

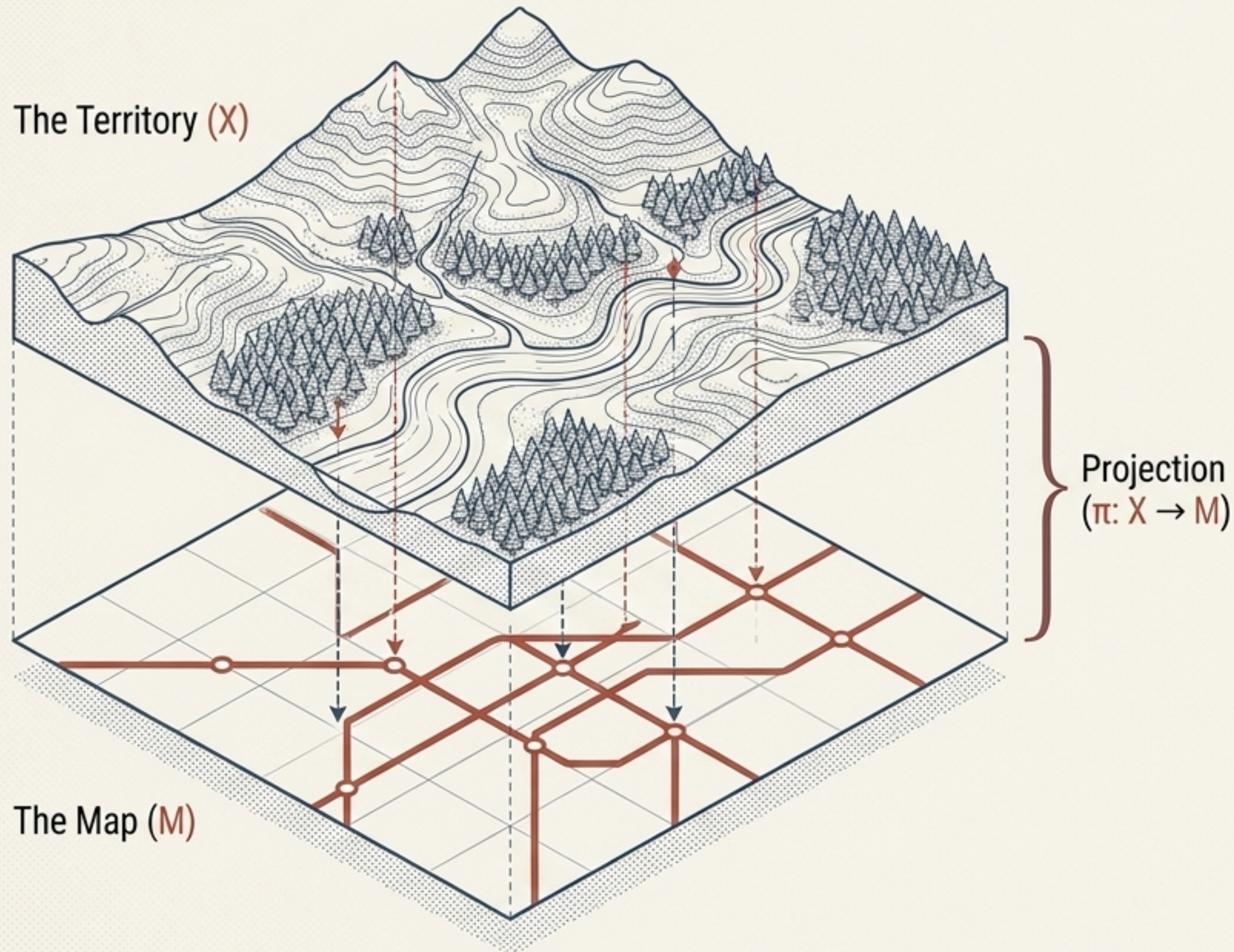
# HOLONOMIC SPACE

THE GEOMETRY OF ADMISSIBILITY



FIGURE 1.1: ADMISSIBLE PROJECTION  
DIMENSIONAL REDUCTION  
GEOMETRIC CONSTRAINT MAPPING  
TERRACOTTA REGION INDICATES ADMISSIBLE STATE

# Observation is not access. Observation is projection.



Every useful representation discards information. A map is useful precisely because it has a nontrivial kernel—the set of features it fails to distinguish.

## The Math of Forgetting

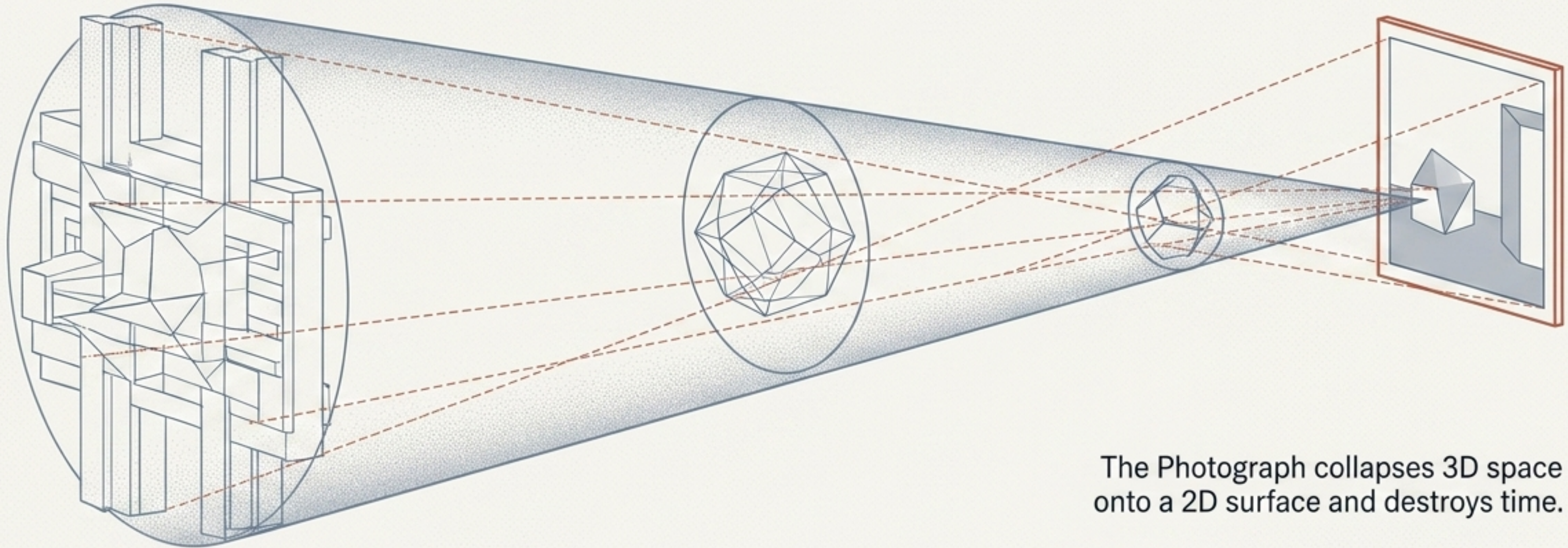
- **Projection ( $\pi$ )**: Preserves structure, discards the rest.
- **Kernel ( $\ker(\pi)$ )**: The measure of what is lost.
- **Quotient Space ( $X/\sim_\pi$ )**: The territory as the map represents it.

