamma}}^j = 0,$$

[illegible] Γ^k_{ij} [illegible] g.

[illegible] $\mathcal{A}$ [illegible] $\mathcal{L}(p,\dot{p}) = \frac{1}{2}g_{ij}(p)\dot{p}^i\dot{p}^j$. [illegible] $\mathcal{A}$ [illegible]

$$\frac{d}{dt}\big(g_{k\ell}\dot{\tilde{\gamma}}^\ell\big) = \frac{1}{2}(\partial_k g_{ij})\dot{\tilde{\gamma}}^i\dot{\tilde{\gamma}}^j,$$

[illegible] $g^{k\ell}$ [illegible] $\Gamma^k_{ij} = \frac{1}{2}g^{k\ell}(\partial_i g_{j\ell} + \partial_j g_{i\ell} - \partial_\ell g_{ij})$ [illegible]. $\square$

[illegible]

4.2 [illegible]

[illegible] $\mathcal{P}$ [illegible] $V : \mathcal{P} \to \mathbb{R}$, [illegible] V [illegible]

[illegible] 4.2 [illegible]

$$\mathcal{J}[\tilde{\gamma}] = \int_0^T \left[\frac{1}{2}\,g_{ij}(\tilde{\gamma})\,\dot{\tilde{\gamma}}^i\,\dot{\tilde{\gamma}}^j + \lambda\,V(\tilde{\gamma})\right]dt,$$

[illegible] $\lambda > 0$ [illegible]

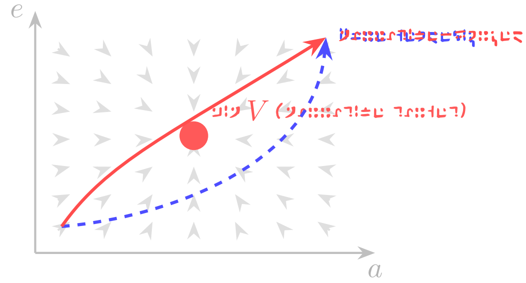
[illegible] 4.3. [illegible] $\mathcal{J}$ [illegible]

$$\ddot{\tilde{\gamma}}^k + \Gamma^k_{ij}\,\dot{\tilde{\gamma}}^i\,\dot{\tilde{\gamma}}^j + \lambda\,g^{k\ell}\,\partial_\ell V(\tilde{\gamma}) = 0,$$

[illegible]

$$\frac{D\dot{\tilde{\gamma}}}{dt} = -\lambda\,\nabla_g V(\tilde{\gamma}).$$

[illegible] 4.4. [illegible] λ [illegible]. [illegible] λ [illegible]; [illegible] λ [illegible].



[illegible] 2: [illegible] V [illegible] a [illegible] e [illegible] $\mathcal{P}$.

5 [illegible]

[illegible] $\mathcal{P}$, [illegible] [4] [illegible].

5.1 [illegible]

[illegible] t_0 [illegible]: $\gamma(t_0^-) \neq \gamma(t_0^+)$. [illegible] $\mathcal{P}$ [illegible] $d_{\mathcal{P}}(\tilde{\gamma}(t_0^-), \tilde{\gamma}(t_0^+))$. [illegible] a, [illegible] o, [illegible] q. [illegible].

5.2 [illegible]

[illegible]:

$$I_t = (1 - \alpha(t))\, R(\gamma_1(t)) + \alpha(t)\, R(\gamma_2(t)), \quad \alpha : [0,1] \to [0,1].$$

[illegible] $\Pi(\gamma_1)$ [illegible] $\Pi(\gamma_2)$ [illegible] $\mathcal{P}$.

5.3 [illegible]

[illegible] $\gamma = (\gamma_1, \gamma_2, \ldots, \gamma_k)$ [illegible]

[illegible] e [illegible] r), [illegible] $\mathcal{P}$ [illegible]
r–[illegible] a [illegible] o [illegible].