Preserved	Lost (The Kernel)
<ul style="list-style-type: none"><li>• Connectivity</li><li>• Relative position</li><li>• Route structure</li></ul>	<ul style="list-style-type: none"><li>• Elevation</li><li>• Physical texture</li><li>• Temporal history</li></ul>
<b>Reconstruction possible:</b> Partial. Combining multiple map types	

# The Inverse Problem of Observation

## The Admissibility Class ( $[I]_p$ )

A photograph does not reveal a unique world. It defines an equivalence class of possible worlds consistent with the observed image:  $P(W_1) = P(W_2) = I$ .

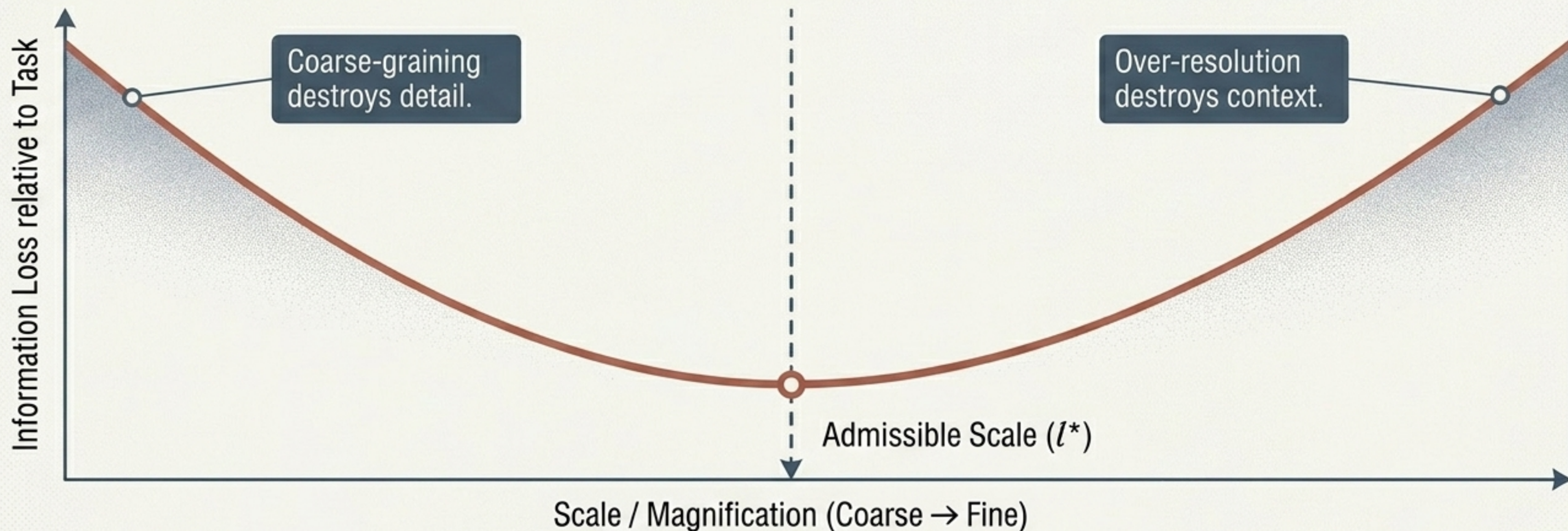


The Photograph collapses 3D space onto a 2D surface and destroys time.

## The Goal:

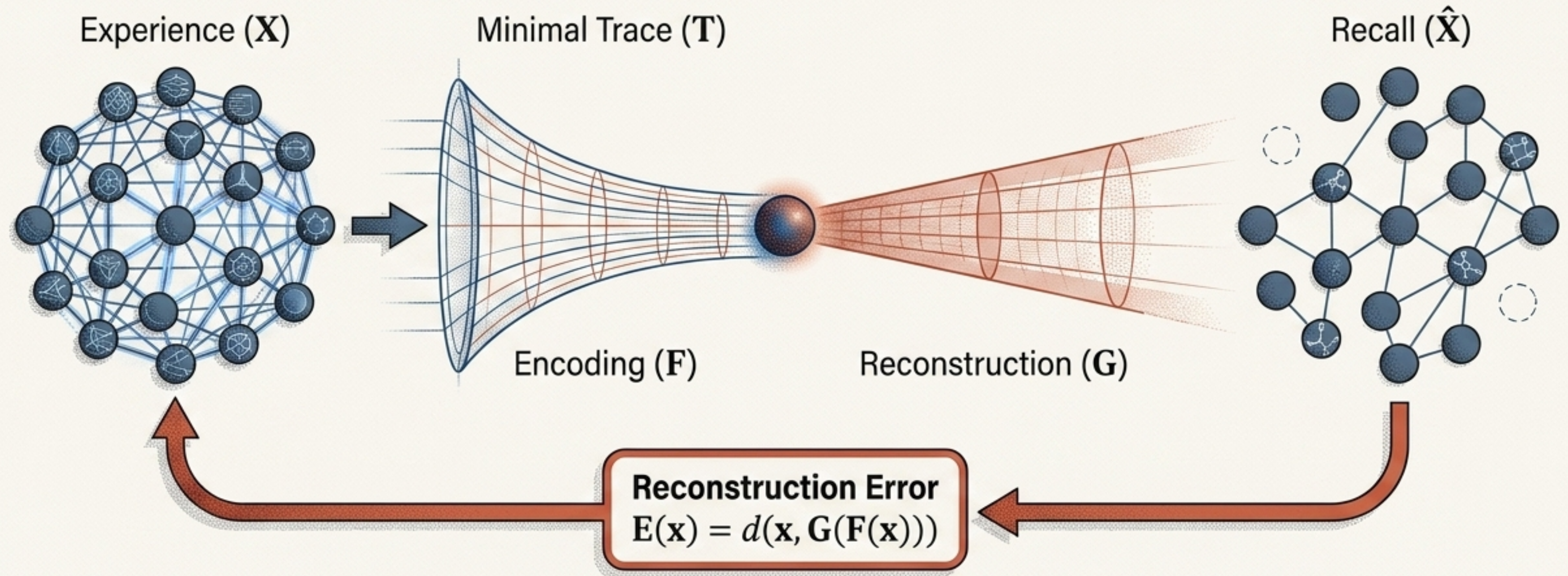
Reconstruction requires additional conditions to choose among the admissible worlds.

# Bidirectional Information Loss



The Microscope Paradox	Task Relevance
At high magnification, context disappears. Observation gains detail while losing organization.	The right level of observation is determined by the reconstruction task, not by the system alone. Useful information $\neq$ maximum information.