5.4 [illegible]

[illegible] 5.1. [illegible] $\mathcal{P}$ [illegible] o. [illegible]

[illegible] $\ell \subset \mathbb{R}^3$, [illegible] ℓ [illegible] ℓ [illegible] o [illegible] g–[illegible] $\mathcal{P}$. [illegible] o [illegible] $\{o > 0\}$ [illegible]. $\square$

5.5 [illegible]

[illegible] t_0 [illegible] $\|\tilde{\gamma}_1(t_0^-) - \tilde{\gamma}_2(t_0^+)\|_{\mathcal{P}} < \varepsilon$ [illegible] ε. [illegible] a [illegible] $\Delta(\mathcal{X})$ [illegible] $\mathcal{P}$.

6 [illegible]

6.1 [illegible]

[illegible] g [illegible] $\mathcal{P}$ [illegible] a [illegible] $P_t \in \Delta(\mathcal{X})$ [illegible] P_t [illegible] $\theta \in \Theta$, [illegible]

$$g_{ij}^F(\theta) = \mathbb{E}_{P(\cdot;\theta)}\left[\frac{\partial \log P(x;\theta)}{\partial \theta^i}\frac{\partial \log P(x;\theta)}{\partial \theta^j}\right].$$

[illegible] $\Delta(\mathcal{X})$ [illegible] [5, 1].

[illegible] 6.1. [illegible] g [illegible] $\mathcal{P}$ [illegible] $\Delta(\mathcal{X})$. [illegible] γ [illegible] $t \mapsto P_t$ [illegible].

[illegible] P_t [illegible] a [illegible] $\int_0^T g^F_{ij}(P_t)\dot P^i_t \dot P^j_t \, dt < \infty$, [illegible] $\Delta(\mathcal{X}) \hookrightarrow \mathcal{P}$, [illegible] $\mathcal{A}[\tilde\gamma]$. $\square$

6.2 [illegible]

[illegible] P^- [illegible] P^+ [illegible]

$$W_2(P^-, P^+) = \inf_{\pi \in \Pi(P^-,P^+)} \left(\int_{\mathcal{X}^2} d(x,y)^2 \, d\pi(x,y) \right)^{1/2},$$

[illegible] π [illegible] P^- [illegible] P^+ [15]. [illegible] $W_2(P^-, P^+)$ [illegible]

[illegible] 6.2. [illegible]

[illegible]

7 [illegible]

7.1 [illegible]

[illegible] $B_t : \Omega \to \mathbb{R}$, [illegible] $\Omega \subset \mathbb{R}^2$ [illegible] $\gamma_{\text{[illegible]}} : [0, \tau] \to \Omega$ [illegible]

$$B_{t+dt}(x,y) = B_t(x,y) + \int_0^{dt} K(x - \gamma_x(t),\, y - \gamma_y(t))\, dt,$$

[illegible] K [illegible]

7.2 [illegible]

[illegible] $\nabla I = (I_x, I_y)$ [illegible] $I : \Omega \to \mathbb{R}$, [illegible] $G_x = S_x * I$, $G_y = S_y * I$, $|\nabla I| = \sqrt{G_x^2 + G_y^2}$. [illegible] $T = (-I_y, I_x)$, [illegible] I, [illegible]

$$\frac{d\gamma_{\text{[illegible]}}}{dt} = \alpha\, T(\gamma_{\text{[illegible]}}(t)) + \beta\, \nabla I(\gamma_{\text{[illegible]}}(t)).$$

[illegible] (α) [illegible] (β) [illegible]

7.3 [illegible]

[illegible]

$$J = \begin{pmatrix} I_x^2 & I_xI_y \\ I_xI_y & I_y^2 \end{pmatrix}$$

[illegible] [13, 9].

[illegible] 7.1. [illegible] J [illegible] J [illegible] Ω. [illegible]

[illegible] [9] [illegible]

8 [illegible]

8.1 [illegible]

[illegible] [8]. [illegible]

[illegible] $\gamma_1, \gamma_2, \ldots, \gamma_n$ [illegible] B, [illegible]

$$B_{t+1} = F(B_t, a_t),$$

[illegible] a_t [illegible] t. [illegible] F: [illegible]

8.2 [illegible]

[illegible]

[illegible]

8.3 [illegible]

[illegible] [10] [illegible]

[illegible] [12] [illegible]

9 [illegible]

9.1 [illegible]

[illegible] $K = (V, E)$ [illegible] $\phi : V \to \mathbb{R}^2$. [illegible] K [illegible]

[illegible] 9.1 [illegible] $w = w_1 \cdots w_n \in \Sigma^*$, [illegible] $P(w) = (\phi(w_1), \phi(w_2), \ldots, \phi(w_n)) \subset \mathbb{R}^2$.

[illegible] 9.2 [illegible] $P : \Sigma^* \to \mathcal{T}$ [illegible] $(\Sigma^*, \cdot)$ [illegible] $(\mathcal{T}, \cdot)$ [illegible]

[illegible] $w = uv$ [illegible] $P(w) = P(u) \cdot P(v)$ [illegible] $P(u)$'[illegible] □

9.2 [illegible]

[illegible] w [illegible]

$$L(w) = \sum_{i=1}^{n-1} \|\phi(w_{i+1}) - \phi(w_i)\| \quad \text{[illegible]},$$

$$C(w) = \sum_{i=2}^{n-1} |\theta_i| \quad \text{[illegible]},$$

$$X(w) = \text{[illegible]},$$

$$\kappa_{\text{[illegible]}}(w) = 1 - \frac{\|\phi(w_n) - \phi(w_1)\|}{L(w)} \quad \text{[illegible]}.$$

[illegible] 9.3 ([illegible]). [illegible] $w, u \in \Sigma^*$ [illegible] $d_F(\widetilde{P}(w), \widetilde{P}(u)) < \varepsilon$, [illegible] $\widetilde{P}$ [illegible] P [illegible] d_F [illegible].

9.3 [illegible]

[illegible]	[illegible]	[illegible]
[illegible] / [illegible]	[illegible]	[illegible] $K_{[illegible]}$
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	$X(w) \geq 1$
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible], [illegible] L
[illegible]	[illegible]	[illegible] L, [illegible] C
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

[illegible] 9.4. [illegible] $\mathcal{L} \subset \Sigma^*$ [illegible] P, [illegible].

[illegible] $w \mapsto$ ([illegible], [illegible], [illegible]). [illegible] ^{2} [illegible] [11], [illegible].

10 [illegible]

[illegible].

[illegible] 10.1 ([illegible]). [illegible] $\gamma_1, \gamma_2 \in \mathcal{T}$ [illegible] $\mathbb{R}^2 \setminus \mathcal{O}$, [illegible] $\mathcal{O}$ [illegible] ([illegible], [illegible]).

[illegible]:

$$X(\gamma) = \text{[illegible]},$$
$$\omega(\gamma) = \text{[illegible]},$$
$$\kappa_{[illegible]}(\gamma) = \text{[illegible]}.$$

[illegible]:

[illegible]	[illegible]	[illegible]
[illegible]	$\omega = 1$	[illegible]
[illegible]	$\omega > 1$	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	$X \geq 1$	[illegible]

[illegible]

11 [illegible]

11.1 [illegible]

[illegible] $p_1, p_2, \ldots, p_n \in \mathcal{I}$, [illegible] $\gamma_{\text{[illegible]}}(t)$ [illegible].

[illegible] [16] [illegible] $\sigma : \mathcal{I} \to \mathbb{R}_{\geq 0}$, [illegible] 4.1.

11.2 [illegible]

[illegible]:

[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]: $\gamma(t)$ [illegible].		