# The Architecture of Memory is Reconstructive



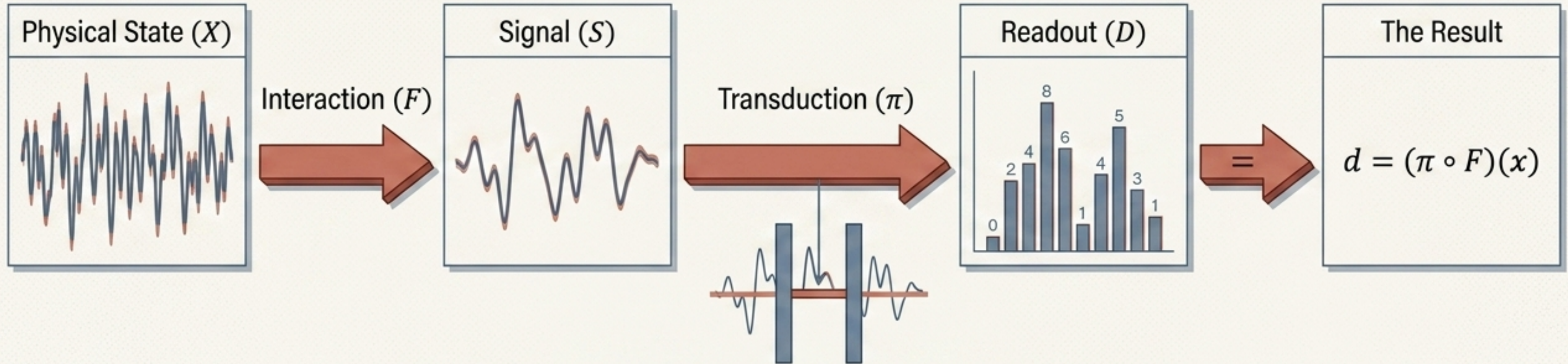
## The Operator

Memory is the composition  $\mathbf{R} = \mathbf{G} \circ \mathbf{F}$ .  
It is almost never exact.

## Semantic Admissibility

Memory intentionally loses sensory precision to preserve narrative and emotional salience. Reconstruction need not be exact to be admissible.

# The Physical Detector is a Two-Stage Projection

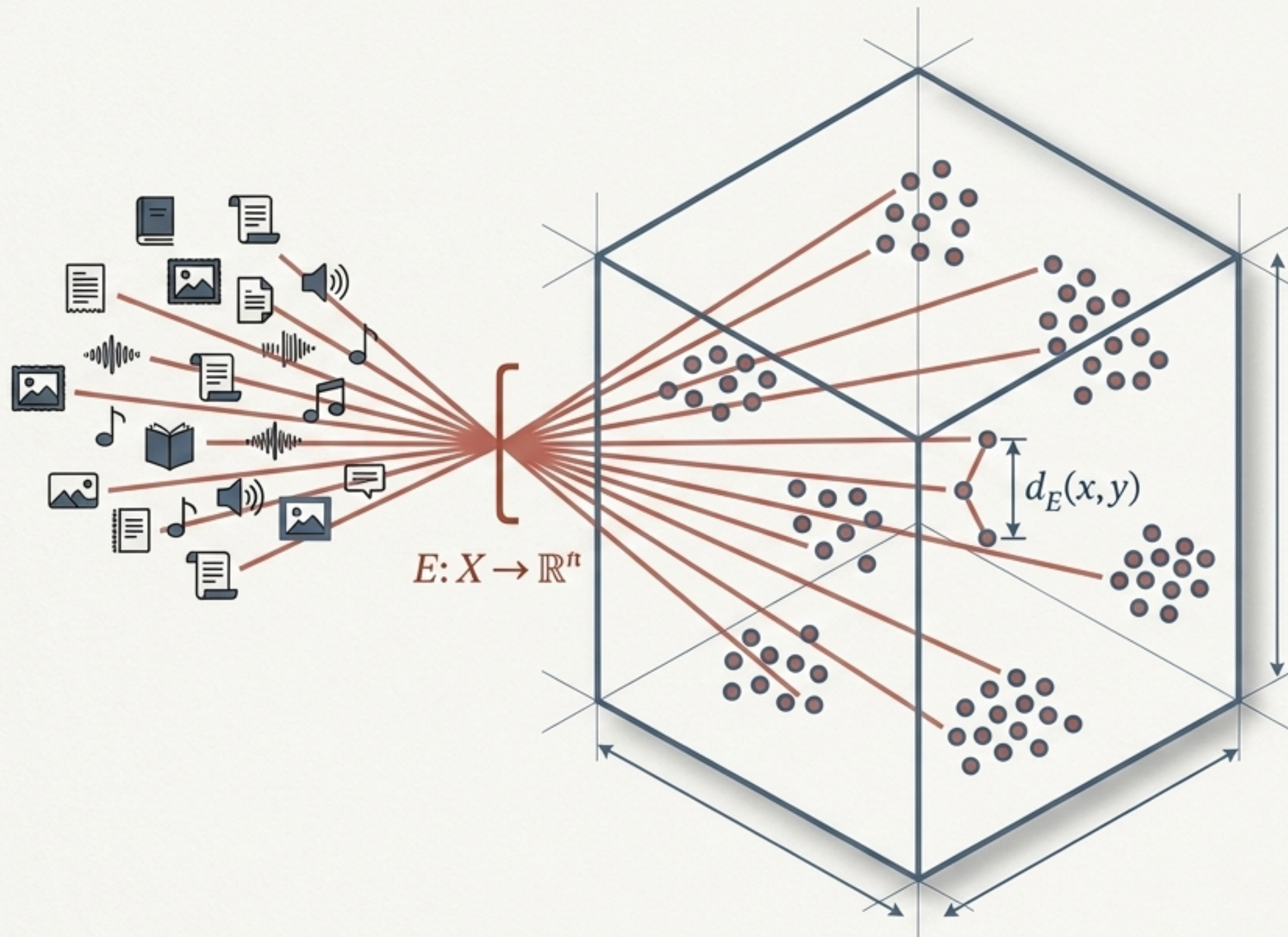


Preserved	Lost
Detector-admissible signal	States outside bandwidth
Equivalence class $[x]_D$	Sub-noise-floor variations
Readout $d \in D$	Non-commuting observables

## Detector Equivalence

Two physically distinct states that produce the same readout are identical to the observer ( $x_1 \sim_D x_2$ ).  
 Measurement is **not access**; it is an **interface** that **hides sub-noise** variations.

# Embeddings as Controlled Compression



## The Mechanism

An embedding replaces a complex object with a vector. The compression is admissible when  $d_X(x, y) \approx d_E(x, y)$  for distinctions relevant to the task.