[illegible]

11.3 [illegible]

[illegible] [6]. [illegible] $m_{t+1} = F(m_t)$, [illegible] [3] [illegible]:

$$a_1 \to a_2 \to \cdots \to a_n \to a_1,$$

[illegible]

[illegible]:

> [illegible]

12 [illegible]

12.1 [illegible]

[illegible] $\gamma_{[illegible]} : [0, T] \to \Omega$ [illegible] $(\gamma) = \{\gamma(t) : t \in [0, T]\}$. [illegible] γ [illegible] $\mathcal{S}$, [illegible]

[illegible]:

[illegible]	[illegible]	[illegible]
[illegible]	[illegible] K	[illegible]
[illegible]	[illegible] B_t ([illegible])	[illegible]
[illegible]	[illegible] σ	[illegible]
[illegible]	[illegible] $\mathcal{S}$	[illegible]
[illegible]: $\gamma(t)$ [illegible] F.		

12.2 [illegible]

[illegible]

[illegible]

12.3 [illegible]

[illegible]

13 [illegible]

[illegible]

[illegible] 13.1 ([illegible]). [illegible] $\tilde{\gamma}(t) = \Pi(\gamma(t))$ [illegible]. [illegible] t [illegible]

$$\kappa(t) = \frac{\|\dot{\tilde{\gamma}}(t) \times \ddot{\tilde{\gamma}}(t)\|}{\|\dot{\tilde{\gamma}}(t)\|^3}.$$

[illegible]

[illegible]	[illegible]
[illegible]	$\kappa(t)$ [illegible]
[illegible]	[illegible] $\kappa(t)$
[illegible]	[illegible]
[illegible]	[illegible] κ [illegible]
[illegible]	κ [illegible] V-[illegible]

[illegible] 13.2. [illegible] $\kappa(t)$. [illegible] κ [illegible] κ [illegible] $\mathcal{P}$ [illegible] Π.

[illegible] $\int_0^T \kappa(t)\,dt$ [illegible] $\mathcal{P}$ [illegible] $\mathcal{S}$ [illegible]

14 [illegible]

[illegible]

[illegible]: [illegible] $G = (R, E)$ [illegible] R_i [illegible] E [illegible].

[illegible]: [illegible] $I(x, y)$. [illegible]

[illegible]:

$$L_o(x,\omega) = L_e(x,\omega) + \int_{\Omega} f_r(x,\omega',\omega)\, L_i(x,\omega')\, (\omega' \cdot n)\, d\omega'.$$

[illegible]

[illegible]

15 [illegible]

[illegible]

[illegible] $\tilde{\gamma}_1, \ldots, \tilde{\gamma}_N$ ([illegible]), [illegible]:

([illegible]) [illegible] g_θ [illegible] $\mathcal{P}$ [illegible];

([illegible]) [illegible] V_ϕ [illegible];

([illegible]) [illegible] Π_ψ [illegible].

[illegible] $\gamma(t)$ [illegible] $\tilde{\gamma}(t)$ [illegible] $\mathcal{J}[\tilde{\gamma}]$ [illegible].

[illegible] 15.1. [illegible]

[illegible]

16 [illegible]

[illegible]

[illegible] (4.1) [illegible] $\mathcal{P}$, [illegible] V. [illegible] (6.1) [illegible] $\mathcal{P}$: [illegible] 180 [illegible] (5.1); [illegible]

[illegible] (9.2) [illegible]

[illegible]

[illegible]

[illegible]

[1] [illegible], 2000.

[2] [illegible], 1954.

[3] [illegible], 2007.

[4] [illegible], 2013.

[5] [illegible], 1982.

[6] [illegible], 2016.

[7] [illegible], 1992.

[8] [illegible], 1977.

[9] [illegible] 1998. [illegible], 1998, [illegible] 453-460.

[10] [illegible], 1969.

[11] [illegible] ^{2}: [illegible] 2004. [illegible], 2004, [illegible] 43-52.

[12] [illegible], 1947.

[13] [illegible] 12.7 (1990): 629-639.

[14] [illegible], 2019.

[15] [illegible], 2009.

[16] [illegible], 1967.