## The Tradeoff

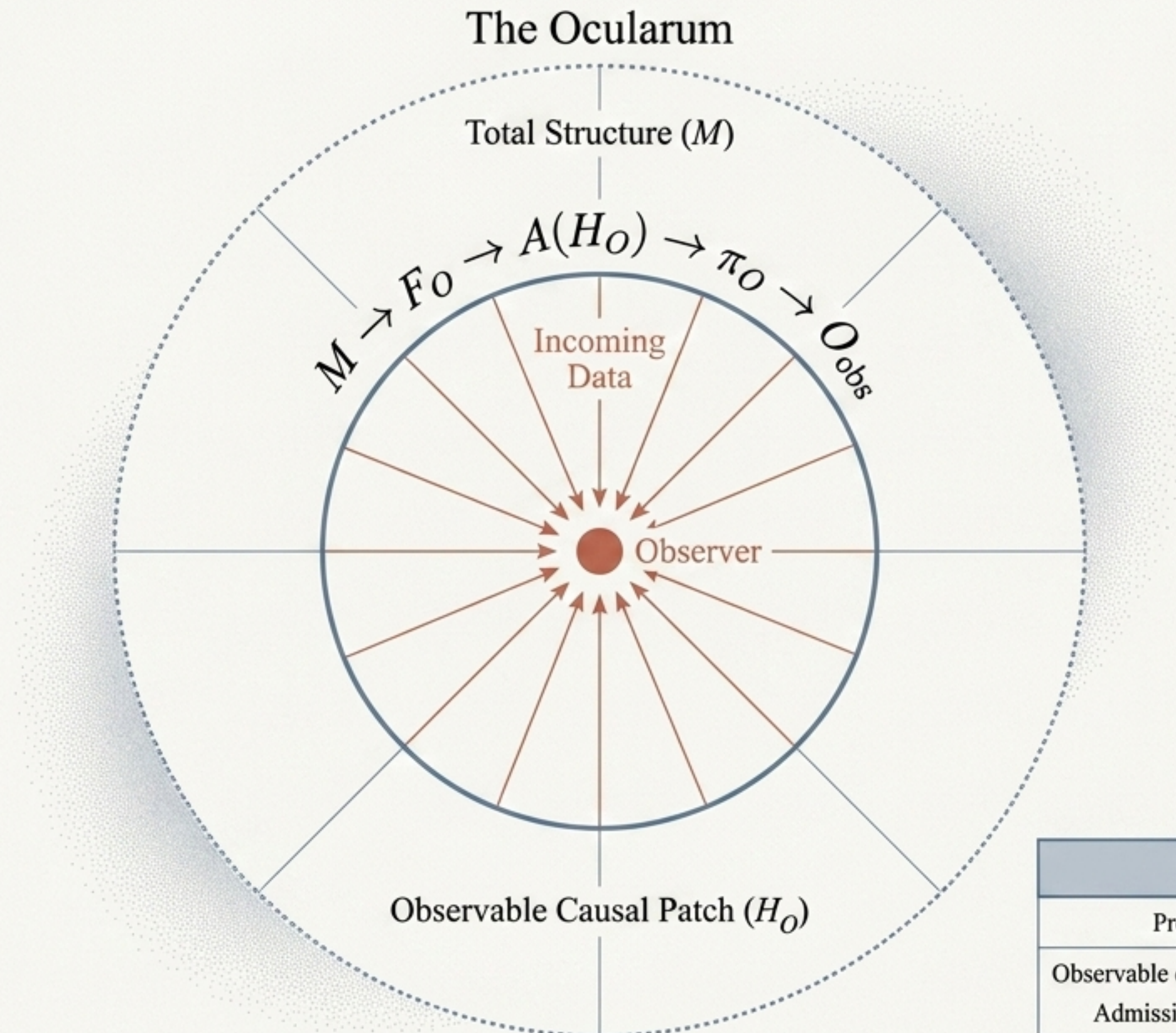
Embeddings purposely discard task-irrelevant variation to preserve structural similarity.

Preserved	Lost
Task-relevant similarity	Exact identity
Approximate metric structure	Irrelevant variation

# Cosmic Horizons Enforce the Ultimate Projection

## The Ocularum

The horizon is not a failure of observation; it is the physical instantiation of the kernel imposed by the finite speed of signals.



## Horizon-Relative Reconstruction

Different observers have different horizons. Local reconstructions can only become global when their overlapping restrictions agree.

Causal Horizon Projection	
Preserved	Lost
Observable causal patch ( $H_O$ ) Admissible local data	Beyond-horizon structure Trans-horizon correlations

# The 7 Forms of Forgetting

System	Projection	Preserved	Lost (The Kernel)
Map	Road Network	Connectivity	Elevation & Texture
Photograph	Image Plane	2D Geometry	Depth & Time
Microscope	Scale Window	Local Detail	Global Context
Memory	Trace Encoding	Narrative / Semantics	Sensory Precision
Detector	Readout Function	Above-Threshold Signal	Sub-Noise States
Embedding	Feature Vectors	Task-Relevant Metric	Exact Identity
Horizon	Causal Boundary	Local Patch ( $H_O$ )	Trans-Horizon Correlations

**Synthesis Note:** Every row studies the exact same mathematical object:  $\ker(\pi \circ F)$ .  
Every act of observation is a study of what falls into this kernel.

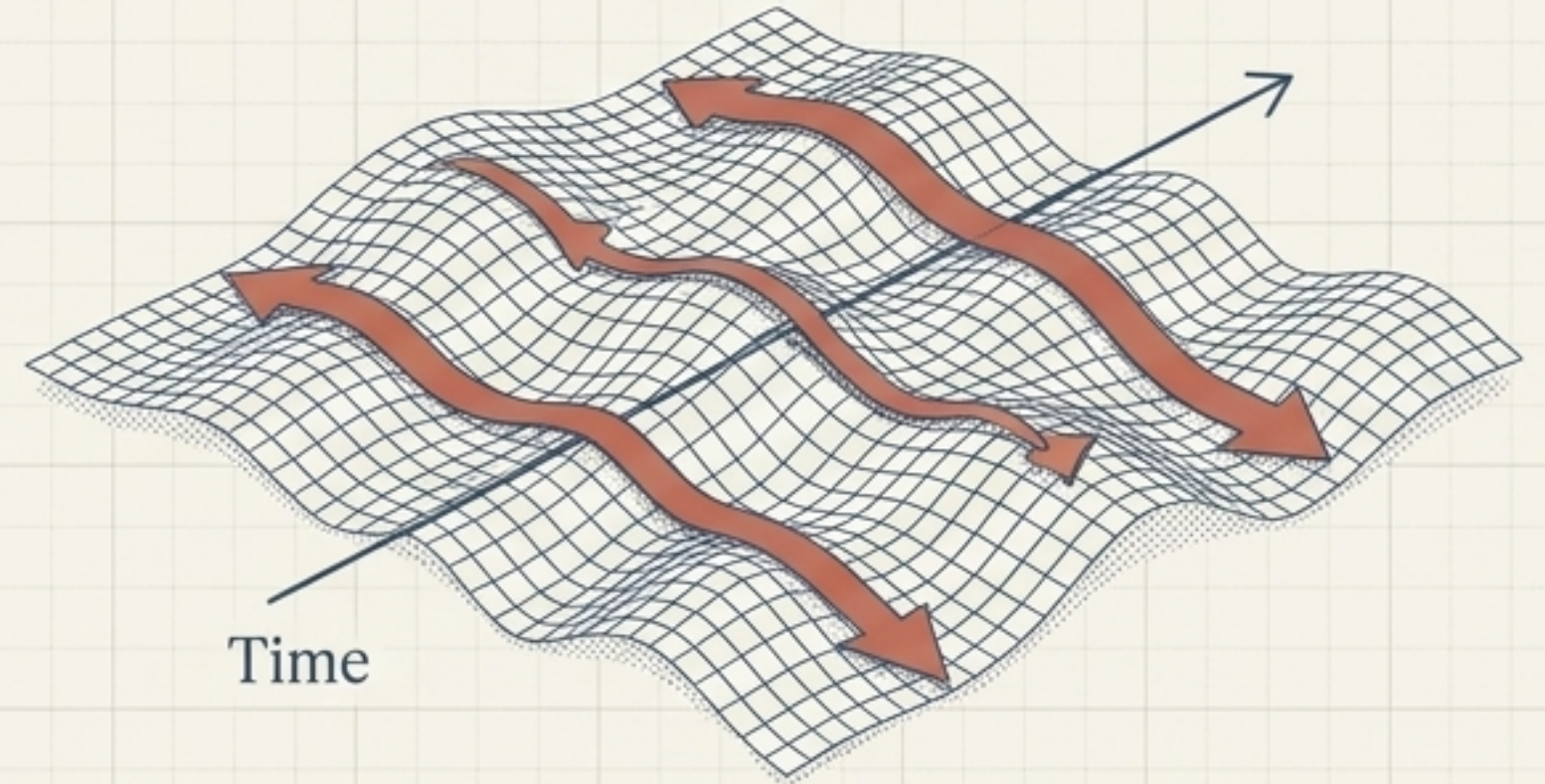
# From Static States to Dynamic Fields

State



A snapshot at a moment. A state assigns a single value to a system.

Field

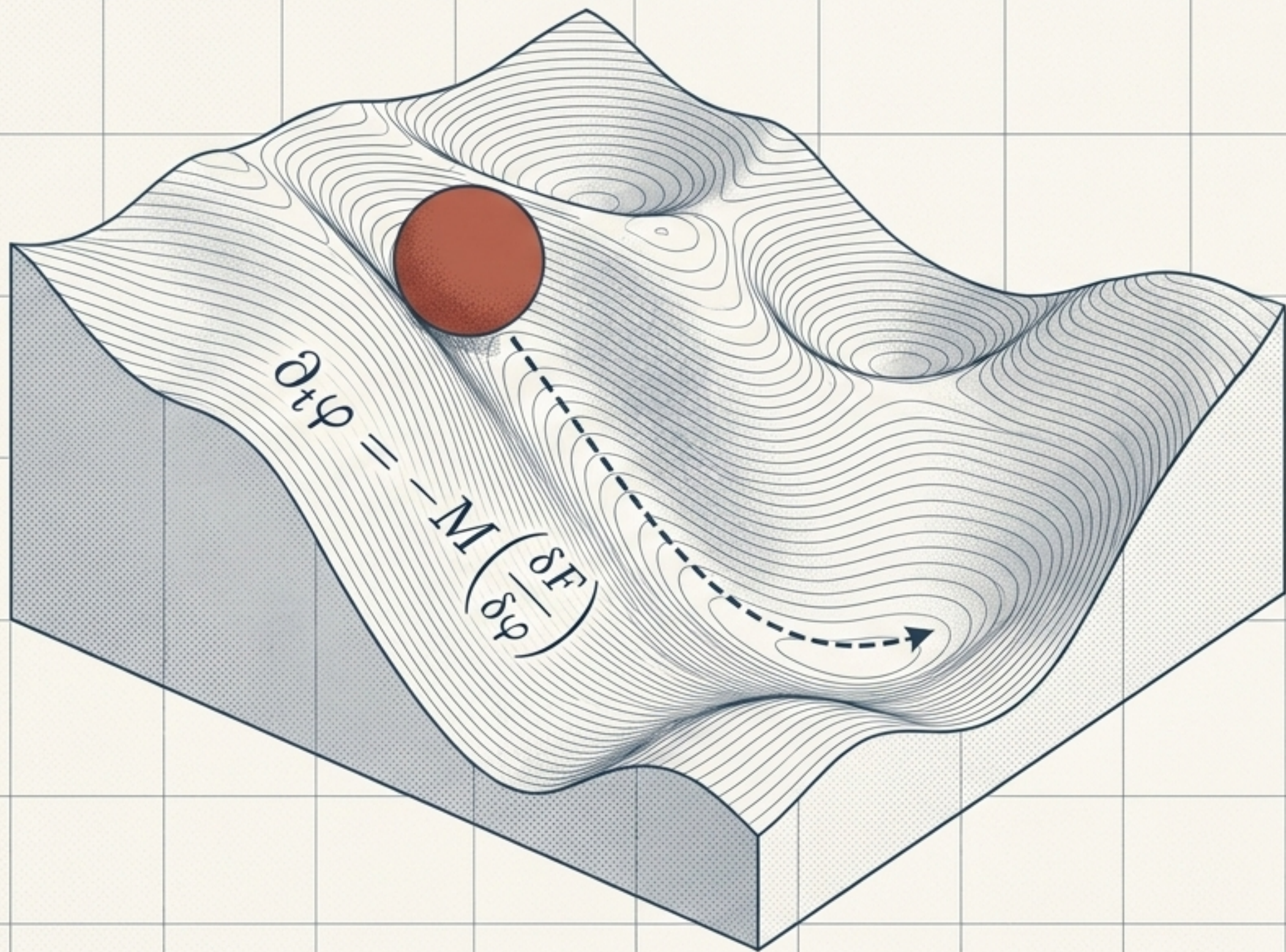


A continuous evolution. A field ( $\phi$ ) assigns a value to every point of a domain at every moment.

## Conservation

The continuity equation ( $\partial_t \phi + \nabla \cdot \mathbf{J} = \sigma$ ) dictates how these fields evolve. If  $\sigma = 0$ , the structure is strictly conserved. If observation is static projection, the underlying reality moves via these dynamical fields.

# Free-Energy Descent: Why Systems Relax



- **Gradient Flow**

Systems naturally descend a free-energy functional ( $F[\varphi]$ ). The dynamics constantly seek a lower energy state.

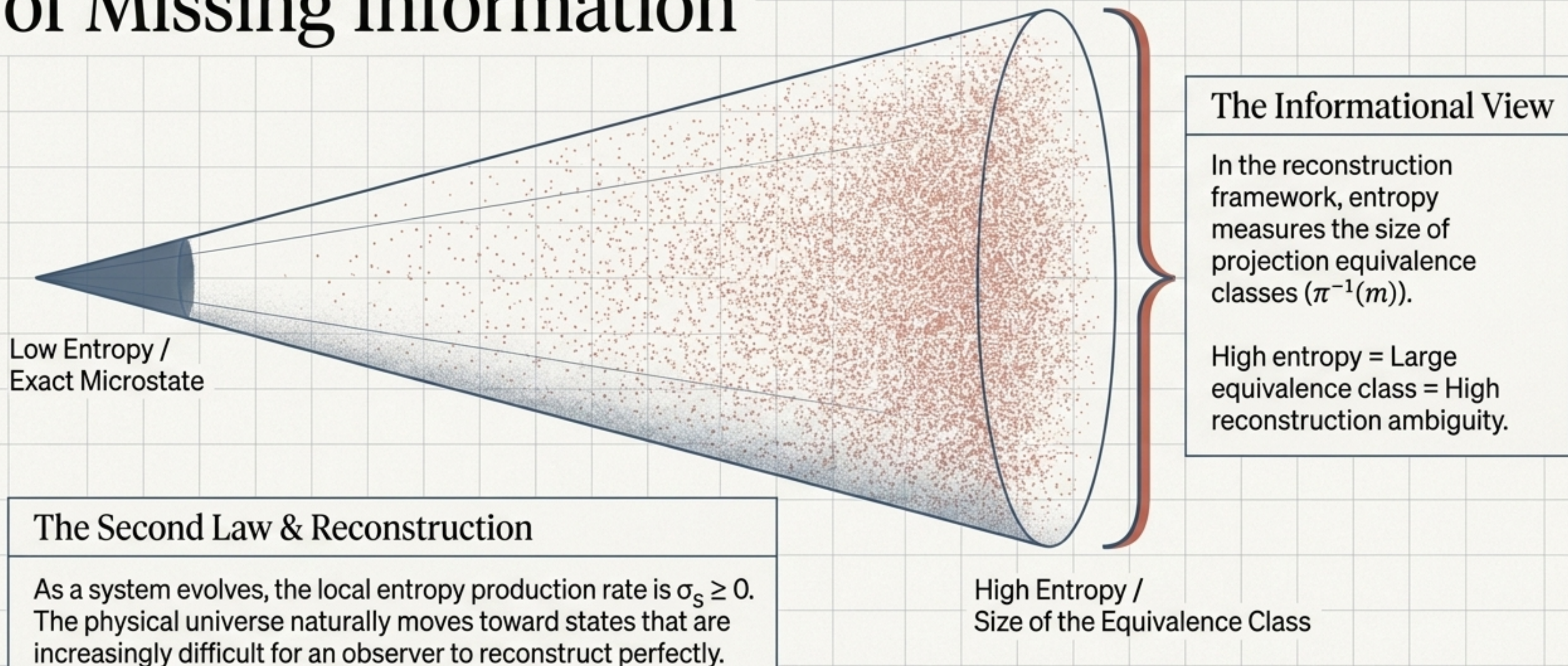
- **The Math of Relaxation**

Along these gradient flows, energy strictly dissipates:  $dF/dt \leq 0$ .

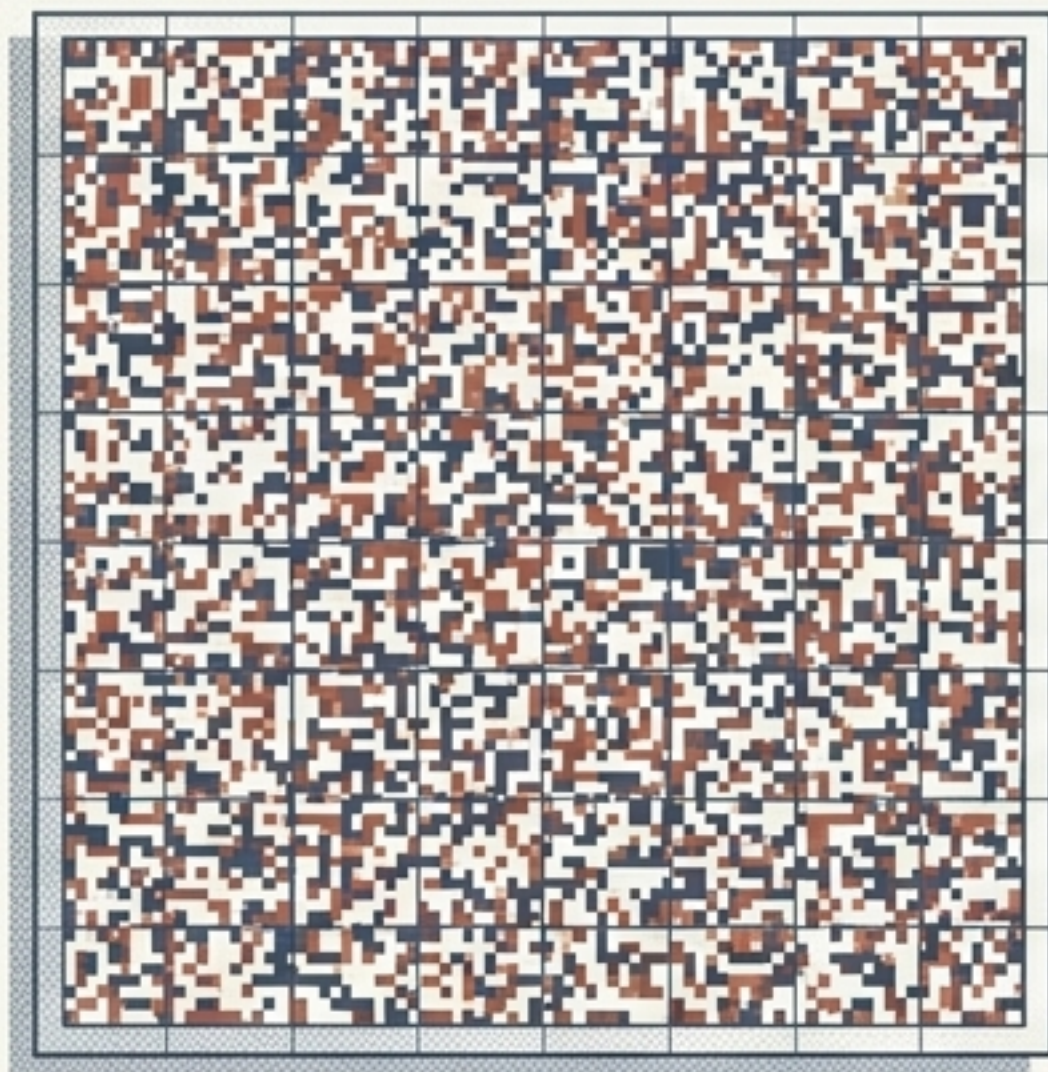
- **Metastability**

Systems do not always reach global equilibrium quickly; energy barriers trap them in slow-relaxing metastable states.

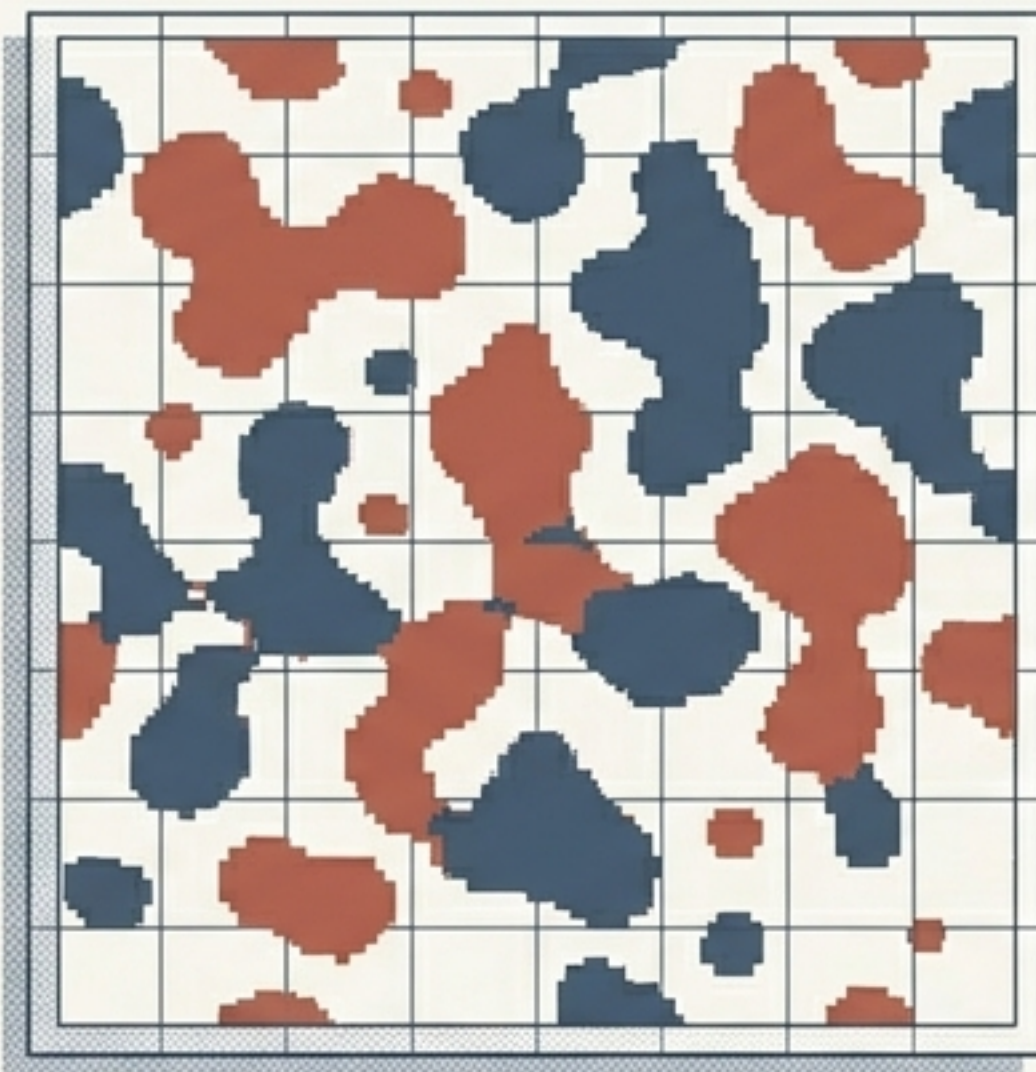
# Entropy is the Measure of Missing Information



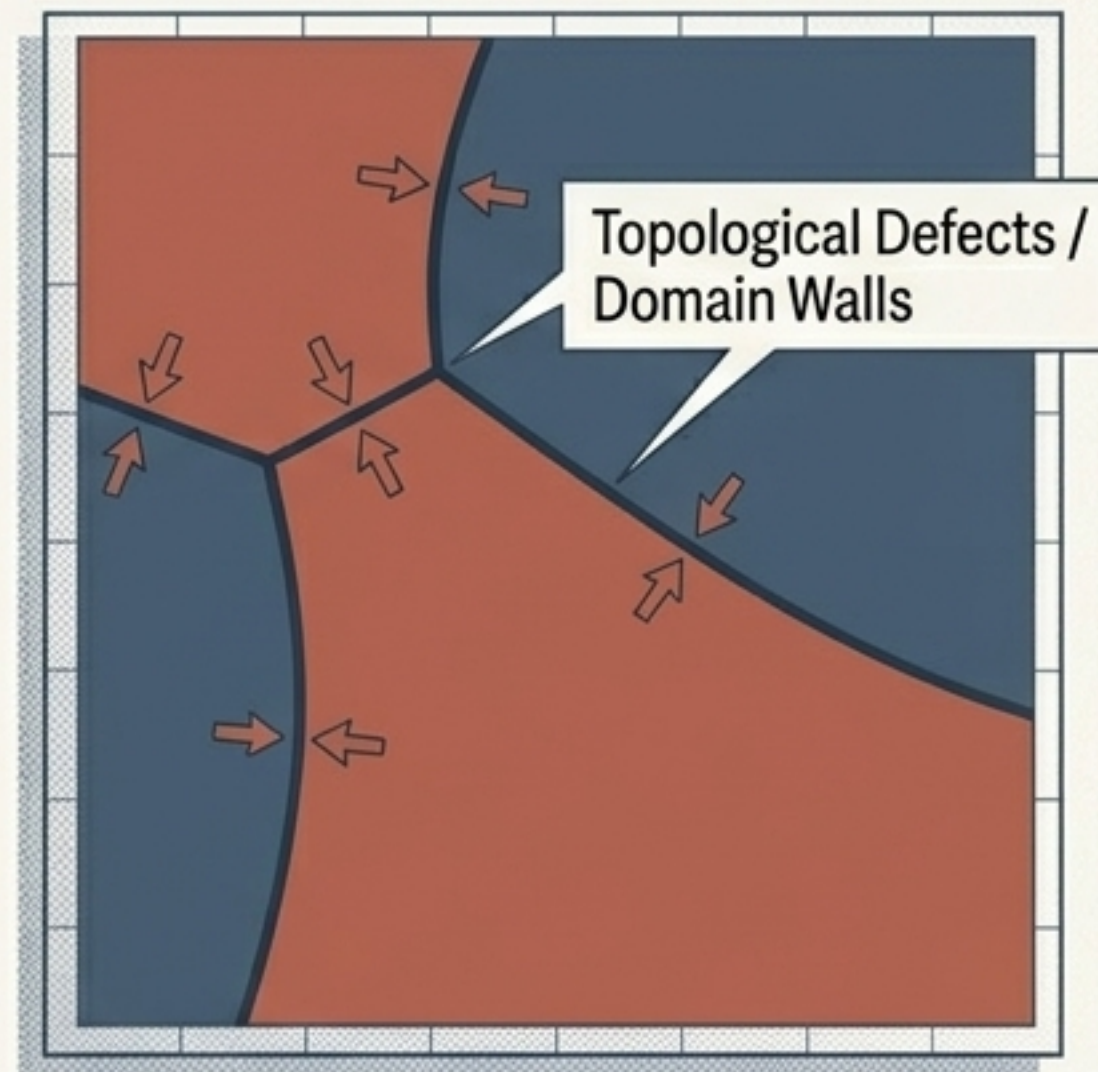
# Coarsening and Domain Formation



**Time 1:** Random Scatter



**Time 2:** Emerging Domains



**Time 3:** Coarsened State

## Symmetry Breaking

Systems with symmetric rules naturally evolve into asymmetric, structured states (e.g., the Ising Model ground states). Continuous fields break symmetry to form structured reality.

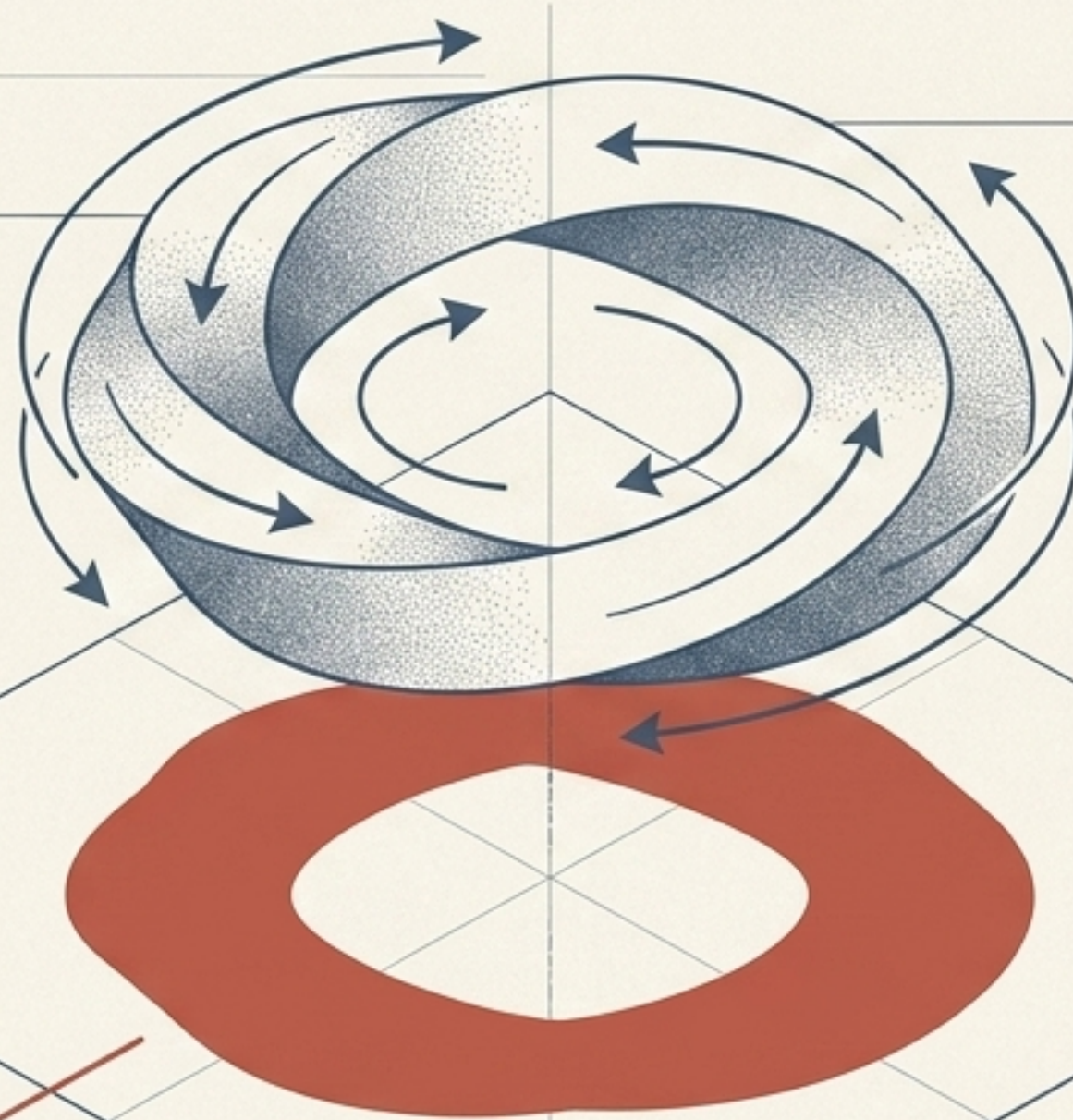
## Defects & Walls

As systems relax, they form distinct phases separated by topological defects and interfaces. The universe we observe is built upon these structural boundaries.

# The Holonomic Perspective

## 1 The Invariant Structure

Two states that differ only by an observation-preserving rearrangement are “holonomically close”.



STATIC / UNCHANGING

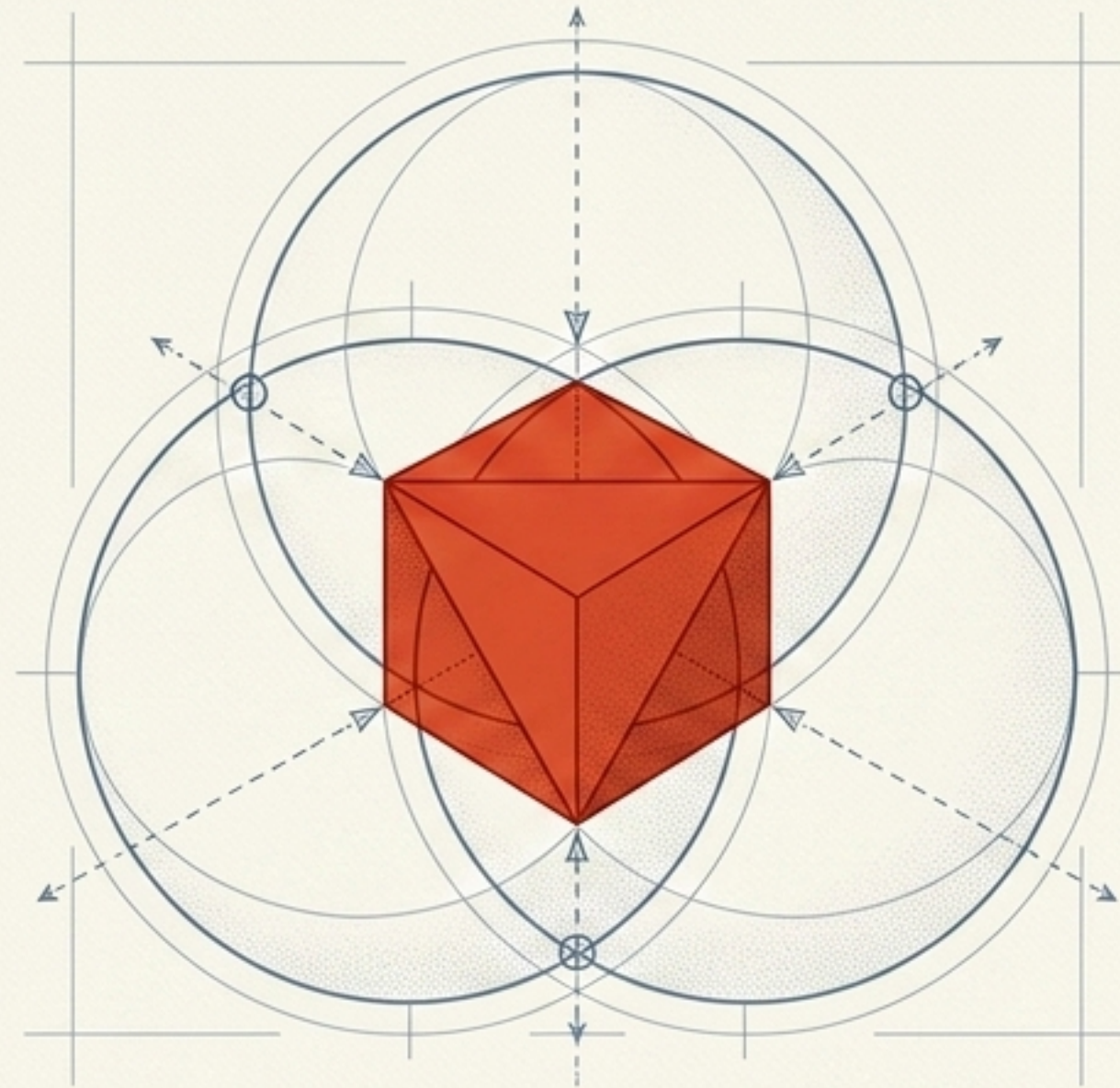
## 2 The Definition

The holonomy of a representation is the group of transformations that leave all observations unchanged.

## 3 The Takeaway

A large holonomy group means a severely underdetermined reality. Every act of observation is a study of this hidden invisible group.

# What Survives the Projection?



## The Synthesis

No single projection is complete. Complete information about a multiscale system is never accessible from any single scale.

## The Final Statement

Reality is reconstructed only when independent, incomplete, and overlapping observations agree on their boundaries.

**Observation is not access. Observation is admissible projection.**